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Sustainability through the Dynamics of Strategic Dilemmas

in the light of the coherence and visual form of the Mandelbrot set

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Introduction

This exploration endeavours to frame the concerns of two earlier associated papers in terms of the insights of dissipative systems and the Mandelbrot set (hereafter referred to as the M-set). The first paper (*Being Positive Avoiding Negativity: management challenge -- positive vs negative*, 2005) was concerned with the appropriate handling of "positive" and "negative" from a strategic perspective and as a judgement on the relevance of feedback. The second paper (*Cardioid Attractor Fundamental to Sustainability: 8 transactional games forming the heart of sustainable relationship*, 2005) sought to demonstrate the importance of a set of 8 patterns of interaction in defining a coherent pattern within any system of relationships -- highlighting the role of the cardioid in that pattern, following the work of Edward Haskell (*Generalization of the structure of Mendeleev's periodic table*, 1972).

Given the prime importance of the cardioid in representation of the M-set, the argument that follows is initially descriptive in clarifying an explanation of dissipative systems in terms relevant to the strategic challenge of interpersonal and intergroup relationships of those papers. There is an extensive body of literature of varying levels of technicality that explains the M-set and related issues. The concern here is the potential relevance of those insights to contexts which have not as yet been a prime concern. Reference is therefore only made to the technicalities where they suggest insights of relevance to the strategic challenge that might otherwise go unrecognized.

The purpose here is to explore imaginative leads and framings -- possibly primarily metaphorical -- that may be a guide to more concrete interpretations. In that respect the isomorphism with Haskell's cardioid may bear a less than rigorous relationship to that discussed here. [This question is currently under investigation by Kent Palmer]

This approach is consistent with that advocated by Ralph H. Abraham (*Human Fractals: the arabesques in our mind*. [text]

> To many pure mathematicians, especially those to whom fractal geometry itself is not mathematics but heresy, these applications of new mathematical ideas to anthropology will seem anathema, vulgarization, fractal evil itself. In my perspective, however, they are the first steps of a major paradigm shift, a critical renewal arriving in timely fashion, of an entire area of cultural studies. Let us encourage this trend, which could be advanced spectacularly by a new generation of students well-trained in mathematics as well as in a social or human science.

Relationships between "incommensurables"

The stimulus for this discussion came from the dynamic between "positive" and "negative" and the developing widespread movement in favour of "positive" thinking and in opposition to "negativity" (see J K Norem and E C Chang. *The positive psychology of negative thinking,* 2002). Negativity may even be condemned as "bad", even sinful. The earlier paper (*Being Positive Avoiding Negativity: management challenge -- positive vs negative*, 2005) endeavoured to show that in both strategic and practical contexts there was clearly a need for both modes -- if only in the cybernetic terms of positive and negative feedback required for systems control.

This polarity can however be seen as merely a rather "pure" and obvious example of many other forms of "incommensurables" between which an operational relationship has somehow to be ensured in practice and in daily life. Other examples range from economy vs environment, through peace vs conflict, female vs male, and including abstract vs concrete. They also include the ordering of the many interpersonal and strategic dilemmas faced in society (cf *Value polarities*).

The (re)discoverer of the M-set, Benoit Mandelbrot, recognized repeating patterns on all scales in numerous phenomena -- cotton prices, clouds, and coastlines. Whilst his research showed that the changes were unpredictable -- namely random -- the sequence of the changes was independent of scale. This means that the variation in each case is no more of a period of centuries, than over decades or years -- so-called scale invariance. This applied equally to shapes such as clouds, trees or earthquakes and resulted in the formulation of the concept of fractals as a measurement of roughness or irregularity that demonstrated self-similarity on all scales. In natural systems, the structure of the whole system is often reflected in every part of it -- especially when similar forces act at many levels of scale. Natural forms tend to reveal transformed copies of the whole in every part. Fractals are therefore widely found in nature. So in many ways fractal structures are potentially more relevant than more conventional idealized scientific concepts.

It was later established that chaos is a feature of many nonlinear dynamical systems. Their deeply cyclic structure does not however imply that the cycles repeat exactly. Whilst the amount of the variation within such cycles is constant, the variations with variations makes them inherently unpredictable at every level of scale. These nested cycles may be simulated by iterative procedures.

It is appropriate that relating the apparently incommensurable should be achieved through "fractal" techniques in contrast with the techniques of algebra" that have proved so appropriate to relating the commensurable. *Algebra* derives from the notion of "binding together", in contrast with *Fractal* that derives from "breaking into irregular fragments".

The term "Mandelbrot set" is used to refer both to a **general class** of fractal sets and to the **particular instance** of such a set derived from the quadratic recurrence equation $z_{n+1}= z_n^2 + c$. In general, a Mandelbrot set marks the set of points in the complex plane such that the corresponding Julia set (J-set) is connected and not computable. It is the particular instance that is discussed in what follows (and referred to as the M-set). [more]

Although the M-set is indeed perhaps the best known fractal, there are many other types. In its more general form, the power in the quadratic recurrence equation may be increased from the standard "squared" form (with one symmetry axis) to the cubic form (with two such axes), the quadric (with three) -- with any number of "poles" by suitable choice of exponent. These raise fruitful questions which are not however addressed in what follows. **It is the particular instance that is the focus here because it is the simplest that gives rise to an object of such great complexity**.

The M-set fractal corresponds to the simplest nonlinear function -- but is also as complicated as a fractal can get. It distinguishes the simplest boundary between chaos and order. It is recognized as the simplest non-trivial example of a holomorphic parameter space. Given the significance of pi in defining a circle as a simpler object, the generation of the M-set by iteration may be compared to the iterative calculation of pi (cf Alex Lopez-Ortiz. *How to compute digits of pi ?*; Dave Boll. *Pi and the Mandelbrot set*) [more]. One method, with a striking formal resemblance to that required for the M-set, is: $z_{n+1}= z_n + \sin(z_n)$, especially if initialized to $z_n = 3$ [more].

In the search for solutions to complex equations, experiments with iterations by computer have highlighted intricate global properties related to nonconvergence and the stability of convergence. The behavior of quadratic functions, as the simplest of all nonlinear mappings, combines ease of calculation with sufficient generality to illustrate most of the abstract properties of iterations. Just as using complex variables often clarifies the properties of functions of a real variable, studying complex iterations can be expected to generalize and illuminate real nonlinear mappings as well.

The complex space in which the conceptual and value concerns are significant has been usefully described by Vladimir Dimitrov (*Glimpses at Mathematics and Physics of Social Complexity*)

> We use the concept of strange attractor to describe emergence of meanings in the mental space of an individual or a group (organization). The phase space where meaning emerges is the 'multi-dimensional' mental space of an individual or a 'swarm' of

> individuals - a non-material ("transcendental" in Kantian term) space energized by continuously generated thoughts and feelings. Meaning has fractal structure - once a certain dynamical sign makes sense to somebody, s/he can 'zoom' deeper and deeper into the meaning of this sign. Although each level ('scale') of meaning-exploration may differ from any other level, there is similarity between the levels, as they all relate to the dynamics of one and the same sign interpreted by one and the same individual. The strange attractor of meanings can exhort human actions. Although, the actions may appear randomly skipping around, they relate to the attractors of meanings, which propel them. If there is no attractors of meanings behind one's actions, the actions are simply meaningless; they are running at physical level only. The lack of intelligent support, be it mental, emotional or spiritual, is incompatible with one's growth as a holistic individuality.

Thesis

Dissipative systems, and the M-set, offer a language through which to explore and identify viable patterns of sustainable relationship between essentially incompatible modes of behaviour or anti-thetical modes of thinking. It is these which are typically fundamental to the strategic dilemmas in pyscho-social systems -- whether intrapsychic, interpersonal or intergroup. It is the continuing search for the resolution of these dilemmas that characterizes the dynamic of such systems. Typically however the resolution is of four types:

- stable (perhaps exemplified by the "constancy of the heart"),
- unstable (exemplified by nonviable projects of every kind),
- a form of periodic stability (exemplified by cyclic patterns of interaction),
- chaotic variety (as with the many weird and wonderful, "improbable" relations between real people).

This approach offers a pattern language to explore the complexities of the periodic resolution to strategic dilemmas -- the space of not-this, not-that (the *neti neti* of Sanskrit). The emergent patterns there are those which characterize a multitude of dynamically stable experiential resolutions of strategic dilemmas. These dynamic resolutions can be depicted (through the M-set) as characteristic patterns of great variety. The set of all such patterns (the M-set as a whole) is of a coherent form that is reflected in many ways (isomorphically) in their detail ("when two or three are gathered together in my name, there am I").

The pattern language is of significance because it enables agonizing psycho-social dilemmas, such as employment vs unemployment (environment vs employment, "affairs of the heart", etc) to be addressed in new ways -- unconstrained by the conventional binary logic and the logically excluded middle. In effect it is a language for exploring the viable patterns of the "middle way". It gives form, space and locus to particular dynamic resolutions of strategic dilemmas. The viability of these patterns, and the challenge to their comprehension, arises, however, from their characteristic dynamic -- in contrast with the stability normally sought in non-dynamic resolutions to such dilemmas.

The characteristic form taken by the set of patterns as a whole is also of particular significance because of the way in which its aesthetic potential can be used to mnemonic advantage. As with delightful melodies, it offers memorable features that reinforce the coherence of the pattern in practice. In addition, as depicted, these emergent patterns are in many respects intuitively recognizable and familiar rather than being alien to the human psyche. It is in this respect that they may echo -- and be echoed by -- cultural symbols of great archetypal significance. In these senses, "M-set" might more usefully be understood as the "Memorable set" or the "Mnemonic set". But the challenge to comprehension -- through "iterative "*re-membering*" of it as a gestalt -- might then be understood in the light of Antonio de Nicolas' poetic title (*Remembering the God to Come: a book of poems*, 1988).

