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Periodic Pattern of Human Knowing

implication of the Periodic Table as metaphor of elementary order

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Introduction

The focus here is on the possible psychosocial implications of any new global understanding of the Periodic Table of Chemical Elements (originally formulated by Dmitri Mendeleev in 1869) in relation to other global frameworks. It is another approach to an argument initially developed in *Periodic Pattern of Human Life: the Periodic Table as a metaphor of lifelong learning* (2009). The Periodic Table is especially significant in that it is considered to be one of the most comprehensive generalizations of science.

Of particular interest is the evolving understanding of the chemical elements and the Periodic Table as recently articulated (Denis H. Rouvray et al. *The Mathematics of the Periodic Table*, 2005), notably in the light of the:

- continuing recognition of the central and fundamental role of elements and their organization
- processes of science in advancing knowledge and superceding deprecated simplistic formulations (possibly of educational value)
- application of a variety of developing mathematical tools to modelling elements and their periodicity
- much deeper probabilistic understanding of the nature of "elements", notably in terms of quantum machanics
- increasing abstruseness, even incommunicability, of emerging insights
- acknowledgement that complete understanding of elements and their periodicity remains elusive

The concern here is not with such patterns of numerical order in their own right, nor for their biological significance, but with what they might imply for the relationships between seemingly disparate modes of cognition. This is increasingly justified by recognition of the role of metaphor in mathematical creativity -- together with challenging questions as to the degree to which mathematics (notably as applied to the Periodic Table) is itself to be considered as a metaphor.

Of particular relevance is the evolving understanding of an "element" -- due to new possibilities of distinguishing it within mathematical abstractions -- and consequently its relationship to distinction of any fundamental "category". In contrast with past assumptions regarding the concreteness of "chemical elements", there is a shift from assertion of the nature of the reality constituted by "elements" to hypothesizing their nature in terms of new abstractions of ever more generic insights. This occurs in a period in which the cognitive role of metaphor in relation to mathematical understanding is of increasing significance.

With respect to pattern, the concern here follows from earlier explorations (*Representation, Comprehension and Communication of Sets: the Role of Number*, 1978; *Patterns of N-foldness; comparison of integrated multi-set concept schemes as forms of presentation*, 1984; *Examples of Integrated, Multi-set Concept Schemes*, 1984). The question here is not the validity or status of specific initiatives (discussed below), but rather what such forms of order might in future imply for psychosocial organization and ways of knowing. This concern also follows from an earlier exploration (*Navigating Alternative Conceptual Realities: clues to the dynamics of enacting new paradigms through movement*, 2002).

Although such frameworks are seemingly quite distinct, even the preoccupation of quite distinct disciplines, the argument here is that there is every possibility, notably in the light of the role of isomorphism in general systems theory, that some kind of relationship is to be discovered between them. Just as the elements in a Periodic Table are distinct, there is an underlying pattern relating their structure. Is it more probable -- after many millions of years -- that present patterns of cognition ("ways of knowing") would rely on forms of which there is no past trace, or rather that those patterns would be conditioned by those of the past -- of which they would then be new instances in some way?

More fundamentally the question may be how we think about what we distinguish and the ordering of it that we consider appropriate. Formally this relates to issues arising from the calculus of indications as initiated by G. Spencer-Brown (*Laws of Form*, 1969), otherwise known as boundary algebra.

Whilst the "elements" and their periodic organization are increasingly presented by disciplines using very sophisticated and widely incomprehensible methodologies, it is vital to recall that a unique working comprehension of them by everbody is effectively fundamental to the biological processes of their daily life. In this sense humans operate out of a profound understanding of the "elements" and their periodic order at every moment of their lives. That understanding might be said to be fundamental to sustainability of human life.

As explored here, the disconnect between such understandings of such a periodic pattern plays itself out in:

- deprecation by specialists of past understandings, or understandings that are incompatible with their currently favoured methodologies
- what remains incomprehensible or inexplicable, namely how life is ordered within a context characterized by a degree of ignorance
- ignorance of how science of the future will understand that pattern and its systemic implications, regretting the oversimplistc framings of the present
- failure to derive valuable insights of relevance to ordering the disparate "elements" of cognitive experience and social organization
- opportunitistic use of metaphors based on partial ("ridiculous") understanding of such periodic patterning, and deprecation of such use

The exploration was triggered by: the seeming lack of relationship between proposals for numerical solutions describing the fundamental and atomic number sequences of the periodic table; other mathematical explorations of their periodicity; proposals for a periodic table of mathematics; the fundamental role played by the Gaussian copula with respect to the financial crisis of 2008-2009; the fundamental importance attributed to symmetry group discoveries; and the continuing quest for a Theory of Everything. The question implied by each such approach to a "comprehensive" framework is how might any such framework affect cognition, especially if there is any implication that "coherence" calls for a cognitive relationship between them -- namely some kind of Rosetta stone, with the integrative comprehension that itself implies -- and from which it originates in some way. The challenge would be all the greater if such frameworks were held to be of profound significance in alleviating uncertainty as some form of ultimate explanation or solution.

Periodic Table -- as instance of a pattern

The Periodic Table is therefore understood here as the instance of a pattern and, as such, is an indication of what might be comprehended as a periodic pattern of life. But the psychosocial dynamics of how such a pattern is intuited, recognized and apprehended is as much a part of the preoccupation.

However the concern here is less with how any such formulation is held to be true in some way by universal consensus (if only amongst specialists). Rather, given their challenge to average comprehension, it is with the interrelationship between:

- the nature of any pursuit of an integrative dream
- ways of attributing or claiming coherence
- the tendency to attribute greater coherence and validity to what may only be "potentially" or "partially" true, notably the coherence inferred by potentially inappropriate comprehension of particular formulations and their limitations
- what a comprehensible Rosetta stone might look like, especially if it holds the relationship between different ways of knowing
- the number of ways of knowing that can be brought into play to comprehend it, given the incommensurability of their respective perspectives and the probable relevance of their complementarity
- how the discontinuities between such ways of knowing are to be held, especially if some are deprecated
- the extent to which particular formulations exemplify or obscure such distinctions
- the significance of the special claims made for particular ways of knowing, especially by their formulators
- the extent to which comprehended coherence, engaging the human mind, is conditioned by characteristics of the human mind
- the extent to which the ways of knowing, and the emergent formulations, hold (if only implicitly) any of these cognitive challenges, namely some degree of self-reflexivity
- the emergent understanding of order through progressive learning about the pattern as a whole -- namely the "overtones" -- and what they signify

Briefly the issue is with how one engages with the complexity of abstract formulations -- beyond one's capacity -- especially when that complexity purportedly holds a higher degree of order and significance, whose integrity one can only partially intuit, if at all. This is a challenge in a context in which those associated with any such formulations often have their own peculiar and seemingly dysfunctional dynamics. This concern follows from earlier involvement in two complementary projects within the framework of the *Encyclopedia of*

World Problems and Human Potential whose subject matter was organized in a periodic pattern explicitly inspired by the Periodic Table (see *Functional Classification in an Integrative Matrix of Human Preoccupations*, 1982). The complementary projects of relevance here, each with an integrative focus, were: Human Development Project and Integrative Knowledge Project.