The particular concern here is with how the geometry of the dynamic pattern is sensed experientially -- how the "geometry is felt" (using the "computing" and "graphics" capability of the brain) -- rather than with the technicalities that are important to its rigorous description. The challenge is to ensure that the latter serves in improving the quality, richness and viability of experience in engaging with strategic dilemmas. As a mathematician, Ron Atkin (*Multidimensional Man; can man live in 3-dimensional space?*, 1981) has addressed how geometry may be "felt" in a communication space (see *Social organization determined by incommunicability of insights*). .

The Mandelbrot set is not an invention of the human mind; it was a discovery. *Roger Penrose*

Methodological approach

The following points endeavour to provide a rationale for the approach taken:

- Everything in the technical description that follows is the coherent expression of one "thing"
- As a fundamental description of dynamic relationships, it is in a significant sense already "known" to the reader
- As such it is characteristic in some way of human living and being
- This is despite the curious mathematical technicalities through which it is described and which, as an artificial language, may be difficult to comprehend
- To the extent that it is in some way already intuitively known and recognized by the reader, it may resonate with archetypal symbols and patterns of quasi-similar form from different cultures
- Such resonance can be usefully distinguished from popular enthusiasms for "fractal thinking"
- There is a case for exploring the more rigorous mathematical descriptions to determine whether particular features and properties support valuable insights relevant to challenging psycho-social issues and dilemmas

In support of this approach, for example, Chris C. King (*Fractal and Chaotic Dynamics in Nervous Systems*, 1991) presents a review of fractal and chaotic dynamics in nervous systems and the brain, exploring mathematical chaos and its relation to processes, from the neurosystems level down to the molecular level of the ion channel.

Challenging aspects of this exploration

This exploration offers an intriguing challenge in attempting to render comprehensible some rather subtle insights. For mathematicians the M-set is recognized as one of the most complex objects -- whilst at the same time claiming that its intricacies are basically accessible to those with a background in high school mathematics. For many, like this author, exposure to mathematics at that level may no longer be meaningfully remembered -- effectively grouping them with those who have not had that exposure. On the other hand, as visualized through dramatic fractal displays, the object lends itself to easy exploration and has aroused much enthusiasm -- supposedly avoiding the need for any mathematics.

However the purpose of this exploration is to benefit to a higher degree from the mathematics, without getting lost in its technicalities, and to focus on its implications for offering an ordering for psycho-social insights that may have been acquired or intuited through other disciplines. That said, there remains the problem of how to structure this exploration so as to offer a link to the mathematics for those who may have some willingness to benefit from it (and be reassured by its formal features) -- without disturbing the flow of the argument and distracting from its integrative commitment.

Clearly the argument is primarily speculative -- a right-brain exercise. The mathematics may offer a left-brain framework for some. It must also be said that, for the author, endeavouring to make the technical arguments of mathematicians meaningful proved to be an extremely valuable exercise in triggering such intuitive right-brain insights.

This paper therefore carries the speculative argument. Extensive links to introductory explanations elsewhere are provided in the table below.

It should be noted that with respect to any "non-mainstream discipline", any reference to it here is not to be considered as an endorsement of that perspective. Its significance may however lie in the size of the constituency holding that view -- namely in the political and cultural implications of the dynamic arising from such alternative views in a global system. The purpose here is to raise issues for imaginative exploration, not to seek premature closure.

Dissipative systems and their illusory continuity

A very useful articulation of the challenge is in terms of dissipative systems about which the remarks of Kent Palmer (*Steps to the Threshold of the Social: the mathematical analogies to dissipative, autopoietic, and reflexive systems*, 1997) seem the clearest and most relevant for the above purpose. For him (pp 587-588):

> Dissipative systems hold two strands of illusory continuity together. They concern the situation where there are two orders that are in imbalance so that one order is displacing the other. Notice that if there is only one order there cannot be a dissipative system. Also if the two orders are in balance or stasis there cannot be a dissipative system. A dissipative system is when there are two different orders or ordering mechanisms that are out of balance with each other so that one ordering mechanism is disordering the other and creating a boundary between the two ordering mechanisms where one is dominant and the other is being dominated.

Such language would seem to be a helpful way of handling the many fundamental strategic dilemmas that affect both the coherence of global debate and the experience of interpersonal relationships. The challenge is indeed one of two different "ordering" mechanisms, whether these are culturally defined (Huntington's "*Clash of Civilizations*", Snow's "*Two Cultures*", political cultures ("right vs left", "mainstream vs alternative"), gender defined ("*Men are from Mars and Women are from Venus*"), or in terms of epistemological mindsets (*Systems of Categories Distinguishing Cultural Biases*, 1993).

As Palmer argues, this situation can be approached using the "imaginary" qualities of complex numbers, stressing the nature of the "illusion" involved:

> This case has the basic form of vector arithmetic or the complex number system that holds the order of the real numbers together with the ordering of the imaginary numbers. The complex number system includes both real and imaginary numbers. The differentiation between the two is indeed imaginary because either number could be designated as real and the asymmetry between imaginary and real numbers is an illusion which comes directly from their conjunction not from the numbers themselves. In the case of the complex number system the reals are dominant and the complex numbers are subservient.

In the other previous paper (*Cardioid Attractor Fundamental to Sustainability: 8 transactional games forming the heart of sustainable relationship*, 2005), the challenge of "positive" and "negative" was handled through a coordinate system developed by Edward Haskell to map pairs of interacting biological species in terms of the nature of their transaction or "game". This gave rise to a "coaction cardioid". But as Palmer indicates in endeavouring to map out such relationships:

> We only actually see the relation between the two if we place the complex axis at right angles to the real axis. When we look at the field of these numbers what becomes apparent to us is the form of the mandelbrot set. The Mandelbrot set is the most complex mathematical object known to man. This set is composed by iteratively taking each point and multiplying it by itself and measuring the rate at which it escapes toward infinity. All real numbers escape toward infinity at the same rate. The numbers that represent the intersection between real and complex have different rates of escape toward infinity. We will... call each of those

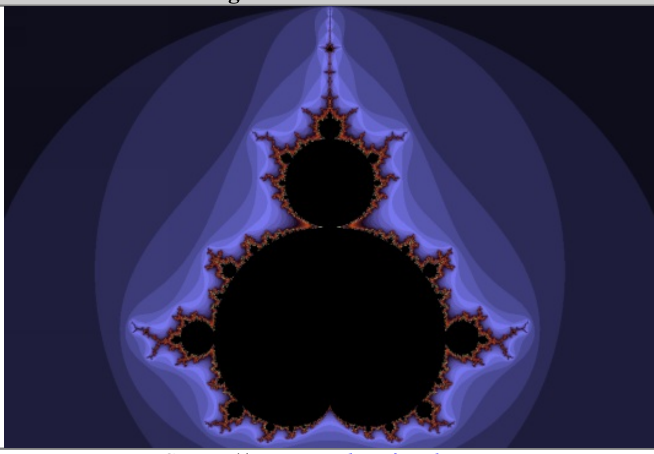
> escape velocity weights the line of flight of a particular point. Dissipative systems have an interface between their two orderings (that of the system and that of the environment) which is very complicated. It involves myriad lines of flight that produce an infinitely complicated pattern which is still determinate.

It was suggested that the cardioid intrinsic to Haskell's approach could possibly be understood as that feature of a M-set. It is indeed the case that the systems to which Haskell's coaction cardioid was applied could be understood as dissipative systems -- even though he did not use the axial representation conventionally used for complex numbers (as described by Palmer).

Structure of the visual representation of the Mandelbrot set

In order to offer a framework for any more detailed discussion of some of the technicalities of how the M-set emerges as a coherent pattern -- and its significance for the above purpose -- it is useful to provide a focus through the features of a visual image to which reference can be made.

Figure 1: Mandelbrot set



Generated by *Xaos: realtime fractal zoomer*

As a representation of the M-set, Figure 1 is rotated 90^{o} from that used conventionally. This is an orientation similar to that used in the earlier paper on the cardioid as an attractor. It is also that favoured by the many who compare it to the seated Buddha, especially when coloured to highlight the concentric "auras" (as in Figure 1). It could be rotated a further 180^{o} to give prominence to the cardioid effect, for those who associate more strongly with representations of the heart.

Selected Web Resources on Mandelbrot and Julia Sets
There are extensive resources available (especially on fractals in general) -- of different quality and duration, since many are developed as student projects

Static images / Animations	*Xaos*	.
	Mandelbrot Explorer Gallery	.
Interactive applets	*Mandelbrot Fractals*	.
	Julia and Mandelbrot Sets	David E. Joyce
	Mandelbrot Applet	.
	The Mandelbrot/Julia Set Applet	James Denvir
	Mandelbrot and Julia sets	Alexander Bogomolny
	Fractal Microscope	http://www.shodor.org/master/fractal/software/mandy/
	Java Julia Set Generator	Thomas Boutell (Boutell.Com)
	Mandelbrot Set	Eckhard Roessel
	Mandelbrot Explorer	Panagiotis Christias
	Mandelbrot Fractals Generator	Salvatore Sanfilippo
	The Fractal Microscope	Alton Patrick.
	The Mandelbrot Set Iterator	James Denvir
	Julia and Mandelbrot Explorer	Julia Thrower (Texas Tech University)
Overview / Features / Tutorials	*The Mandelbrot Set Explorer*	Systematic overview by Robert L. Devaney
	The Mandelbrot and Julia sets Anatomy	Evgeny Demidov
	Xaos	Separate tutorial (associated with a demonstration)
	Studying Mandelbrot Fractals	Suzanne Alejandre:
	Overview	SUNY (Binghamton)
	The Mandelbrot Set and Julia Sets	Yale University
	The Mandelbrot Set	University of Utah
	Introduction to the Mandelbrot Set	David Dewey (A guide for people with little math experience)
	Chaos Hypertextbook	Glenn Elert
	Chaos and Fractals	Dale Winter (introductory course, notably Julia sets and Mandelbrot set)
	Fractal Explorer: Mandelbrot and Julia sets	Fabio Cesari
Downloadable	*Xaos: realtime fractal zoomer*	Excellent open source demonstration software (free), with many interactive facilities and an excellent tutorial -- focused on Mandelbrot and Julia sets
		Allows user to explore functional iteration, including Mandelbrot and Julia sets

interactive demos	*Winfeed fractal exploration program*	(Exeter University)
	Fractal Explorer	Free. See also tutorial
Glossaries	*Mu-Ency - Mandelbrot Set Glossary and Encyclopedia*	Robert Munafo
	Mandelbrot Set Explorer	.