Of particular interest, given the challenge of representing higher order patterns, is how to talk about them without rendering them even more meaningless to many. In this sense the relationship between the processes whereby such understanding is attempted are themselves indicative.

Moving axes of comprehension and communication

The challenge of the previous paragraph was the theme of earlier explorations (*In Quest of Mnemonic Catalysts -- for comprehension of complex psychosocial dynamics*, 2007; *Conditions of Objective, Subjective and Embodied Cognition: mnemonic systems for memetic coding of complexity*, 2007; *Comprehension of Appropriateness*, 1986). The first notably endeavoured to map the associated dynamics onto a single diagram (*Imagining the Real Challenge and Realizing the Imaginal Pathway of Sustainable Transformation*, 2007). The assumption is that there is a need for a degree of self-consciousness and self-reflexivity in advancing and endeavouring to comprehend formulations that claim to be of a more integrative order.

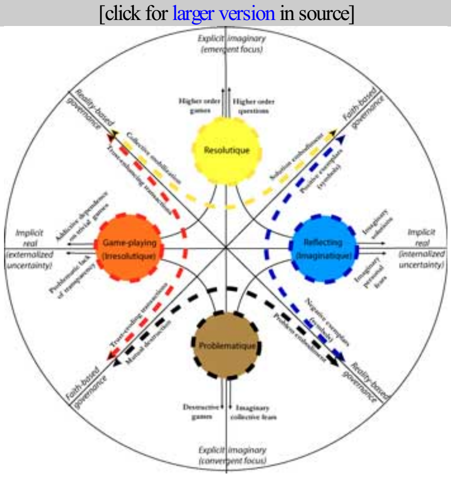
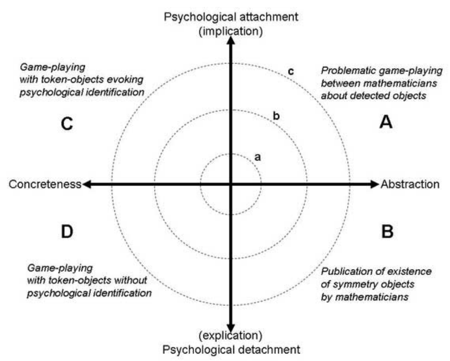
A further exercise, in anticipation of the challenge of the formlations highlighted below, is to consider the following "psychosocial" processes which typically are irrelevant to the composition of such formulations from a mathematical perspective (in contrast with judgements on the quality of the mathematics) -- and deprecated in any mathematical lexicon. Terms commencing with "co-" are used to offer a provocative mnemonic set.:

- comprehension
- consensus
- cooperation (collusion)
- correspondence
- configuration (composition)
- concentration

- communication
- connectivity
- consciousness
- complexity
- congestion (eg information overload)
- condescension

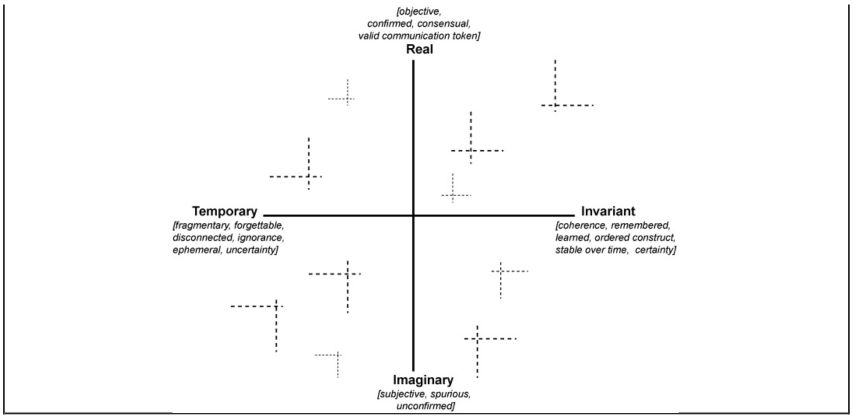
- competition
- coincidence
- copyright
- coherence
- co-arising
- controversy

In earlier explorations the following figures were used to present the interrelationships between conventional categories and processes and those which were notably characteristic of lived reality. (*Dynamics of Symmetry Group Theorizing: comprehension of psycho-social implication*, 2008)

Fig. 1: Interrelating problematique, resolutique, "imaginatique" and "irresolutique" [click for larger version in source]	**Fig. 2: Indication of relationship between dimensions relating to engagement with symmetry** [click for larger version in source]
	

Rather than use the real and imaginary axes of the complex plane as in the figures above, **in the figure below** they are treated as extremes on the same axis, with a second axis based on temporary/invariant. This allows any approach (of an individual or a group) to challenging formulations to be positioned within the arena -- as illustrated by the dashed smaller diagrams ("crosshairs" of identity) -- each structured to reflect its position in the arena. Learning, as collectively recognized, then involves movement into the top right quadrant. Much daily subjective experience is in the bottom left quadrant. More interesting is that the dashed constructs are not static but may move around the arena, alternating between various positions -- with the representative construct adjusting accordingly. The challenge with any definitive formulation (of the type identified below) is that it is a characteristic outcome of processes in the top right quadrant. The "co-" processes (above) are characteristic of the other quadrants, notably in imagining (intuiting) their possibility or recognizing one's ignorance when faced with learning about them.

Fig. 3: Mapping the comprehension/communication challenge



Formulations characteristic of the top right quadrant are effectively set in "cognitive stone". In navigating towards them through a learning process, delicate associations of connectivity are imagined between phenomena. These may be understood as "pathways" but possibly best described as "elven pathways" negotiable only by the light and swift of cognitive foot, as discussed elsewhere (*Walking Elven Pathways: enactivating the pattern that connects*, 2006; *Climbing Elven Stairways: DNA as a macroscopic metaphor of polarized psychodynamics*, 2007).

The most remarkable tale regarding such unconventional connectivity is that relating to the discovery of the Monster Group of symmetry, a feature of the discussion below (Mark Ronan, *Symmetry and the Monster: one of the greatest quests of mathematics*, 2006). Curiously there are many mathematical papers explaining the associated theory of "monstrous moonshine", as discussed separately (*Potential Psychosocial Significance of Monstrous Moonshine: an exceptional form of symmetry as a Rosetta stone for cognitive frameworks*, 2007).

The issue is what degree of connectivity is necessary to move into the top right quadrant, and are other degrees of connectivity appropriate for other purposes -- justifying any reluctance to aspire to move there. Part of the response is to be found in the nature of meaningful "correspondences" -- an issue at the heart of the controversial "moonshine", as discussed separately (*Theories of Correspondences -- and potential equivalences between them in correlative thinking*, 2007). One interesting formulation of the challenge of appropriate connectivity is the famed 10 ox-herding images of Zen Buddhism, as discussed elsewhere (*Progressive integration of the shadow of non-self-reflexivity*, 2007). In such terms, at its simplest perhaps, should formulations (as with those discussed below) be progressively interrelated, namely:

- be variously framed and separately positioned (like icons) on the walls of one's cognitive domain?
- should there be some design to their placement, with some suggestion of complementarity or sequence?
- should their placement configure (or imply) some larger cognitive whole, together functioning as a form of cognitive antenna?
- is there some sense in which progressive engagement with them is personally transformative (as attributed to mandalas)?
- etc

Interweaving threads: a pattern that connects?

There are indeed traces of connectivity between the formulations that are more readily comprehensible. However these traces are most evident in a language that is (necessarily) highly abstract. In the spirit of the argument above the question is the psychodynamic challenge constituted by such abstraction.-- and the accessibility that it precludes at a time when it might be said that there is a desperate need for higher orders of connectivity.

David Corfield (*Mathematical Kinds, or Being Kind to Mathematics*, *Philosophica*, 2004) makes the point clearly:

> In the case of the philosophy of science, for instance, one may either be a reductionist and take a few physical properties and their laws as fundamental or natural, or else think that some chemical or biological properties are not reducible, and so argue for a disparate array of local laws. The harder step is to attempt to patch together these local studies.... Towards the end of the century, a few were beginning to realise that set theoretic reductionism ignores distinctions between specific kinds of reasoning and went in search of local particularity...but in selecting local studies, and later in fitting them together, we needed to be guided by a larger conception of mathematics. **Here the huge obstacle looms: we have no overview of mathematics as a whole**. *[emphasis added]*

With regard to this challenge, Corfield cites an insight of Vladimir Arnol'd:

> One... characteristic of the Russian mathematical tradition is the tendency to regard all of mathematics as one living organism. In the West it is quite possible to be an expert in mathematics modulo 5, knowing nothing about mathematics modulo 7. One's breadth is regarded as negative in the West to the same extent as one's narrowness is regarded as unacceptable in Russia. (1997)

and (indicative of what mathematicians are conditioned to consider as relevant), Arnol'd also notes:

> All mathematics is divided into three parts: cryptography (paid for by CIA, KGB and the like), hydrodynamics (supported by manufacturers of atomic submarines) and celestial mechanics (financed by military and by other institutions dealing with

> missiles, such as NASA. (1999)

Corfield indicates that towards the end of the twentieth century several mathematicians proposed overarching schemes to organize the facts they considered most significant. He briefly discuss three of these schemes (those of Vladimir Arnol'd, Michael Atiyah, and of John Baez with James Dolan), before drawing some philosophical consequences from their attempts. He argues that we are dealing here with a more open-ended sense of conceptual growth. He illustrates his theme by discussing the elaboration of algebraic structures designed to measure symmetry.

Corfield worries about the (above-mentioned) ignorance that presents a "huge obstacle" for a philosophy of mathematics sensitive to what mathematicians have discovered. However he notes:

> Where a philosopher of physics will be able to offer you some kind of sketch of the whole domain of physics, for most outside of mathematics, including most philosophers of mathematics, there is little more to work with than the idea gained from the generic view that everything is expressible set theoretically.

His succinct review of the organization schemes leads him to conclude:

> So we have a series of interrelated schemes covering large tracts of mathematics, but what to make of them? There is a temptation to relate these forms of mathematical classification to types of classification in other sciences: physical particles, periodic table of chemical elements, animal species.... We saw several organisational schemes in mathematics from... **Are they pointing to some ultimate classification**, or are there a vast range of schemes shaped by our interests. For instance, computational complexity classes may be seen as not intrinsic characteristics, but as part of a classification due to an interest in what we can compute with our present kinds of technology. *[empahsis added]*

Given the concern expressed above, what is unsatisfactory about his concern is that it seemingly excludes any reference to the experiential implications of mathematics and classification -- even though he refers readers to his more extensive philosophical treatment (*Towards a Philosophy of Real Mathematics*, 2003), and he himself has a professional involvement with psychology. Whatever the capacity of expression "set theoretically", this is not indicative of comprehensibility or experiential relevance.

Mathematical knowledge management

Corfield's expression of concern has been powerfully reinforced by the challenge of managing the volume of mathematical knowledge as articulated in the keynote speech to a Mathematical Knowledge Management meeting in 2003 by Michiel Hazewinkel (*Mathematical knowledge management is needed*) who notes:

- total number of pages published in mathematics so far makes up a stack of about 60km height.
- how much redundancy there is in all that is anybody's guess; at present there not even preliminary ideas how to estimate that.
- the universally used *Mathematics Subject Classification* scheme is a tree with some 5500 leaves; most of these are large enough to drown a whole (super)specialism in.
- there were some 25,000 mathematical articles published in 2001.
- the number of new theorems (as defined by their authors) published per year has been estimated at 200,000, this does not include lemmas, propositions, scholia, constructions, definitions, etc

Hazewinkel's framing of the situation is that: "*We don't even know how much we know that we don't know we know*"?? This is reminscent of the preoccupation of the notorious "poem" of Donald Rumsfeld, as discussed separately (*Unknown Undoing: challenge of incomprehensibility of systemic neglect*, 2008).

The meeting was associated with the Mathematical Knowledge Management Network, funded for a year as a project under the Knowledge Technologies action line of the European Union's Fifth Framework. That event has been associated with a series of conferences (Austria, 2001; Italy, 2003; Poland, 2004; Germany, 2005; UK, 2005; Austria, 2007; UK, 2008) assembling valuable contributions. However it is difficult to detect much preoccupation with any overarching "overview" (as stressed by Corfield) as opposed to improving "access".

- James Harold Davenport and Bruno Buchberger (Eds.). Mathematical knowledge management: second international conference (MKM 2003, Bertinoro, Italy, 2003). Springer, 2003
- Andréa Asperti, Grzegorz Bancerek, Andrzej Trybulec. Mathematical knowledge management: third international conference, MKM 2004 ... Mathematical knowledge management: third international conference, MKM 2004. Springer, 2004

A MKM Interest Group (Mathematical Knowledge Management) has since been created as a loose network to focus on the intersection of mathematics and computer science. Specifically it focuses on the need for efficient, new techniques - based on sophisticated formal mathematics and software technology - for deriving benefit from the enormous knowledge available in current mathematical sources and for organizing that knowledge in a new way. On the other side, due its very nature, the realm of mathematical information would seem to be the best candidate for testing innovative theoretical and technological solutions for content-based systems, interoperability, management of machine understandable information, and the Semantic Web.

The challenge is also recognized, and variously addressed, as for example:

- a proposal submitted to the NSF Cyber-enabled Discovery and Innovation (CDI) Program (Jim Pitman, et al., *Bibliographic*

Knowledge Network, 2008)
- a proposal for for a wiki for mathematical knowledge by Christoph Lange (*A Semantic Wiki for Mathematical Knowledge Management*, 2006; *SWIM : A Semantic Wiki for Mathematical Knowledge Management*, 2007).
- the Open Archives Initiative, as explained by Antonella De Robbio and Dario Maguolo (*Mathematics Subject Classification and related schemes in the OAI framework*, 2002)

With respect to the argument here, however, **there seems to be little concerted effort to focus on the order implied by the field of mathematical knowledge as a whole**, as previously considered (*Is the House of Mathematics in Order? -- are there vital insights from its design*, 2000). One such attempt by Dave Rusin (*The Mathematical Atlas: a gateway to modern mathematics*, 2000) does indeed offer a modest map providing links into the *Mathematics Subject Classification* (MSC), discussed below. This offers a visual index to the subfields of mathematics. The question is whether more is possible -- benefitting from the sophisticated understandings of mathematics.