As noted by Len Warne (*A Meditation on the Mandelbrot Set, via Walt Whitman*):

> The Mandelbrot Set emerges from the behavior of a famously simple mathematical function. The Set itself is like a black hole in the abstract space it inhabits. Most of that space is a vast, featureless void. But the points near the boundary of the Set are torn between the temptation to join the Set and the lure of infinity. When their behavior is coded in colors, the result is a beautiful filigree of infinite depth and complexity.

Interpreting features of the M-set

For detailed descriptions of features of the M-set, see the web resources in the above table.

The M-set can be divided into an infinite set of figures (typically represented as black, as in Figure 1) with the largest figure (in the center) being a cardioid. An (infinite) number of circles are in direct (tangential) contact with the cardioid -- but they vary in size, tending asymptotically to zero. Each of these circles has in turn its own infinite set of smaller circles in contact with it, and these surrounding circles also tend asymptotically in size to zero. Repeatedly indefinitely, this branching produces a fractal. In addition the M-set is characterized by filaments or tendrils within which some new cardioids appear, not attached to lower level "circles". [more]

1. **Complex plane / numbers**:
   The M-set is not represented graphically on a plane in a normal 2D space. The nonlinear dynamics to which it points can only be effectively represented on a complex plane. Mathematically one dimension is then "real" and the other "imaginary". These dimensions are discussed with respect to the axes, to be followed later by their possible psycho-social implications. Complex numbers (instead of "real" ones) are required to define the position of a point on a complex plane. Each point on that plane represents a single complex number of the form x + yi, where y is the distance left or right from centre line (negative when left, positive when right) and x is the distance above or below the centre line (negative when below, positive when above) and i is the root of -1. The widespread interest in the M-set derives from the simplicity of the iterative formula giving rise to such a variegated object. If the formula was only based on "real" numbers it would give rise to an uninteresting picture.

2. **Axes**: Conventionally the "real" dimension is represented on the horizontal axis (x-axis), whereas the "imaginary" is represented on the vertical axis (y-axis). It is important to recall Palmer's articulation above regarding the illusory quality of what is being described. The axes are given their significance by arbitrary convention regarding "x" and "y", "vertical" and "horizontal", "positive" and "negative". In the representation used here, the vertical axis (above the horizontal) holds the positive values of real numbers, the negative below; the horizontal axis holds the positive imaginary values on the right, the negative on the left.

   The axes of the complex plane on which the M-set is represented may be usefully compared to the experiential significance of being "crossed". This expression tends to be used to describe the encounter with a mode of behaviour that is inconsistent with the logic which one normally used. It reflects a different mode of organization. This distinction might, for example, be used to describe the relationship between "right" and "left" in politics, or between "mainstream" and "alternative" development strategies -- or even between "female" and "male".

3. **Points:** The axes of the M-set permit various complex numbers (having a real and an imaginary component) to be positioned in relation to one another in a systematic manner. Specially named points include:
   - **Origin**: This is the point at which the axes cross, defined as (x=0; iy= 0) and on the basis of which the M-set is generated. It might be usefully compared with the birth place of person, one's home, the starting point of the dynamics between two irreconcilable positions, etc. It may be related to the centre of gravity of the body (*hara*), as in martial arts.
   - **Complex points:** These are the positions that complex numbers taken up in terms of the axes. They effectively mark the condition of a nonlinear dynamic relationship, such as the status of an argument or a relationship relationship
   - **Fixed points:** This is the point at which transformation ends, despite any further iteration. In conditions of nonlinear dynamics, behaviour in a person's life may be governed by:
     - **attracting fixed points**: where behaviour tends to be attracted to a single fixed point of focus (family **or** job), holding the notion of eventually "returning home" or "coming back". In the complex plane, the origin (0) and 1 are considered special fixed points.
     - **repelling fixed points**: where behaviour is repelled by a single fixed point (a place, a person, a perspective, etc)
     - **attracting periodic points**: where behaviour tends to alternate between attraction to two or more such fixed points (family **and** job **and** sport)
     - **repelling periodic points**: where behaviour tends to alternate between repulsion by one or more such fixed points (family **and** job **and** sport)
     - **Lyapunov-stable fixed points**:
     - **neutral (nonhyperbolic) fixed points**: ****

     Infinity is also treated as a fixed point since complex numbers near infinity (far from 0) stay near infinity (far from 0).
   - **Critical point:** This is the starting point through which the dynamic function is tested to determine whether it results in connected or disconnected sets. It might be understood as the critical point through which the coherence of a discussion

or an initiative is tested. It might also be understood in terms of *kairos* as the dramatic moment at which the future outcome of an interaction is effectively mapped out -- a moment of "destiny".
- **Centre of gravity**: The M-set is symmetrical around the real axis, on which the center of gravity is therefore located. Its imaginary coordinate is therefore 0. [more]

4. **Iterative generation:** Iterative generation of the M-set happens in the "dynamical plane", with the set of all possible parameters is the "parameter plane". For most functions there are areas of the parameter plane (i.e. certain parameter values) for which the iteration exhibits the same properties as the quadratic function by which the M-set is generated, and those regions of the parameter plane contain shapes that look like the M-set.
Given the manner in which "iteration" is associated mathematically with fractals, it might be assumed that it is unrelated to natural phenomena. In fact it is exceptionally common in that it simply means using the current state of a system to create the next state, and repeating this process many times. It is a feature of any standard office procedure, of driving a vehicle, of selling to a client, or of courtship behaviour. In each case it is typically impossible to predict with any certainty -- however well-controlled the process -- the outcome of that process.
    - **Function of M-set**: The simple function used to generate the M-set is as follows: $z_{n+1} = z_n^2 + c$. In it, z starts out with both the real and imaginary parts set to zero (namely z = 0 + 0i). Then c is initialized to the complex number representing the point to be calculated -- its real portion is its vertical distance from the centre of the plane, its imaginary portion is its horizontal distance from the centre of the plane. The result of each iteration is fed back into the function.
    - **Outcome**: The iterative process is characterized by a "dynamic interaction" between z and c. If z is larger than one, when it is squared it jumps outwards -- breaking towards infinity. But if c is located in the opposite direction, then when c is added in the function z is pulled back. On the other hand, if z is smaller than one, squaring it makes it even smaller. In this way c will push z inwards or outwards -- unpredictably towards 2 or falling back to 0.
    - **Function of J-set**: Exactly the same function is used as for the M-set. Now, however, z is initialized to the current point, and c is initialized to a seed value -- another complex number typically taken from the M-set. Through the iterative process, the fate of all possible seeds for that fixed value of c are considered -- with those those seeds whose orbits do not escape forming the filled Julia set of $x^2 + c$. For each different value of c, an entirely different Julia set is generated. Given that there are an infinite number of values for c, there are an infinite number of J-sets. Each J-set can be zoomed into to any level level of magnification. The filled Julia set is a picture in the dynamical plane, not the parameter plane.
    - **Iterations:** The maximum number of iterations (N) used in testing points in the generation of the M-set can be selected as desired, for instance 100. Larger N will give sharper detail but take longer. In effect iterative generation of the M-set never actually draws the M-set completely, only an approximation to it that is the better the larger the number of iterations. [more | more]
    - **Processes**: Human life may be understood as characterized by iterative processes. Physiologically these include breathing and the pumping action of the heart. Vision is based on rapid eye movement (REM). The circadian rhythm of the waking/sleeping cycle can also be understood in this way, as can the cycle of consumption/excretion. Many habits are characteristically iterative, as is engaging in sex. The succession of human generations, through which society (and the planetary surface) is populated, may also be considered iterative. A number of religions hold strong convictions regarding reincarnation, itself an iterative process, through which eventually individuals evolve into "buddhahood", for example. Within society there are many regular processes that can be usefully seen as iterative: rituals, regular meetings, festivals, etc that provide benchmark points indicative of its status. The most fundamental debates have an iterative aspect as the same points are explored again and again. It might be argued that in order for sustainable consensus to emerge amongst divergent perspectives a pattern of iterations is required to engender a form isomorphic with graphical representations of the M-set.

5. **Surfaces and volumes**: The M-set can be represented on a complex plane surface or on a complex sphere. Humans make extensive metaphorical use of a supposedly flat "surface", whether to describe the "domain" or "territory" to which they lay claim ("my land") or which forms part of their "homeland", "nation" or "empire". This usage is reflected in the competitive relations between corporations and (organized criminal) gangs -- even when the "territory" is a range of products and services rather than tied to a particular geographical surface. The territory may be effectively associated with a "sphere" of operations and may be understood in "global" terms. Such understandings of territory are fundamental to the highly dynamic relations between academic disciplines -- even with respect to their specialization into "fields". It is useful to stress that any such "surface" understood through such metaphorical usage is not as stable as is implied by efforts at demarcation by surveying and mapping techniques. As well demonstrated in the Middle East, even the demarcation of the land surface may be highly disputed. Such distorting dynamics are even more evident in competitive relations and conflicting between commercial interest groups, ideologies, religions or academic disciplines.

6. **Scope:**
    - **Size**: ***
    - **Boundedness**: An orbit is said to be bound if it does not escape to infinity on iteration.
    - **Boundary of the M-set:** The zone of the M-set containing all of the chaotic behaviour, namely all points that iterate indefinitely without a cyclic period. It is here that the variety of the M-set is evident. It is infinitely convoluted, but can be mapped onto a circle.
    - **Dimension**: Fractal dimensions are entirely different from conventional understandings of dimension and raise very serious questions regarding the limitations of customary understanding. Fractal dimension is a measure of the irregularity or roughness of a shape -- the degree to which the shape "fills space". The boundary between points within the set and

outside it is so convoluted, folded, and detailed, that it is considered to have fractional dimension. When doubling the magnification, the length of the curve, and hence the area covered, does not simply double. Any previously visible portions of the curve double in length, but new features forming the boundary become visible and add to the length. The M-set has a fractal dimension of two -- meaning that each doubling of magnification, the length of the boundary increases four times. Human experience may be understood as lying within the world of polarization and duality. The coherence and integrity of human experience -- any sense of unity -- therefore emerge within the framework of that duality.