Missing "map" of mathematics: a self-reflexive "periodic table"?

Despite such recent progress, and in the light of Corfield's earlier conclusion, **a potentially interesting question is why there is seemingly no mind map -- even a crude one -- showing the entailment of the different mathematical approaches, and notably to any Periodic Table** (given its potentially focal status). In how many innovative ways might such a map be represented and visualized (as previously illustrated)? For example, Guillermo Restrepo and Leonardo Pachón (*Mathematical Aspects of the Periodic Law*. *Foundations of Chemistry*, 2007) indicate:

> Mathematics employed to study the periodic system includes number theory, information theory, order theory, set theory and topology. Each theory used shows that it is possible to provide the Periodic Law with a mathematical structure. We also show that it is possible to study the chemical elements taking advantage of their phenomenological properties, and that it is not always necessary to reduce the concept of chemical elements to the quantum atomic concept to be able to find interpretations for the Periodic Law. Finally, a connection is noted between the lengths of the periods of the Periodic Law and the philosophical Pythagorean doctrine.

Albert Khazan (*Upper Limit in Mendeleev's Periodic Table: Element No.155*. *Svenska Fysikarkivet*, 2009) focuses on a hyperbolic law whereby the content of an element in different chemical compounds can be described by the equation of an equilateral hyperbola.

Alain Connes (*Noncommutative Geometry*, 1994) argues with respect to the successful classification of the elements in the periodic table:

> The theoretical explanation of this classification, by Schrödinger's equation and Pauli's exclusion principle, is an equivalent success of physics in the 20th century, and, more precisely, of quantum mechanics. One can look at this theory from very diverse points of view. With Planck, it has its origins in thermodynamics and manifests itself in the discretization of the energy levels of oscillators. With Bohr, it is the discretization of angular momentum. For de Broglie and Schrödinger, it is the wave nature of matter. These diverse points of view are all corollaries of that of Heisenberg: physical quantities are governed by noncommutative algebra.

A. John Coleman (*Groups and Physics: Dogmatic Opinions of a Senior Citizen*, *Notices of the AMS*, 1997) notes that the hamiltonian of an atom is invariant with respect to the rotation group in 3-space. A knowledge of the possible dimensions of irreducible representations of SO(3), together with the implications of the Pauli Principle, leads again to an explanation of the periodic table of the elements. He stresses that: *It was variations on these themes involving quite delicate mathematics which filled the early papers and books on group theory and quantum mechanics...*

The comprehensive review compiled by Denis H. Rouvray and R. Bruce King (*The Mathematics of the Periodic Table*, 2005) includes twelve presentations highlighting often-neglected mathematical features of the Periodic Table and several closely related topics. It considers predictions of what the ultimate size of the Periodic Table will be (D. H. Rouvray, *The Ultimate Size of the Periodic Table*) and examines the nature of its periodicity (N. N. Khramov et al., *Statistical Modeling of Chemical Periodicity and Prediction of the Properties of the Superheavy Elements*; P. G. Mezey, *Syncopated Periodicity of Atoms in Molecules*). The nature of such a table is next considered in dimensions other than two (H. Hosoya et al., *n-Dimensional Periodic Tables of the Elements*) . The natural clustering of the elements into groups is then considered by three different but complementary routes:

- the topological structures of the groups (G. Restrepo et al., *Topological Study of the Periodic)*,
- the self-association of the elements as evidenced by neural network studies (J. Fayos, *Self-Associative Periodic Table of Elements by Neural Networks*),
- information theoretical analysis of the behavior of atoms (G. P. Shpenkov, *An Elucidation of the Nature of the Periodic Law*; D. Bonchev, *Periodicity of the Chemical Elements and Nuclides: An Information-Theoretic Analysis*).

Following a detailed investigation of the mathematical basis for the periodicity seen in atomic and molecular spectroscopy (K. Balasubramanian, *Mathematical Basis of Periodicity in Atomic and Molecular Spectroscopy*), three separate presentations delve into many different aspects of the group-theoretical structure of the Periodic Table:

- O. Novaro, *Group Theory of the Periodic Table*
- M. R. Kibler, *A Group-Theoretical Approach to the Periodic Table: Old and New Developments*
- V. N. Ostrovsky, *Group Theory Applied to the Periodic Table of the Elements*)

These features of a possible entailment map are necessarily narrowly focused on the Periodic Table and do not appear to take account of:

- the manner in which some of the mathematical tools applied are themselves part of a much larger array of tools. In this sense their application to the Periodic Table is the exploration of a concrete instance where more abstract and generic cases may be of greater theoretical interest, notably in the case of group theory. The question is then to what degree these generic insights have any implication for understanding of the Periodic Table, especially to any Periodic Pattern of Human Life
- the manner in which the Periodic Table has inspired the quest for such tables in a diversity of other arenas, in many of which mathematics may be important, although possibly of little theoretical interest to mathematicians (see *Related explorations and precedents*). The question is then what special insights into periodicity emerge from these other contexts which might entail other mathematical approaches. It is perhaps appropriate to note the recent popularity of the periodic table as a metaphor to order understanding (*Periodic Table of Metaphors*; *Elements of Information Control*; *Periodic Table of Vulgarity* -- including some which are interactive *Periodic Table of the Internet*; *Periodic Table of Visualization Methods*; and Michael Swanwick's *Periodic Table of Science Fiction*

Both points are of course relevant to the psychosocial implications of such periodicity as considered here. **It is naturally tempting to consider that a more generic Periodic Table -- or one of psychosocial relevance -- might be elegantly self-reflexive in that the entailment map might be fruitfully structured as a Periodic Table -- of ways of knowing and understanding a Periodic Table.**

Origin of mathematics and the periodic table -- in human cognition?

The question of the respect in which "mathematics" is independent of the human mind is an old one. It is a question which many mathematicians consider to be irrelevant to their interests. For others the order discovered by mathematics is simply an exemplification of some understanding of the divine order within which human cognition has emerged.

For example, Reuben Hersh (*What is Mathematics, Really?* 1997) notes that most philosophers of mathematics treat it as isolated, timeless, ahistorical, inhuman. Hersh argues the contrary, that mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context. He reveals mathematics as seen by professionals, debunking many mathematical myths, and demonstrating how the "humanist" idea of the nature of mathematics more closely resembles how mathematicians actually work. This follows from his earlier collaborative investigation (Philip J. Davis and Reuben Hersh, *The Mathematical Experience*, 1981).

More recently Hersh has collected disparate essays on understandings of mathematics (*18 Unconventional Essays on the Nature of Mathematics*, 2005) that are tackling, from various points of view, the problem of giving an accounting of the nature, purpose, and justification of real mathematical practice -- mathematics as actually done by real live mathematicians. His concern is with the the nature of the objects being studied and what determines the directions and styles in which mathematics progresses (or, perhaps, degenerates).