7. **Sets and connectedness -- J-set and M-set:** The J-sets and M-set emerge in a 4 dimensional space that is populated by iterations. Two of the dimensions correspond to the (real and imaginary) values of the parameter c from which the iteration is started. Two correspond to the (real and imaginary) z values resulting from the iteration. J-sets are therefore cross-sections in the z plane, whereas the M-set is a cross-section in the c plane passing through the origin. **The J-sets reside in the dynamical plane (namely the z-plane where the iterations take place), whereas the M-set resides in the parameter plane (namely the c-plane).** The Julia set can also be defined as the boundary of the filled-in Julia set, which consists of the points z whose orbits stay bounded.
    - **Julia set (J-set)**: [more] The shape of a J-set is entirely governed by the complex parameter c. If c = 0 the J-set is the unit circle. If c = -2 the J-set is a straight line between -2 and +2. For all other values of c, the Julia set is a fractal. A J-set might be described as an event horizon within a phase-state description of a discrete nonlinear dynamic process. With respect to human behaviour and understanding, a J-set might be usefully described as a "pattern". A distinction can then be made between three kinds of pattern:
        1. Disconnected J-sets ("Cantor dust"): Essentially unstable patterns that persist (or exist) only briefly, if at all. These may include behaviours which seem to be part of an enduring pattern but more or less quickly prove not to be. Equally they are the modes of thought which may briefly appear to be consistent, but quickly prove not to be.
        2. Connected J-sets: These are patterns which are essentially habitual and unvarying, consistent with a single general pattern of behaviour of which they are an exemplification.
        3. ***
    - **Connectedness**: A set of points is connected if, for any two points in the set, there is at least one path consisting entirely of points in the set, which leads from one point to the other. The bands beyond the M-set corresponding to points of 2, 5, 10, or more each form complete loops around the M-set. Whilst this is true for higher numbers of iterations, their bands are so convoluted as to be difficult to trace around the multitude of features. Within the M-set (the parts usually represented in black), the (infinitely) many miniature copies of the M-set set are each connected to the central cardioid part by one, and only one, infinitely thin filament that may never be visible. Its presence can be detected by noting the constantly thinning bands of colour squeezing in on it from both sides.
    - **Mandelbrot set (M-set)**: As the M-set is the set of all parameters c that give rise to connected J-sets, it is also called the connectedness locus for complex quadratic polynomials.
    - **Resemblance**: The most obvious features of the filaments in an area of the M-set near point p will then resemble the J-set for c=p. Except for a scaling and rotation, the region near p in that J-set will resemble a region near p in the M-set.

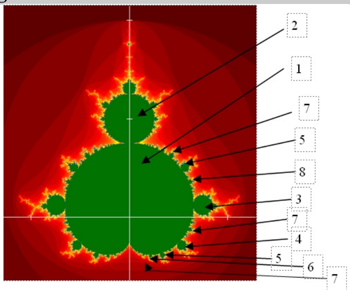
8. **Form:** There is a remarkable similarity between the dynamical pictures of different quadratic maps (J-sets) and the parameter picture of the whole quadratic family (the M-set). A systematic approach to counting the number of features of each type in the M-set has been developed [more]. As noted by Linas Vepstas (*Mandelbrot Bud Maths*, 2000):

    > The shape of the main Mandelbrot cardioid is given by the formula $z = e^{it}/2 - e^{2it}/4$. The largest bud, the west bud, appears at t=2*pi/2. The next largest, the north bud, is at t=2*pi/3, and so on, each at t=2*pi/n.... In fact, the importance of Fibonacci Numbers and Farey Numbers for the description of the Mandelbrot set is well known. (See, for example, R.L. Devaney How to Count).

    - **Self-similarity**: In a fractal image, such as the M-set, the parts are contained in the whole fractal image, but the whole is equally contained in the parts. Smaller scale versions can be viewed as replicas of the whole. *The Mandelbrot Set and Julia Sets The Mandelbrot Set - Small Copies*
    - **Symmetry**:
    - **Cardioid**: This is the main body of the set as represented in Figure 1. Attached to it are "bulbs".
        - http://www.xahlee.org/SpecialPlaneCurves_dir/Cardioid_dir/cardioid.html
        - *Single-factor / Integral*
    - **"Bulbs" (or "circles", or "disks"):**
        - **Primary**: These are the circular features attached directly to the cardioid
            - **"Head"**: This is the principal primary bulb situated above the cardioid (in the seated orientation). Given that it is defined by a cyclic period of 2, it may be usefully associated with binary thinking and polarization.
            - **Other:**
            - ***
        - **Secondary, etc**: These circular features are attached to the primary bulbs.Similarly tertiary bulbs may be attached to the secondary bulbs, etc. [more]
          triadic, quadrilemma, multi-set

    - **Filaments / Tendrils / Antennae**: These are features of the M-set that consist of infinitely convoluted and branching structures that connecting miniature versions of the M-set to each other. It is the filaments that contain most of the variety in the M-set, being remarkably different from each other -- even when they appear similar. Most of the symmetry in the M-set is to be found in the filaments [more].

Figure 3: Periods of M-set "bulbs" and cardioid



9. **Orbits**: An orbit is the trajectory of a point through a succession of iterations. In the M-set representation, the outer zone represents unbounded orbits (escaping to infinity), the central cardioid zone represents fixed points (to which the orbit converges), the other circular features represent of distinct cyclic periods. The thin boundary zone around the figure represents chaotic orbits. The chaotic regions appear to be restricted to the boundary of the M-set and to a portion of the real axis (represented vertically at the top of the figure). [more] [demo]
   - **Period of attracting cycle**: This may be understood as the number of distinct states that an iterative system cycles through. Demonstrations are provided by interactive applets (James Denvir. *The Mandelbrot Set Iterator*; Alexander Bogomolny, *Iterations and the Mandelbrot Set*, 2005, after bypassing the sponsor's advertising). Iterations within the M-set evolve differently depending on the value of c. Where the starting point c, is within the cardioid, iterations converge -- period 1. For c inside the "head", the iterations converge to a cycle of period 2. For c inside each "bulb" attached directly to the cardioid, the iterations converge to a cycle whose period is determined by the corresponding "bulb". If c lies in the interior of a bulb, then the orbit of z_0=0 is attracted to a cycle of a period n -- it is a multiple of n for c inside any smaller bulbs attached to the primary bulb. The behaviour of the iterations is related to the appearance of the Julia set associated with c. [more]
   - **Period doubling / Bifurcation**: Moving along the real axis (vertically in the seated-orientation), the period of the iterates of the current point keeps doubling. This is admirably illustrated by an applet (*Period doubling and Feigenbaum's scaling*, 1999; *Bifurcation diagram for quadratic maps*, 2002; *Universal period n-tuplings cascade of bifurcations*, 2000)
   - **Attractor / Limit cycle:** This is the value to which a function may converge on iteration, irrespective of the starting point. The number of points in a limit cycle is called the period. Fractal shapes are depictions of attractors. A region of points with attractors, like the M-set, is termed a basin of attraction. This may be associated with processes such as temptation and conversion. A J-set is the boundary of all its attractor basins.
   - **Repellor**: At the boundary of an attractor basin, points inside the boundary are convergent and trapped by the attractor. Points outside the boundary are divergent and escape the attractor --the attractor then acts as a repellor. The boundary of a J-set functions like a repellor -- as the closure of all the repellors.
   - **Mode locking**: This is the tendency to fall back into the behaviour pattern, the attractor, even when external perturbations disturb the pattern momentarily.
   - **Rotation numbers**: (see Robert L. Devaney. *Rotation Numbers and Internal angles of the Mandelbrot bulbs*, 2000)
   - **Unit circle**: This is the circle of radius 1 centred on the origin. Any iterative seed on (or inside) the circle of radius 1 has an orbit that does not escape to infinity. All orbits of x^2 that lie on the unit circle are those that behave in a chaotic fashion.

10. **Features**: Many of the detailed features of the M-set have been given colloquial names, usually descriptive in the light of their resemblance to natural phenomena [more].

11. **Colour**: Three main approaches to colour are used:
   - **Exterior**: The exterior of the Mandelbrot set consists of points for which certain iterations diverge. Normally colours are added to representation of the points that are not inside the set, according to how many iterations were required before the magnitude of z exceeded two. This creates concentric shapes, each a better approximation to the M-set than the last. Other colouring conventions may be available (as with Xaos) based in part on the real and imaginary coordinates.
   - **Interior**: Areas inside the set are usually filled in black, but this is only a convention. Again (as with Xaos), there may be many different ways to show the colour within the set, notably based on the real or imaginary coordinates of the latest orbit, or its angle.
   - **Colour cycling**: This is a technique to automatically shift or rotate the palette of colours through which the M-set is displayed, making the display strikingly dynamic.

12. **Mapping in higher dimensions**: The classic M-set is a map in the complex plane. Software to explore the M-set typically allows the user to more complex fractal structures:
   - **Polynomial maps**: the power in the quadratic recurrence equation may be increased from the standard "squared" form (with one symmetry axis) to the cubic form (with two such axes), the quadric (with three), etc -- with any number of "poles" by suitable choice of exponent. [more] Including the Mandelbrot set (power 2, 3, 4, 5 and 6) in the case of Xaos. Also known as multibrot sets [more | more | more | more]

- **Complexifying the plane**: The complex plane may be rendered more complex using a choice other common "surfaces"
- **Higher dimensions**: Potentially the M-set can also be mapped into higher dimensions. for example, using quaternions the M-set is a 4 dimensional object [more].

As with other types of fractals (eg the Octo fractal, Barnsley's fractals, the Newton fractal, Phoenix and Magnet), these raise fruitful questions which are not however addressed in what follows.