The focus of several more recent studies (some cited by Hersh) indicates the challenge for the philosophy of mathematics at the crossroads of two schools of thought. On one side are the old school mathematicians who see mathematics as a foundation of science. On the other side is a small but growing group of scholars made up of cognitive psychologists, linguists, and neural biologists (and some mathematicians as well) who see mathematics as a function of the brain (John D. Barrow, *Pi in the Sky: Counting, Thinking, and Being*, 1994; Brian Butterworth, *What Counts: how every brain is hardwired for math*, 1999; Keith Devlin, *The Math Gene: how mathematical thinking evolved and why numbers are like gossip*, 2001; George Lakoff and Rafael Núñez, *Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2000).

These new neurobiological / linguistic / cognitive theories promise help in understanding how mathematics is learnt and comprehended. (Ironically the provocative list of "co-" terms above highlights a sense in which mathematics is "born" from the matrix of a "space" that they contribute to defining, as the problematic history of the treatment of mathematical innovators by their peers shows only too clearly)

In particular George Lakoff and Rafael Núñez advocate a cognitive idea analysis of mathematics in terms of the human experiences, metaphors, generalizations, and other cognitive mechanisms giving rise to those ideas. Idea analysis is distinct from mathematics and cannot be performed by mathematicians unless they are trained in cognitive science. They are mainly concerned with proposing and establishing an alternative view of mathematics, one grounding the field in the realities of human biology and experience.

This emphasis accords with the interest in the cognitive status of a Periodic Table as a metaphor of a Periodic Pattern of Human Life. However **the emphasis here is on the manner in which such a periodic table incorporates or embodies the challenges of learning. This implies a degree of self-reflexivity. There is an implication of one in the other.**

Categorification, classification and knowledge management

As further indication of the array of methodologies applied to a "periodic table", John Baez (*Computation and the Periodic Table*, 2008). points out that:

> By now there is an extensive network of interlocking analogies between physics, topology, logic and computer science, which can be seen most easily by comparing the roles that symmetric monoidal closed categories play in each subject. However, symmetric monoidal categories are just the n = 1, k = 3 entry of a hypothesized "periodic table" of k-tuply monoidal n-categories. This raises the question of how these analogies extend.

The work of John Baez and James Dolan introduced the **periodic table of mathematics** in 1995. This identifies k-tuply monoidal n-categories and is said to mirror the table of homotopy groups of the spheres. These describe how spheres of various dimensions can wrap around each other. They are examples of topological invariants, which reflect, in algebraic terms, the structure of spheres viewed as topological spaces (ignoring their precise geometry). Unlike homology groups, which are also topological invariants, the homotopy groups are surprisingly complex and difficult to compute.

Care seems to be necessary in determing whether such explorations do indeed treat the Periodic Table of Elements as a specific instance -- even an extremely trivial instance -- of far more generic considerations, or whether such excitingly integrative possibilities are themselves an instance of a total disconnection from the reality which people are called upon to comprehend and with which they are expected to deal. There does not seem to be any extant explicit bridge between the two understandings of "periodic table". In particular, given the cognitive argument above, it is unclear how such abstractions are a product of human cognition and constrained by it -- and how this might be reflected in any periodic table of ways of knowing explicated or exemplified by mathematical categories and specializations.

More provocatively, to the extent that such abstraction approaches (if only asymptotically) the highest degrees of generality, it might be asked to what degree such wholiness is consistent with the omniscience of a Theory of Everything. Has divinity abandoned humanity?

Assuming a generic understanding of a periodic table (inclusive of the particular case, even if only potentially so), associated with that framing is the development of categorification, an interest which Baez shares with Corfield. They are key figures, with physicist Urs Schreiber, in the instigation of The n-Category Café, a physics / mathematics / philosophy blog. Categorification refers to the process of replacing set-theoretic theorems by category-theoretic analogues. When done successfully, it replaces sets by categories, functions by functors, and equations by natural isomorphisms of functors satisfying additional properties. It might be described as the process of identiying isomorphisms with more abstract (and typically generic) formulations. (John Baez and James Dolan, *Categorification*, 1998; David Corfield, *Categorification as a Heuristic Device*, 2005). Seemingly the focus on higher order categories has shifted away from any periodic table of categories or of mathematics (Eugenia Cheng and Aaron Lauda, *Higher-Dimensional Categories: an illustrated guide book*, 2004).

In its pursuit of higher order categories, categorification needs of course to be seen against decades of research, notably by the documentation sciences, into classification theory in all its flavours (library classification, taxonomic classification, scientific classification of organisms, classification in machine learning, statistical classification, document classification, classification theorems in mathematics). This is of course intimately related to categorization as the process by which ideas and objects are recognized, differentiated and understood, through grouping them into categories. Both are a proccupation of knowledge organization, notably as framed by the International Society for Knowledge Organization and its journal *Knowledge Organization: International Journal devoted to Concept Theory, Classification, Indexing and Knowledge Representation* 1973-), recognizing that there exist different historical and theoretical approaches to, and theories about, organizing kowledge, which are related to different views of knowledge, cognition, language, and social organization.

Categorification and the periodic table of categories

Within this framework, the articulation of John Baez (*The Dimensional Ladder*, 2005) regarding the "Periodic Table" (of n-categories) in the light of a "Ladder of n-Categories" is of particular interest, notably with respect to the relationships to physics (see also John Baez, *Categories, Quantization, and Much More*, 2006). .

The appropriateness of the "ladder" metaphor has of course been challenged from some perspectives, notably by feminists. The same might be said of the "table" metaphor, as discussed separately (*Comprehension of Requisite Variety for Sustainable Psychosocial Dynamics: transforming a matrix classification onto intertwined tori,* 2006). For example, Alison Bailey (*Locating Traitorous Identities: toward a view of privilege-cognizant white character*, In: Deborah Orr, et al. Feminist Politics, 2007) uses language that could appropriately inform a richer understanding of a Periodic Table of Life:

> Not a climb up the ladder, but a discarding of the hierarchies and rigidites implicit in the ladder metaphor. It involves recognition of the plasticity and the thickness of identities, a new understanding of the ways in which identities interlock, being freed from old limitations, and the emergence of new possibilities.... When we pay attention only to statistical ladders, we tend to substitute two-dimensional markers for the multidimensional situations whose changes need to be evaluated. We fail to see the ways in which the ladder conceals the composition of the masses struggling at the bottom. The trick is to see "rising" more multidimensionally: not as progress up defined ladders, but as the yeast that allows the dough to spring back against the hands that knead it -- the pressure that expands. It empowers through change in the structure of our identities and the possibilities inherent in the categories that locate us; change in the categories that we locate; change in our relations to one another. (p. 181) .

Categorification might then be understood as a move towards a meta-process through which disparate schemas may be organized at a higher level of abstraction -- posing the question of whether their advocates subscribe to their being subsumed in this way.

Although seemingly (and perhaps necessarily) abstruse, categorification is currently a focus for interrelating disparate concerns as indicated by the preoccupations of a workshop on *Categorification and Geometrisation from Representation Theory* (Glasgow, 2009). This event is premised on the recognition that:

> For a long time the idea of categorification has been in the background of many ideas in algebraic Lie theory and its connections to geometry. Several hard questions in Lie theory have been solved by translation (often via geometry) into combinatorics. For example, irreducible modules are labelled by combinatorial data and multiplicity formulas can be computed via combinatorially defined polynomials. On the other hand, topological questions are sometimes transferred into combinatorics in order to produce a clean answer: combinatorially defined knot invariants via polynomials; changing of coordinate systems via mutation rules; etc. It is becoming increasingly clear that the connecting principle of many such results in both Lie theory and topology is the idea of categorification. The notion of "ecategorification" goes back to Crane and Frenkel, motivated by mathematical physics, and in

> particular by the hope to construct higher dimensional topological quantum field theories.

Current concerns are to clarify the notion of categorification and its appearance in three different areas of mathematics: algebraic geometry, symplectic geometry and representation theory.

The further development of a generalized periodic table of n-categories is currently an active concern in relation to homology and cohomology, notably as a generalization of the Witt group. This is an abelian group whose elements are represented by symmetric bilinear forms over the field. Two symmetric bilinear forms are equivalent if one can be obtained from the other by adding zero or more copies of a hyperbolic plane (the non-degenerate two-dimensional symmetric bilinear form with a norm 0 vector). The Witt group is considered to be the simplest case of the cohomology of the periodic table of n-categories as recently discussed (Noah Snyder, *The Witt group, or the cohomology of the periodic table of n-categories* 30 March 2009). Homology in mathematics is a procedure to associate a sequence of abelian groups or modules with a given mathematical object. Cohomology can be viewed as a method of assigning algebraic invariants to a topological space that has a more refined algebraic structure than does homology. Both might best be understood as methods for the detection of degrees of isomorphism.

Such developments are of course a delightful exploration of abstractions -- to the extent that anyone (especially this writer) can rise to the challenge of comprehension they represent. It is to be hoped that they give rise to greater insight into how the dramatic differences in society can be related, if this remains of relevance. Unfortunately **it remains unclear as to whether such developments are capable of organizing the representation of the field of mathematics itself in more meaningful and accessible ways** -- the challenge to which Corfield pointed (above) -- especially if there are "competing" views on how this is to be achieved and whether the formulations of such disparate views can themselves be integrated into an overarching theory, through some understanding of "complementarity".

Essentially it would appear that there is a capacity to discover and organize categories more coherently at higher orders of abstraction. The Periodic Table, as commonly known, is then potentially to be understood as a relatively simple instance of this. The question is whether it is the most readily comprehensible pattern which may be indicative of the nature of a possible Periodic Pattern of Human Life -- or whether such a pattern only emerges at yet higher orders of abstraction, beyond average capacity to comprehend it in any useful way.

Whether the Periodic Table can fruitfully be considered as a metaphor of human life, Kenneth Boulding, as cofounder of general systems theory, offers the following insight relating to such use of metaphor in providing an integrative understanding of human life:

> Our consciousness of the unity of self in the middle of a vast complexity of images or material structures is at least a suitable metaphor for the unity of group, organization, department, discipline or science. If personification is a metaphor, let us not despise metaphors -- we might be one ourselves. (*Ecodynamics; a new theory of societal evolution,* 1978)

If "cohomology" is the key to the way forward (or "upward"), then it is appropriate to point out that the provocative checklist of "co-" terms (above) includes terms that bear on the self-reflexive comprehensiveness of any such pattern. Most of the relevant literature is subject to "copyright" and there is a high degree of (deniable) "competition" between mathematicians seeking to affirm their identities through claims on new conceptual territory -- replicating, without addressing, challenges faced by society, as discussed elsewhere (*And When the Bombing Stops? Territorial conflict as a challenge to mathematicians*, 2000; *Einstein's Implicit Theory of Relativity -- of Cognitive Property? Unexamined influence of patenting procedures*, 2007). It might be asked whether there is any greater irony than efforts to copyright the ultimate periodic table of categories.

Towards a periodic table of ways of knowing -- in the light of metaphors of mathematics

In the spirit of Gregory Bateson (*Angels Fear: towards an epistemology of the sacred*, 1988), this theme is explored (in Annex 1) as an exercise based on assumptions detailed there in relation to:

- Towards a periodic organization of the *Mathematics Subject Classification***:** use of the *Mathematics Subject Classification* (MSC) as implying a form of periodic table of mathematial ways of knowing, suggesting the possibility of "groups" and "periods". In contrast with the (above-mentioned) preoccupations of mathematical knowledge management regarding access, the emphasis is placed on employing the insights of mathematics into order and relationship so as better to configure the map as a whole -- if only to enable neophytes to navigate the terrain and discover territory they find meaningful..
- Mathematics and metaphor: cognitive understanding of mathematics, as indicated above (*Origin of mathematics and the periodic table -- in human cognition?*), The topic has been variously explored but it is useful to distinguish some possible "flavours", since some may be far from implying others. Annex 1 therefore cites authors distinguishing: Mathematics as metaphor, Metaphors of mathematics, Mathematical metaphors
- Associating metaphor with formal representation: the *I Ching* of Chinese culture: the mathematically interesting pattern of hexagrams of the *I Ching* of Chinese culture. The specific relevance here being the fact that metaphors are associated with the elements of the pattern to facilitate understanding of each and of the pattern as a whole
- Comprehensive set of ways of knowing: the *All-Embracing Net* of Buddhist culture: Of great relevance to explorations of any ordering of ways of knowing is that formulated in the *Brahmajala Sutta*. This is considered to be one of the Buddha's most important and profound discourses, weaving a net of sixty-two cases capturing all the philosophical, speculative views on the self and the world (Bhikku Bodhi (Tr). *The Discourse on the All-Embracing Net of Views; the Brahmajala Sutta and its commentarial exegesis*. Kandy, Buddhist Publications, 1978).

The emphasis on the cognitive dimension, and the processes and modes of learning, also suggests that in effect such a periodic table (informed by mathematical insight) could be understood as a "periodic table of metaphors". This has a notable justification in that the originating insight for many mathematical innovations typically takes metaphorical form. Such a table therefore provides a vital link to the process of "doing" mathematics in contrast with focusing on the use of what has been formed by others in the past. The "elements" of any such table are then to be understood as generative metaphors "through", or "out of", which ways of knowing are framed.

Questions for future symmetry-related explorations

With regard to Corfield's concern with "ignorance" of what mathematics may have discovered, his focus on the algebra of symmetry offers a useful example. Separately the challenge of comprehending its psychological implications has been discussed, necessarily naively even though that is consistent with his "obstacle" (*Dynamics of Symmetry Group Theorizing: comprehension of psycho-social implication*, 2008). Therein a number of questions were posed (*Possible questions for future symmetry-related exploration*, 2008) as follows:

Regarding the organization of symmetry-related insight:

- does the metaphor of an "atlas" (of finite groups) imply a generically spherical organization of its contents?
- is mathematics sufficiently self-reflexive in applying its most appropriate tools to ordering the mathematical body of knowledge in a manner that honours the relationships between its parts, or does it rely on conventional tools of knowledge classification? (cf *Is the House of Mathematics in Order? -- are there vital insights from its design*, 2000)
- how does the organization of mathematics in its entirety affect its comprehension and the challenges of mathemtical education?