You first plow in the dynamical plane and then harvest in the parameter plane. *Adrien Douady*

Potential implications: orders of abstraction and "explanation"

The emergence of the M-set as a focus of disciplined reflection enables extremely useful distinctions to be made between levels of abstraction and the nature and credibility of any corresponding explanations. Any assumption that the M-set offers a simplistic, single-factor explanation should be carefully examined -- perhaps in the light of the relation between multi-period attractors and multi-factor explanations. The potentials of the M-set relate in fact to the concerns with meta-systemic perspectives, notably those developed by Kent Palmer (*Autopoietic Meta-Theory*, 1998; *Deep Mathematics and Meta-Systems Theory*, 1997; *Vajra Logics and Mathematical Meta-models for Meta-systems Engineering*, 2001).

Distinctions might be made in terms of deductive ("top-down") and inductive ("bottom-up") approaches:

- **Top-down distinctions**: A top-down argument might then point successively to the existence of the following levels:
  - an invariant map of all dynamic functions (the M-set)
  - sets of dynamic functions (the J-sets)
  - dynamic functions
  - dynamic systems understood in systems terms
  - sensible phenomena experienced as dynamic and possible unpredictable

  It is the first of these which is associated with invariance in its most fundamental sense. But as such it is at a level of abstraction that is largely beyond explanation and comprehension. It points to (or "maps") the variety of models of dynamic behaviour, effectively providing an explanatory context for them. We then encounter the level of systems analysis that abstracts from sensible phenomena particular behavioural functions by which the behaviour is modelled. Only finally do we encounter the level of the actual sensible phenomena that are experienced in the confusing variety that is organized by any form of systematic analysis at the preceding level.

- **Bottom-up distinctions**: Whilst the first levels of distinction might be usefully made in terms of scientific disciplines, their concerns shade into the preoccupations of philosophy.

There are a number of sources distinguishing levels of abstraction:

- **Zen**: Perhaps the most succinct and systematic distinctions are made through the classic sequence of *Zen Ox-herding Pictures* [more | more]. The subtle relationships between such standpoints have, for example, been illustrated by two well-known variants of this in which the ox may be interpreted as any (or all) objectified conditions(s) over which mastery is sought.

  The successive levels of comprehension of this task bear the following names in the Kaku-an series: searching for the ox; seeing the traces; seeing the ox; catching the ox; herding the ox; coming home on the ox's back; the ox forgotten, leaving the man alone; the ox and the man gone out of sight; returning to the origin, back to the source; entering the city with bliss bestowing hands. The distinctions between them are variously discussed in earlier papers (*Tao of Dialogue*, 1996; *Dancing through Interfaces and Paradoxes*, 1997; *Configuring Conceptual Polarities in Questing metaphoric pointers to self-reflexive coherence*, 2004).

- **Questions**: One bottom-up approach is the challenge of distinguishing questions of "higher order" through the necessary cognitive "twist" that is called for, and the potential for manipulation of those unaware of such twists (cf *Engaging with Questions of Higher Order: cognitive vigilance required for higher degrees of twistedness*, 2004)

- **Variety of systems of "levels"**: Metaphorically the transition between levels may be described as a cognitive rebirth. An earlier paper (*Varieties of Rebirth distinguishing ways of being "born again"*, 2004) endeavoured to distinguish such levels in a variety of systems clustered into the groups indicated in the table below. These correspond to quite different senses of being "born again". These may interweave to reinforce each other -- or may reflect contrasting, even incommensurable, understandings or experiences. In a larger multi-dimensional scheme all these threads may together constitute a larger fabric of insight to which humanity has yet only partial access. In the table the threads or clusters explored are tentatively ordered in terms of increasing experiential implications for the individual. Two "paths" may be distinguished to relate the clusters.

G. Experiential rebirth (operacy, flow, embodiment of mind, speaking with God, born-again, possession, psychedelic experience, embodiment in song, spiritual rebirth)	F. Cognitive perspective (metacognition, critical thinking, philosophy, aesthetic sensibility, orders of thinking, systematics, orders of abstraction, disciplines of action)
E. Therapeutical rebirth (release from trauma, mentors, self-help, discipleship)	D. Developmental rebirth (education, perspective, initiation, cultural creativity, individuation)
C. Psycho-behavioural rebirth (sin-to-virtue, changing patterns of consumption, conversion,	B. Socio-religious rebirth (birthright, destiny, reincarnation, social status, ceremony, ritual, group affiliation, games, sports)

A. Cultural rebirth (renaissance, aesthetic birth, mytho-poesis)

Levels of abstraction, however clustered, focus two intertwined questions:

- What stays constant, namely what actually is invariant and in what way?
- What is the nature of the identity of that which is understood as invariant? Or, of greater interest, what is the nature of the identity at a more abstract level -- whether in the case of a person or a group? How is that form of identity experienced?

These questions might be discussed in terms of meta-levels and how communication about them can be ensured with minimal confusion (** Palmer). For example, David Wright (*Talking 'about': Languaging, emotioning and communicating knowings*, 2002) asks:

> How do art and anecdote meet analysis in the pursuit of understanding? This paper proposes that we are in a constant process of negotiating genres in recognition of the structural limits to our knowing, further that our knowing is itself in negotiation with those structural limits and that emotion and language are key mediators and moderators in this process.

Whilst mathematics may offer formal distinctions, the mind may be much challenged to comprehend these distinctions. Much use can be made of metaphors in making the distinctions, as is typical of spiritual discourse. These do not necessarily offer an experiential sense of the distinction. It is here that the proposal of Kathleen Forsythe to use the term "isophor" to depict a feeling response to one thing in terms of another is relevant. Forsythe describes an isophor as something that is "experienced emotionally and, (that) as such, define(s) the experience of understanding." (1987)

Kathleen Forsythe (*Cathedrals in the Mind*, 1986) in a paper to a meeting of cyberneticians argues: "Analogy and its poetic expression, metaphor, may be the "meta-forms" necessary to understanding those aspects of our mind that make connections, often in non-verbal and implicit fashion, that allow us to understand the world in a whole way."

Forsythe uses the term isophors for isomorphisms experienced in the use of language. Isophors are distinct from metaphors in that they are experienced directly. With the isophor there is no separation between thought and action, between feeling and experience. The experience itself is evoked through the relation. She suggests that the experience of one thing in terms of another, the isophor, is the means by which domain is mapped to domain and that consciousness of this meta-action, when recognized, lies at the heart of cognition. Forsythe has postulated the development of an epistemology of newness in which learning is the perception of newness and cognition depends on a disposition for wonder leading to this domain of conception-perception interactions. She argues that the notion of metaphor is commonly understood to mean the description of one thing in terms of another -- presupposing an objective reality. This objectivity may be questioned and if, as suggested by Maturana, (objectivity) is placed in parentheses:

> "we can begin to appreciate clearly the role we play in the construction of our own perception of reality. for this reason, the notion of the experience of one thing in terms of another, the isophor, suggests that it is this dynamic constructing ability that involves conception and perception -- unfolding and enfolding, that this gives rise to the coordination of actions in recursion which we know as language." [more]

The challenge in what follows is to determine whether felt experience and insight resonate with formal representation. This is the challenge of aesthetic proportion in general and of sacred geometry in particular. The concern is not to force such an association but rather to provoke an imaginative exploration of possibilities. This exploration is not aimed at closure or reductionistic grasping of a subtle integrative pattern. The concern is more to offer a form (the M-set) that ensures an interplay between suggestive possibilities. It is however important to recognize that the very complexity of the M-set, however well understood intuitively in some way, poses a real challenge to explanation and comprehension of the M-set as a 4-dimensional gestalt. The challenge might in some ways be compared to explaining movement up a spiral staircase, without pictures, to someone who has never seen one. The length of this paper is perhaps a measure of the lack of full understanding of that gestalt by its author!

Meshing mathematical and experiential understanding

It is important to be as clear as possible concerning the challenge of relating what mathematics can discover, the possible beauty of its graphical expression, with reality as known and experienced. Each constitutes a different focus and their relationship is not necessarily evident or rigorously established.

It is perhaps appropriate to recall the early recognition of the paradoxical challenge of explaining chaos as articulated by Heraclitus (540-475 B.C.E):

> Dissatisfied with earlier efforts to comprehend the world, Heraclitus of Ephesus earned his reputation as "the Riddler" by delivering his pronouncements in deliberately contradictory (or at least paradoxical) form. The structure of puzzling statements, he believed, mirrors the chaotic structure of thought, which in turn is parallel to the complex, dynamic character of the world itself.
>
> Rejecting the Pythagorean ideal of harmony as peaceful coexistence, Heraclitus saw the natural world as an environment of perpetual struggle and strife. "All is flux," he supposed; everything is changing all the time. As Heraclitus is often reported to have said, "Upon those who step into the same river, different waters flow." The tension and conflict which govern everything in our experience are moderated only by the operation of a universal principle of proportionality in all things. (Garth Kemerling. *Origins of Western Thought*, 2001)

In contrast with the uniqueness of a particular experienced phenomenon, a fractal models the essence of a species or type, not the appearance of a particular individual. Fractals can be found to fit any set of data, including inherently unpredictable and chaotic systems -- where linear equations fail entirely (as representations of the rate of change of a system at any given point). The question is how to understand the relation to the J-sets and M-set discussed here. Fractals emerge at the border between harmony and dissonance -- when rhythms fall into or out of sync. The human heart and the brain are dynamical systems in motion. In this respect, psychiatrist Arnold Mandell is quoted (by James Gleick. *Chaos: making a new science*, 1986) as follows:

> Is it possible that mathematical pathology, ie chaos, is health? And that mathematical health, which is the unpredictability and differentiability of this kind of structure, is disease? ... When you reach an equilibrium in biology, you're dead.

There is now an interest in "dynamical diseases" -- when fractal rhythms fall out of sync. On this point, Dick Oliver (*Fractal Vision*, 1992) notes:

> Life is rhythm. But it's a special kind of rhythm, a rhythm where resistive friction is always dragging it toward rest and almost-coordinated pushes are always pumping it back into sync. Graphs of heartbeats and brainwaves are not smooth pulsations, but fractals with a dimension nearer to two than one. They thrive on chaos, and sicken with smoothness. when this measure of dimensional roughness falls closer to linearity, a heart attack or seizure is probably on its way.