Regarding symmetry object detection as a result of abstraction:

- is it appropriate to ask how many levels of abstraction/generalization can be usefully recognized?
- are any "meaningful constraints" on the number of levels of abstraction related to the properties of the early (prime) numbers and their role in symmetry through polygons -- or possibly to the form of the Mandelbrot set (*Psycho-social Significance of the Mandelbrot Set: a sustainable boundary between chaos and order*, 2005)?
- what kinds of "entity" tend to become "apparent" as emergent properties at each level of abstraction?
- at what level of abstraction does the nature of such "entities" effectively fail to hold communicable meaning?

Regarding psychological implication in symmetry objects of the mode of knowing associated with abstraction:

- how does the mode of knowing change with each shift in level of abstraction?
- what is the nature of the "fixation" with emergent objects at a particular level of abstraction?
- how is the sense of psycho-social identity challenged by each shift in level of abstraction or level of reality? (cf *Emergence of Cyclical Psycho-social Identity: Sustainability as "psyclically" defined*, 2007; *Varieties of Rebirth*, 2004)
- is such implication related to modes of knowing that have been extensively explored by articulations in other traditions?
- what implication might be associated with the Monster group? (*Potential Psychosocial Significance of Monstrous Moonshine: an exceptional form of symmetry as a Rosetta stone for cognitive frameworks*, 2007)

Regarding categories distinguished by psycho-social systems:

- can distinguished categories, and differences in perspective, be usefully seen as "points" whose relationships may be fruitfully explored by symmetry group theory?
- does such treatment, through abstraction, lead to recognition of higher orders of symmetry through which categories could be related?
- would such an approach to their "virtualization" make evident richer understandings of categories which are typically defined simplistically ("job", "income", "knowledge", "nation", "community", "species", etc)?

It was with these questions in mind that Figs. 1 and 2 were produced of which Fig. 3 is effectively a variant (*Indication of relationship between dimensions discussed relating to engagement with symmetry*).

Given the potentially fundamental importance of any Rosetta stone relating different modes of knowledge, the question would appear to be how to write about such matters meaningfully. How to write about what one does not understand -- especially when the longer the explanation, the greater the alienation of the reader.

Relevant to such questions is the view of Eugenia Cheng (*n-categories with duals and TQFT*, 2007) in discusing the work of John Baez. She stresses her bias in emphasizing the following viewpoints:

- looking at higher-dimensional structures we know are there, and characterising what those are, rather than just coming up with definitions abstractly
- the importance of working things out very precisely in low dimensions, and indeed at all (rather than just making broad sweeping theories, though these are important too)
- the importance of a theory that is actually useable somehow, rather than something so abstract that we have no idea what it looks like

It is in this sense that a succinct periodic table offers mnemonic clues through its patterning. However it would appear that mathematics has its own challenges in this respect. Given the seeming lack of any overarching order to mathematics -- perhaps to be understood as a work in progress -- there is the intriguing possibility that mathematics itserlf might be "organized" as a periodic table (*Is the House of Mathematics in Order? -- are there vital insights from its design*, 2000). .

It is interesting that much benefit has been derived from the recognition of set of ways through which social organization might be represented and understood, notably as articulated by Gareth Morgan (*Images of Organization*, 1986; *Imaginization: New Mindsets for Seeing, Organizing, and Managing*, 1997). Curiously he first distinguishes eight "images": Machine, Organism, Brain, Culture, Political System, Psychic Prison, Flux and Transformation, and Instrument of Domination, and then six "models". Both potentially a helpful identification of the "groups" basic to the ordering of a form of periodic table? Is such "imagining", and its emergence, a key to understanding the nature of any periodic table of human life and knowing?

Comprehensive formulations and their cognitive challenge

This theme is briefly discussed in Annex 2. As noted above, an extensive summary has recently been produced (D. H. Rouvray, *et al.*, *The Mathematics of the Periodic Table*, 2005). The examples relating to the Periodic Table in that Annex are those of Jean-Claude Perez, Jozsef Garai and R. Buckminster Fuller. Their relevance here is that:

- their approaches are much simpler
- they make no reference to each other
- there is no reference to them in the compilation by Rouvray *et al.*
- if they are to be considered overly simplisitic, then this helps to highlight the challenging relationships between simpler (and possibly inadequate) formulations and those which are more complex but necessarily comprehensible to a more limited number of people

Also considered there as "comprehensive" frameworks are the development by Perez of his periodic table equation (into an Equation of Life), the Mandelbrot fractal, the Gaussian copula, and the set of higher order symmetry groups.

The question implied by each is how might any such framework affect cognition, especially if there is any implication that "coherence" calls for a cognitive relationship between them -- namely some kind of Rosetta stone, with the integrative comprehension that this implies. Who might be concerned that there is seemingly no connection between the periodic table of categories, and of mathematics (John Baez, *et al.*) and the preoccupations of Rouvray *et al.*? Given the ambitious undertaking of R. Buckminster Fuller (*Synergetics: Explorations in the Geometry of Thinking*, 1975), and its early indication of an approach to the organization of the periodic table now considered viable (see Annex 2), what questions does this raise with regard to his other explorations of more cognitive significance?

Given the challenge of complexity, potentially relevant is human dependence on the neglected nature of approximations as discussed by Valentin N. Ostrovsky (*Towards a Philosophy of Approximations in the "Exact" Sciences*. *HYLE--International Journal for Philosophy of Chemistry*, 2005). As one of the few mathematicians discussing epistemological issues, he demonstrates that

> ... approximations are in fact in the core of some recent discussions in the philosophy of chemistry: on the shape of molecules, the Born-Oppenheimer approximation, the role of orbitals, and the physical explanation of the Periodic Table of Elements. The ontological and epistemological significance of approximations in the exact sciences is analyzed. The crucial role of approximations in generating qualitative images and comprehensible models is emphasized. A complementarity relation between numerically "exact" theories and explanatory approximate approaches is claimed.

More generally such comprehensive formulations point to the ultimate **cognitive** challenge of a Theory of Everything and how people would be expected to relate to it were it to emerge from ongoing research in fundamental physics (cf Paul Halper, *The Great Beyond: higher dimensions, parallel universes and the extraordinary search for a Theory of "Everything*, 2004). The potential challenge is partially highlighted by the recent controversy regarding the publication of a An Exceptionally Simple Theory of Everything proposing a basis for a unified field theory, named *E8 Theory*, which attempts to describe all known fundamental interactions in physics, and to stand as a possible theory of everything (A. G. Lisi. *An Exceptionally Simple Theory of Everything*, 2007). A more recent example is that of George James Ducas (*Trans-Dimensional Unified Field Theory Physics Theory: a theory that advances the unification of relativity with quantum mechanics and string theory* 1974-2009) which

> ... explains the matrix and periodic table of a multidimensional universe.... Our universe is a multidimensional universe where processes and procedures involving natural physics relate and exist simultaneously in multiple dimensions. Natural occurrences are multidimensional. Historically we identify our existence within three dimensions or vectors of space. However, the matrix of space needs to be redefined as a periodic table of "components" or "vectors" which build up space-time and relate all physics within the relationship of space "component vectors" and "component matrixes".