In a personal communication in 2005 on the questions posed by this paper, Chris Lucas (CalResCo: The Complexity and Artificial Life Research Concept for Self-Organizing Systems) makes the points:

> The problem would be that mathematicians who study fractals concentrate on low dimensional (analysable) systems and do not think their work is applicable to the messy "real world". Those who recognise real world fractals like lungs, trees, fern and coastlines have rarely the mathematical knowledge to create a formula (which probably would be beyond the mathematicians anyway). We can estimate fractal dimension for simple examples like trees, but in itself this says little about the generating function. We can though find a sort of "formula" for real fractals by using Michael Barnsley technique of fractal compression, which finds a "seed" which when iterated recreates the image, some examples of the results of this can be seen in *Fractal Vision*
>
> What we must bear in mind is that nature does not generate fractals by computer iterations, and certainly not in the rather strange way in which we create the M-set (where we iterate disjoint points thousands of times (for a 640x480 screen we do it 307,200 times in total) -- the picture we then see is an artefact of this and not a single ongoing iteration.... This is again the "map is not the territory" distinction, the painting is not the landscape. So even if we ask the questions that you pose, it is not at all clear that they could even have a meaningful answer.
>
> The Julia set for example, although related to the M-set in definite ways is yet another concatenated set of disjoint iterations (which are just adjacent) and has no physical reality as such. In other words, for a tree we go along a trunk and this "branches" fractally, but in neither the M-set or J-set is any part of the picture mathematically connected to the adjoining points, we "construct" the picture analogously to making a mosaic, i.e. the resultant is more like an abstract "phase portrait" - a "map" of state space than anything natural. All connections we make to real world "fractals" are just analogies (as far as I understand it, but bear in mind I'm not a mathematician, some may see it all very differently -- especially those who think the world *is* a vast computer !). It is interesting to speculate *why* we created the M-set in this strange way -- is our love of fractal structure such that we made it in our own image subconsciously ? What other ways of "assembling" such things may be possible... !

The theoretical disconnect from experienced reality has been expressed differently by Dick Oliver (*Fractal Vision*, 1992):

> You obviously can use fractal templates based on affine transformations to model nature. But all this Julia set business seems about as far from nature as you can get. chaotic, nonlinear transformations such as z squared plus c can produce pretty pictures, but they don't seem to have any connection with the physical world at all.... No one is certain how the spirals and branches in the Mandelbrot and Julia sets arise from nonlinear equations, let alone why they follow the archetypal patterns of nature so closely. these topics are at the forefront of current mathematical and scientific research.

It is curious the extent to which so much hangs on the understanding of "iteration". Clearly there is a sense in which the cyclic phenomena studied by Mandelbrot can be usefully understood through iterative procedures. It is also clear that recurrent daily experience -- such as starting the day with the state of one's office as left the previous day -- can be understood as an iterative process. Experience itself may be understood as having been built up through iteration -- repeatedly taking past experience and using it as the configuring seed for experience in the present moment.

An iteration takes all the past -- the past as a whole -- and feeds it into the function in the moment. From any "seed thought" (or intuition), one is then always recomputing the whole -- so the maths of iteration are extremely close to moment by moment thinking (life as "constantly computing z from c"), and our various cognitive habits (see Antonio de Nicolas. *Habits of Mind: An Introduction to Philosophy of Education*, 2000).

It is the pattern recognition (or imposition) ability of the human mind that bridges the mathematical disjointedness to which Chris Lucas refers. The iterative process of recomputing points to constitute an image has similarities to the pointillist, post-impressionist painting style founded by artist Georges Seurat. The semiotic challenge in relation to artificial intelligence has been addressed by Swinton Roof (*MandelBug: A New Project*, 2003).

> There has to be some kind of balance and homeostasis amidst interesting dynamics. Semiotic unity has a similar problem. If symbolic unity is achieved in a brain, how does it do it? A brain is a fantastically complex non-linear system. What enables it to avoid fractally treacherous and chaotic event horizons that rip up any possibility of a unified coalescence of the state vectors. And how does it do it yet still remain flexible and novel? Some sort of critical edge would be good for precise decision making, but conversely, we can't have every minute change affect a system with hair-trigger sensitivity. Symbols have to retain a certain fixed identity yet still connect adaptively to novel changes.
>
> The foundations of mathematics has a similar problem. Recursive statements lead to inconsistency and lack of completeness. Attempts at unified foundations for symbolic systems thus becomes fractured by holes and fork-ed tongued truths. Mathematics, the bedrock of rationality, stands on the brink of bifurcative nonsense. Recursive functions and algorithms in computer programming are notoriously difficult to debug and in some cases impossibly so perhaps.

It is fruitful to look for "levels of abstraction" from experience through to the M-set. But such a ladder is itself problematic as implied above. Furthermore, there is a body of literature stimulated by feminist scholars (cf Carol Gilligan, 1982, 1990) that questions use of "levels" and points to use of a configuration of complementary modes that may be variously accessible (see *Learnings for the Future of Inter-Faith Dialogue : Insights evoked by intractable international differences*, 1993). Such cognitive modes might be understood as attractors of different types. The configuration might be understood in terms of the M-set pattern. Some exploring fractals also question this asymmetrical viewpoint. Although a fractal image of the "lower" parts are contained within the "higher" whole, remarkably the "higher" whole is equally contained in the "lower" parts.[more]

In addition, "below" the experience of dynamics, there is the question of how one participates in those dynamics -- entering into them nonabstractly. And, "above" the abstraction of the M-set, there is the question of how one engages with it and embodies it. Both extremes are beyond the maths (and may merge together forming a "cognitive torus", as with the Ourobouros). The M-set may be effectively understood as an experiential standing wave.

These considerations may require a decision from the reader as to how to approach suggestions in this paper. The broader issues may indeed make it "too flaky" for mathematicians and decision-makers, and too formal for experiential people. But as such it does hold the dynamic of the dilemma their unrelatedness constitutes. Given the ambition of the paper in relation to strategic dilemmas, it is to be expected that this would be in some measure reflected in how its content is explored. Those with a relative dominance of left-brain over right-brain would seek any order offered by the mathematical abstractions in response to chaos, whereas those with a relatively dominant right-brain would be more persuaded by the aesthetic continuities of the patterns and what they imply for participative experience.

Possible psycho-social significance of the M-set

See Annex 2

- Potential implications in terms of religious symbolism
- Potential mytho-poetic implications
- Potential experiential implications in terms of concentration and meditation
- Potential implications for self-awareness, relationships and psychotherapy
- Potential implications: fractal quasi-similarity of patterns

Imagination, Resolution, Emergence, Realization and Embodiment: iterative comprehension

See Annex 3

- Progressive "embodiment" of M-set through iteration
- Differentiating features within the M-set by colour
- Iterative comprehension of the M-set as an ordering template

Potential implications for interdisciplinary and intersectoral initiatives

Julie Thompson Klein (*Interdisciplinarity and complexity: An evolving relationship*. 2004) concludes her review of the challenges to more integrative thinking as follows:

> Contests of legitimacy over jurisdiction, systems of demarcation, and regulative and sanctioning mechanisms continue, and perceptions of academic reality are still shaped by older forms and images. Yet, boundaries are characterized by ongoing tensions of permanency and passage. Simplified views of the complex university only add to the problem of operational realties that outrun old expectations, especially older definitions that depict one part or function of the university as its "essence " or "essential mission "... Repeating the same metaphors... adds to the confusion, impeding understanding of new knowledge, new relationships, and nonlinear, non-vertical perspectives that are multi-dimensional and multi- directional. A wider range of physical and topological or architectural metaphors are being used to describe relations of elements that make up innovations and their contexts -- dimensions, joints, manifolds, points of connection, boundedness, overlaps, interconnections, interpenetrations,

> breaks, cracks, and handles. And ... we might add ... a Mandelbrot set.

Because of the relevance of fractals to many fields, they tend to raise questions about the limited specialized boundaries of science, thus facilitating a more integrated approach. As such hey require a different type of understanding than is typically associated with scientific understanding. The conventional approach is still based on the rational paradigm (which is limited in many ways). Fractals are claimed to require a deeper holistic appreciation involving both reason and intuition. [more]

Managing intractable differences: relevance to particular polarities

As noted above with regard to "incommensurables", the stimulus for this investigation was associated with the the challenge of dealing with irreconcilable perspectives, notably a focus on the "positive" (as the "good") and an avoidance of the "negative" (as the "bad"). Using the axes of the complex plane, to position perspectives reflecting different kinds of "positive" and "negative", reframes the dynamics of the dramatic polarization on which much attention is un fruitfully focused. Furthermore this framing of the dynamics that characterize the encounter between polarized perspectives indicates the possible existence of various zones that merit greater attention:

- **arid void:** With respect to those dynamics, as noted poetically by Len Warne: "Most of that space is a vast, featureless void". This is indeed a description of what typically characterizes polarized dialogue.
- **boundary**: But the points near the boundary of the M-set are torn between the temptation to join the set and the lure of infinity.

The question is whether the M-set is indeed indicative of a zone of stability relevant to understanding of other paradoxically opposed, value-charged perspectives, such as:

- **knower vs known**: namely the classical debate between schools of philosophy, notably those sensitive to the concerns of spiritual development
- **ethical vs unethical**: namely the fundamental challenge of society in handling the grey areas between morality and criminal activity that characterize so much of social life
- **head vs heart**: as exemplified by the polarization between rational discourse and the reasons of the heart, whether romantically or compassionately inspired
- **matter vs energy** [more]
- **individualism (freedom) vs community**: notably as explored in relation to the M-set by T R Young (T R Young. *Chaos and social change: Metaphysics of the postmodern*, 1991) through his concept of ultrastability, which he associates with "our contradictory desire for freedom and community". He uses the M-set as an intricate planar region with a fractal boundary as his principal image of ultrastability. Will C. van den Hoonaard and William W. Hackborn (*Chaos as Metaphor for the Study of Social Processes in the Post-modern World: a Bahá'í Illustration*, 1994 / 2002) compare Young's understanding with their concept of unidiversity.

In other words, does the M-set then perform a kind of "keystone" function sustaining a space. This challenge of balancing polarities has been explored in previous papers, notably in the light of the metaphor provided by tensegrity structures (*Implementing Principles by Balancing Configurations of Functions: a tensegrity organization approach*, 1979; *Transcending Duality through Tensional Integrity: From systems-versus-networks to tensegrity organization*, 1978), and subsequently explored by management cybernetician Stafford Beer (*Beyond Dispute: Invention of Team Syntegrity*, 1994).

"Real" vs "Imaginary"

In each case, the question is whether the tensions between the value-charged strategic polarities can be fruitfully dissociated into "real" and "imaginary" components such that the dynamics engender a sustainable boundary vital to psycho-social coherence -- without collapsing the dramatically opposed perspectives that characterize the polarity.

In a strategic context, "real" is associated with factual data. But as is evident in practice, proponents of opposing initiatives have divergent interpretations of "real" and of the weight to be attached to different "facts", held to be "true". Each is then free to accuse the other of responding to "imaginary" interpretations -- and this tends to be very sharply stated in debate (caricaturing the opposition with terms such as "dreamers", "deluded", "unrealistic", etc) regarding what is "false". In a sense each sees the opposition as responding to an unreal "image" of reality. It is the dynamics of disagreements of this nature that need to be held with a framework of requisite complexity -- transcending relativism -- in order for governance to articulate strategies that are sustainable.

There is a certain irony to the tendency of strategic proponents to plead for more "facts" (monitoring, research, etc) prior to action -- or to call for more "imaginative" thinking to respond more effectively to new kinds of crises or the inadequacies of previous strategies.

The argument here is that the kind of sustainability that would be sustainable -- rather than being itself a victim of these dynamics -- is at a level of abstraction to which the M-set usefully points. As a framework, it in no way denies the existence of the dynamics between constituencies with different understandings of what is real and what is imaginary. Rather the recognition of the M-set depends on those dynamics -- just as the 2D polarities within a tensegrity are essential to the emergence and viability of the 3D structure resulting from their configuration.

In this light the question becomes how to recognize and distinguish the strategic elements contributing to recognition of such an M-set. The need is to offer clearer understanding of the role of "real" and "imaginary", recognizing that "real" to one group may be "imaginary" to another. This reinforces the point made with regard to transforming the axes between "real" and "imaginary", or between "positive" and "negative".

Considering once again how such distinctions would be made in the absence of the cartesian understanding of axes, it is worth reflecting again on the notation used in the thinking basic to the *I Ching*, namely the system of trigrams configured in the *Ho Tu* or *Lo Shu* arrangements of the Ba Gua mirror basic to the discipline of Feng Shui (as discussed earlier). There the focus is on "directions" rather than axes. But clearly two distinct digrams (rather than trigrams) would provide an adequately complex notation to distinguish "positive" and "negative" on a "real" axis, with a second two to distinguish "positive" and "negative" on an "imaginary" axis. In this connection, it should not be forgotten that it was the exposure of Gottfried Wilhelm von Leibnitz to the *I Ching* that inspired his development of the binary coding system.

The nature of the relationship between "real" and "imaginary" can be further considered in the light of the Chinese categories of "yang" and "yin" which are not fruitfully treated as "opposites". All relationships based on yin and yang are considered as relative. Mutual interaction must be considered, therefore, nothing can be defined as strictly yin or strictly yang. Yin and yang are symbolically represented by the Liang-I (two symbols). The Yang-I is represented by a continuous straight line and the Yin-I is represented by a broken line. The conditions to which they refer cannot be considered as permanent states. There is always dynamic movement which is encoded through combinations. The first group of these is called Szu-Hsaing. These four figures (digrams) are formed by combining the Yin-I and the Yang-I. The Szu-Hsaing represent the maximum number sets that can be formed by combining two differing elements in sets of two.

This development is framed in a much-cited passage in the *I Ching* that relates these abstractions to the strategic challenges of human affairs:

> In the Changes there is the Supreme Ultimate (T'ai Chi), which produced the Two Forms (yin and yang). These Two Forms produced the four emblems (Szu-Hsiang), and these four emblems produced the eight trigrams (Pa Kua). The eight trigrams serve to determine good and bad fortune (for human affairs), and from this good and bad fortune spring the great activities (of human life).

As noted earlier, the question is then how to relate such digramatic codes to M-set axes in the light of greater insight into the contrasts between the *Ho Tu* and *Lo Shu* arrangements and understandings of "real", "imaginary", etc in the M-set mapping. There is a case for exploring these arrangements in the light of use of polar coordinate mappings of the M-set, notably in relation to the rotation number of "bulbs" (cf Linas Vepstas: *Douady-Hubbard Potential*; *Mandelbrot Bud Maths*; *Parameter Ray Atlas*. 2000) [more].

If it was also necessary to distinguish conditions that were "positive-real" from "negative-imaginary", for example, then indeed trigrams would be necessary. Finer distinctions (as with the compass directions SSW or NNE) could then be made by adding an extra line position to the digram notation. The 64 hexagrams of the *I Ching* (*"The Book of Changes"*) provide the notation for a Chinese approach to the dynamics that emerge as the M-set from the mapping onto the complex plane. There is an extensive body of literature exploring the significance of the mathematical underpinnings of the *I Ching* [more | more | more], notably that of Stephen M Phillips (*I Ching and the Eight-fold Way*). It is appropriate to reflect on the justification for the incorporation of hexagrams into Version 4.0 of the Unicode standard [more]

In contrast with the mathematics of the M-set, this Chinese system was designed to embody qualitative value contrasts (rather than purely quantitative value contrasts) and was notably used in the clarification of strategic options by the emperors of China. The *I Ching* was in fact required reading for the Chinese civil service for about 1,000 years. One might well ask what tools of comparable complexity and scope are currently used with respect to global governance.

Of particular interest is the polarity between "objective" approaches ("real") and "subjective" approaches ("imaginary"). This is notably evident in the "objective" attitude of mathematicians to complexity -- in comparison with the "subjective" attitudes associated with the psycho-social phenomena noted above. However it is the "real" nature of the 40 religious conflicts around the world -- driven by a sense of "positive" ("good") and "negative" ("evil") in this "imaginary" dimension -- that can be contrasted with the "unreality" of mathematics to those engaged in those conflicts.

Both Physicist David Bohm (*Wholeness and the Implicate Order*, 1980; *The Undivided Universe: an ontological interpretation of quantum theory,* 1993) and those referring to his work, tend to cite the fractal nature of the M-set. In this light, the strategic challenge of "real" and "imaginary", from a psychological perspective, is helpfully discussed by Nicholas Williams (*Psychological Entropy*, 2003):

> Non-linear dynamics challenges our pre-conceptions to the very core - in fact it reverses our basic expectations about the world. If we did not know better, we would naturally assume that positive feedback (amplification of errors) can lead to nothing but meaningless static, zero information in other words, but what the study of non-linear systems shows us is that the reverse can happen - instability can lead to the system regrouping on a more highly organized level, not the other way around. There is an information jump, not a drop.... We think of linear systems, on the other hand, as being progressive in the sense that they are continually moving in the direction of increased information content. The reverse of this is true - **no new information ever comes into a system exhibiting linear change, they are utterly tautological and never say anything new**.
>
> The psychological implications of this are immense - we are forced to confront David Bohm's spooky vision of the rational (i.e. linear) mind as a dealer in illusions, forever offering up visions of progress, new ways of looking at the world which are actually only "reshuffled" versions of the same old thing. The "no win situation" that is neurosis arises precisely out of this disguised tautology because all the "solutions" that our rational mind offers up are in fact reshuffled versions of the original problem. The (purely rational) attempt to come up with and apply a solution is the very root of the problem that the neurotic mind seeks to eradicate -- the attempt to fix the problem is the problem, not the problem itself. *[emphasis added]*

Relevance to strategic dilemmas

Notably with respect to the challenges of sustainability, at every level of society, the above argument raises the question as to whether the M-set offers a means of addressing the value-charged strategies that are currently so divisive in public debate. The nature of these strategic dilemmas was documented in relation to the concerns of the 1992 Earth Summit (*Configuring Strategic Dilemmas in Intersectoral Dialogue*, 1992). The possibility of configuring such dilemmas was explored, using the strength of opposition between polarized perspectives as a design feature, as a means of framing a new kind of space (*Configuring Conceptual Polarities in Questing: metaphoric pointers to self-reflexive coherence*, 2004). The work of Stafford Beer reflects this possibility, notably his articulation of the practice of adaptive "problem jostling", or "problem jostle", for team syntegrity. In this light, there is a case for exploring the relation between the complex dynamics of a spherical tensegrity structure and its fractal organization in seeking equilibrium (cf Donald E. Ingber. *Tensegrity II. How structural networks influence cellular information processing networks*, 2003).

From the perspective of management cybernetics, a key principle is that of the requisite complexity necessary for the management of a complex global society. This principle is known as Ashby's Law. As the most complex mathematical object known, and given the understanding of mathematics as the science of relationships, the M-set could therefore be understood as the most complex relational object that could prove to be a suitable candidate in that respect. There is otherwise the danger that "sustainability" will be sought -- and purportedly found -- at a level of abstraction at which it cannot be sustained.

It should not be forgotten that the principal management-related arena to which the fractal perspectives of chaos theory so far have been applied is that of the financial markets (see resources at Orlin Grabbe. *Chaos and Fractals in Financial Markets*, 2003; *Fractal Finance*; *Chaos, Fractal Theory in the Financial Markets*). Fractal equations are well-suited to the wildly random world of financial trading, where price fluctuations have been resistant to traditional mathematical models. This is also an application which Benoit Mandelbrot has himself explored (*The (mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward* by Benoit B. Mandelbrot and Richard L. Hudson, 2004).

There appears to be no reference to the relevance of the M-set to strategy in other strategic domains. With respect to the concept of sustainability, again research seems to have had the same preoccupation (Benoit B. Mandelbrot. *Fractal financial fluctuations; do they threaten sustainability?*). Joanne Tippett (*A Pattern Language of Sustainability Ecological design and Permaculture*, 1994) refers to Mandelbrot's work but does not elaborate its relevance.

But, with respect to city planning, for example, Verna Nel (*Complex Adaptive Systems and City Planning*) argues that:

> Globalisation implies that countries and cities are tangled in webs of connectivity. Simple reductionist explanations are no longer adequate. A new paradigm, appropriate for the new order is required. This paper argues that the metaphors of complex adaptive systems can meet the demands of city planning both in a global context and in South Africa today.

The M-set is however considered to be of much greater strategic relevance by Moshe S. Landsman (*Toward a Fractal Metaphor for Liberation of Palestinian Women*, 2001) in a discussion of stages of liberation where he considers that the fractal metaphor plays a significant part in both understanding the process and in planning intervention strategies. He considers that the fractal characteristics of M-set have at least a metaphorical potential for enhancing conceptualization of multilevel social processes. Among them are the following:

> 1. The level of intervention is not as important as the strategy. Since structural and functional changes will eventually interact and cause changes on other levels of the process, the interventor.. may begin at the level that is simply the most convenient...
>
> 2. As hinted above, the strategy of intervention, which enhances multilevel change, is one that fosters awareness of parallel processes above and below the level of intervention and attempts to address them as well. If they cannot be addressed at this particular point in the process, they should at least be monitored, as changes at one level can teach us lessons at another.
>
> 3. Not only are changes at one level portentous for another, but any phenomenon occurring at one level of intervention gives up potentially valuable information for understanding another. Therefore, when the interventor reaches an obstruction at the present level of intervention, observation of processes at other levels may often uncover ways of dealing with the obstruction.
>
> 4. The fact that fractals are borderline phenomena may foster conceptualization of strategies for social change. Non-fractal changes in the terrain may be an indication that we have achieved meaningful social change. On the other hand, meeting the same fractal terrain may tell us we are not out of the woods, and may be going around in circles.

This suggests that the M-set may have wider implications for social change in situations fraught with strategic dilemmas. How to understand sustainability under such dynamic conditions may be intimately related to the challenge of understanding the M-set. For example, if the multiplicity of conceptual models (through which the dynamics of change are envisaged) were to be understood as J-sets, and represented by them, what significance would be associated with the corresponding M-set -- and how would it be understood?

As noted by Julie Klein (above) with respect to interdisciplinary approaches to knowledge generation of relevance to complexity, incoherent advocacy of distinct models, as currently practiced, fragments strategic initiatives and isolates their proponents (cf *Dynamically Gated Conceptual Communities: emergent patterns of isolation within knowledge society*, 2004). There is clearly a case for reviewing such disjunction in terms of the language of attractors and repellors -- and event horizons.

The difficulty in seeking to apply strategies based on such models is that few of them take account of the existence of other models -- understood by their proponents as reflecting more adequately the priorities of alternative strategies. The existence of competing models engenders a dynamic in the dialogue relating to governance, especially at the global level. This dynamic is seldom based on rational discourse. In fact it is typically characterized by irrational argument and can usefully be described as constituting a nonlinear dynamic system.

The question is whether the M-set offers pointers to mapping the dynamic between alternative strategies. Given the structure of the M-set, this might then both distinguish and interrelate strategies that could be described (after an "iterative" succession of budgetary cycles) as characteristically:

- unconvergent and essentially incoherent (perhaps typical of short-term crisis management without any commitment to long-term coherence)
- convergent on a "fixed" perspective (perhaps typical of faith-based and for-profit strategies)
- periodic or alternating (perhaps typical of those emergent in democracies where governance alternates between the policies of distinct coalitions, or periodic alternation between centralization and decentralization, for example)
- "chaotic" (perhaps typical of strategic nimbleness, highly adaptive strategies, and what is caricatured as "bumbling through")

The argument above suggests that it is precisely the dynamic between the "real" and the "imaginary" dimensions of such strategies that engenders the dynamic stability mapped by the M-set. The question is whether these dimensions satisfactorily hold the "reality" of radically opposed proponents in the light of the "imaginary" characteristics that they attribute to those whom they oppose (or by whom they are opposed).

Such concerns have been fundamental to initiatives to profile the many thousands of problems, strategies and values of international constituencies (*Simulating a Global Brain: using networks of international organizations, world problems, strategies, and values*, 2001). In particular such profiles endeavoured to capture understandings of the problem (or the strategy) as "real" -- in contrast with "imaginary" or in some way misleading. It is such differences in perspective that are the fundamental drivers of global dialogue.

In mapping dynamics onto a complex plane, the M-set suggests the value of mapping the dynamics between the complete range of human activities onto such a surface. This contrasts with current approaches -- even when based on mapping those activities onto what amounts to a generalization of the periodic table (*Functional Classification in an Integrative Matrix of Human Preoccupations*, 1982; *Alternation between Development Modes: reinforcing dynamic conception through functional classification of international organizations and concerns*, 1982). This was partially inspired by Edward Haskell's original work (discussed earlier).

Enhancing insight through audio-visual techniques

Seismologists, meteorologists, economists, chemists, hydrologists, and every kind of engineer, have been confronted with visual patterns that were more elegant than predictable. There is a trap to the graphical representation of the M-set and enthusiastic explorations of the strange imagery. This is partly indicated by the much-cited statement of Alfred Korzybski (*Science and Sanity - an introduction to non-aristotelean systems and general semantics*, 1933): The map is not the territory [more]. In distinguishing the M-set from J-sets, this phrase is especially apt.

The M-set is above all significant to individual and collective navigation of a complex reality -- to the extent that it can be embodied in a meaningful way (*Navigating Alternative Conceptual Realities: clues to the dynamics of enacting new paradigms through movement*, 2002). How such "embodiment" is achieved is a preoccupation of many of the symbol systems and disciplines referenced above. It also reflects the concerns explored by enactivism (notably that of Francisco Varela et al. *The Embodied Mind: Cognitive Science and Human Experience*, 1991).

At its simplest, "looking at" the M-set representation sets up a knower-known polarity without seeking to reframe the associated dynamics. The point is well made in the Chinese tradition by the Ba Gua Mirror (see above). This uses the 8 complementary trigrams to frame a mirror in which the observer is confronted with the real challenge to understanding -- the Delphic "know thyself".

"Looking at" should be challenged by "sensing the geometry" as discussed earlier. One approach to this is through shifting to a 3D representation of the M-set (cf Ralph Abraham. *Complex Quadric Dynamics: A Study of the Mandelbrot and Julia Sets,* 2001). Another approach is by understanding the use of the facilities of M-set fractal browsers as cognitive operations in their own right (cf (*Computer Use as Philosophy in Operation: Metaphors of the Inner Game*, 2003).

It is also useful to recognize the extent to which strategic thinking is trapped into the linearity of textual explanations and verbal discourse -- even though it may be endeavouring to encompass nonlinear dynamic phenomena. One response is to animate representations of institutions and their programmes (cf *Animating the Representation of Europe*, 2004). A more integrative approach is however called for -- justifying exploration of the potentials of the M-set.

A quite different approach is through sonification of the M-set, notably the production of "Mandelbrot music". For example, David Spondike (*Hearing the Mandelbrot Set*) presents an experiment in sonifying the M-set noting:

> There are many ways in which the M-set can be mapped to sound. These examples demonstrate but one. The purpose here is two-fold. 1. To demonstrate the feasibility of sonifying (as opposed to visualizing) scientific data, and 2. To demonstrate the possibility of finding musical structures in the M-set. To compose music implies creative manipulation. This "composition" was kept to a minimum in these examples, perhaps lessening the musical qualities of the examples, but in an effort to preserve objectivity toward the scientific data.

There are a number of other experiments in giving musical form to the M-set [more | more]. Related arguments are presented with regard to giving musical form to traditional Chinese conceptual coding systems (*Musical Articulation of Pattern of Tao Te Ching Insights: Experimental sonification based on magic square organization*, 2003).

The case for sonification of scientific data has been articulated by the US National Science Foundation and the International Community for Auditory Display (*Sonification Report: Status of the Field and Research Agenda*, 1997). For example, Davide Rocchesso (*Audio effects to enhance spatial information displays*, 2002) has given consideration to use of sonification in representing the complex plane. There are many experiments in relation to "fractal music" [cf Fractal Music Lab].

It is worth reflecting on the role of overtone chanting, notably in Tibetan Buddhism, as a means of articulating higher forms of order [more]. There are extensive web references by those concerned with the possibility of extraterrestrial intelligence (notably in the SETI community) to the role of M-set representations -- especially in the light of crop circles purportedly of that form [more | more | more]. Clearly the ability of a civilization to recognize the full significance of the M-set could be considered an "entry qualification" for such communication (see *Communicating with Aliens: the Psychological Dimension of Dialogue*, 2000).

As noted in an earlier paper (*Enhancing the Quality of Knowing through Integration of East-West metaphors*, 2000), the embodiment of knowledge, in the light of the chanted hymns of the *Rg Veda,* has been explored by Antonio de Nicolas (*Meditations through the Rg Veda*, 1978), using the non-Boolean logic of quantum mechanics (Heelan, 1974). The unique feature of the approach is that it is grounded in tone and the shifting relationships between tone; it is through the pattern of musical tones that the significance of the *Rg Veda* is to be found. As de Nicolas indicates:

> "Therefore, from a linguistic and cultural perspective, we have to be aware that we are dealing with a language where tonal and arithmetical relations establish the epistemological invariances... Language grounded in music is grounded thereby on context dependency; any tone can have any possible relation to other tones, and the shift from one tone to another, which alone makes melody possible, is a shift in perspective which the singer himself embodies. Any perspective (tone) must be "sacrificed" for a new one to come into being; the song is a radical activity which requires innovation while maintaining continuity, and the "world" is the creation of the singer, who shares its dimensions with the song." (p. 57)

There is a case for exploring how this classical perspective is to be combined with that of enactivism in the light of possible articulation of experiential dynamics through the M-set.

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