Possible cognitive implications

In the following comments it is important to recognize that:

- the inadequacies of any implied emphasis on a periodic "table" (see *Pattern of learning as a whole: life is not a "table"*) are a consequence of its convenience for representation in printed form (at the end of the print era). More complex representations of such periodicity are possible, but (as yet) these pose a challenge to comprehension.
- the periodic table is explored as a different "language" through which cognitive implications can be articulated, benefitting from the greater subtlety of mathematical formalsim. However, that articulation is no more "true" thyan the periodic table is itself "true" as a representation of the reality of the relationship between the elements
- the implication explored here is that human cognition constrains the representation of complexity in any domain. Thus even the mathematical insights, purportedly exclusively focused on the physical reality of the array of elements, are especially significant in

terms of the limitations on how cognition can credibly order patterns emerging from such research -- to whatever improbable subtleties they succeed in giving legitimacy. Potential insights from mathematical perspectives on periodicity have therefore been included below despite the complexity of the mathematics from which they derive.

- **whilst exploration of complexity is typically considered inappropriate to understandings of human life and development, it remains a matter of experience that preferences for simplification (even over-simplification) result in painful, even violent, situations which are then held to be "inexplicable"**. Insight from more complex frameworks therefore merits a degree of consideration in developing more appropriate approximations for simpler framings.

Any insights from mathematics included here are based on statements in the compilation by Denis H. Rouvray et al. (*The Mathematics of the Periodic Table*, 2005). Their selection (page numbers are given below where relevant) and interpretation are in themselves a reflection of extremely partial competence in these highly specialized matters (on the part of this writer). This exercise is in effect a provocation to those with the necessary competence to "re-read" such a compilation -- possibly substituting references to "atom" by references to more complex understandings of human "identity", for example (cf *"Re-reading" patterns of concepts*, 1995; *.Principles of Re-reading and Rapplication*, 2001).

The possibility of ordering global understandings of human living and knowing in terms of a periodic pattern calls for consideration and exploration of the following, which may well already have traces of periodic features. Possibilities include:

- **Varieties of knowing**:
  - **disparate set of ways of knowing**: this remains a challenge to determination of the pattern of underlying order. The efforts devoted to the periodicity of chemical elements indicate that early closure on a complete explanation is liable to be premature, whatever its adequacy for some purposes
  - **styles of thinking, intelligence and cognitive bias**: given the various explorations of these, it is possible that they might be usefully related in terms of a periodic pattern (*Systems of Categories Distinguishing Cultural Biases*, 1993)
  - **psychological typing**: implications for existing patterns of psychological typing and, even more interesting, the "competitive" relationships between them as exemplars of contrasting ways of knowing
  - **patterning implications**: given the fundamental nature of human knowing, of interest is the possibility of the "emergence" of related patterns: a ***periodic table of philosophies***, a ***periodic table of beliefs***, a ***periodic table of human development.***
  - **"tuning" a periodic pattern**: any elaborated pattern is an approximation in time to current understanding (by an individual or group), especially in terms of the significance attributed to terms used in the articulation of that pattern. In this sense, as with any instrument, the pattern calls for continuing attention to its "tuning", as previously discussed (*Tuning a Periodic Table of Religions, Epistemologies and Spirituality -- including the sciences and other belief systems*, 2007). There is a sense in which the "struggling" of modes in relation to one another may be compared to exploration of the self-classification elements and properties as neurons in neural networks (p. 101, 117). It is interesting that management cybernetician Stafford Beer has elaborated on this process, as "problem jostling", in syntegration.
- **Experiential understanding**:
  - **experiential emphasis**: although the discussion above has emphasized mathematical formalisms, the underlying argument points not to such formalisms but to the experiential process of "doing", even in the case of mathematics. The argument is that the mathematics is suggestive of a richer variety of ways of engaging with the various experiences of "doing". It is in this sense that although the ongoing debate regarding quantum consciousness, quantum psychology and quantum mind is of interest [more], the emphasis here is not on whether such insights offer valid explanations from an "external" (objective) perspective but rather whether they offer insights for those engaged with such experiences (subjectively).
  - **implicit knowledge**: whilst explanations of a periodic pattern, such as exemplified by the periodic table of elements, tend to be incomplete or inadequate to some degree, it is a fact that living is effectively a demonstration of capacity to understand that pattern (at some fundamental level) to a far higher degree than is represented by mathematical exploration to date.
  - **dynamics of a periodic pattern as a "dance"**: living might appropriately be said to be the capacity to dance to the music of the table of elements -- to "dance on the table". It is in this sense that widespread engagement with music might be understood as a rehearsal of complex understandings which are cumbersomely represented by conventional mathematical articulations, an argument explored from an epistemological perspective by Antonio de Nicolas (*Meditations through the Rg Veda*, 1978). As indicated in the earlier paper (and in Annex 1), an interesting bridge is suggested by the use of the codification of the *I Ching* hexagrams as a score for drumming (Michael Drake, *I Ching: The Tao of Drumming*, 1997; Melinda Maxfield, *Drumming the I Ching*, 1991). Such a perspective raises questions regarding "composition", "choreography" and "orchestration".
- **Learning**:
  - **complexity**: a pattern offers a sense of the directions of development of human understanding, even if such understandings are far from being immediately accessible
  - **learning pathways**: implications for sequential learning through periods and stages, and the possibility of shifting across the array of modes of knowing (as illustrated by "snakes and ladders"). This then offers a way of understandings phrases such as a "man for all seasons" and the remark of Valdimir Arnol'd (quoted above) that: *One's breadth is regarded as negative in the West to the same extent as one's narrowness is regarded as unacceptable in Russia*.
  - **completion of periods of learning**: the structure of the periodic table helpfully indicates how stages of learning contribute to "completion" of a course of learning, enabling a new "period" of learning to be undertaken
  - **ignorance, non-understanding, the unknown**: a periodic pattern enables what is not (yet) understood to be tentatively positioned implying that it may come to be understood -- at the same time recognizing that it may well be understood implicitly even when explanations are inadequate, whatever their sophistication (cf *Varieties of the "Unsaid" -- in sustaining psycho-social community*, 2003; *Unknown Undoing: challenge of incomprehensibility of systemic neglect,* 2008)

- **comprehension vs understanding**: the manner in which the periodic table breaks out of simplistic patterns is suggestive of a fruitful distinction between "superficial" comprehension and "deep" learning -- with the former potentially modelled by "outer" shell development and the latter by "inner" shell development, and with the former needing on occasion to await for completion of the latter before progressing further
- **disconnection and alienation**: a periodic pattern offers a useful means of understanding how any movement "up" the table to more complex modes of understanding (using categories of a "higher order") may readily ensure a disconnection from earlier positions in a learning pathway, notably in the light of the articulations from that "higher" position and the impatience with the over-simplification of "lower" positions (*Dynamics of Symmetry Group Theorizing: comprehension of psycho-social implication*, 2008). The periodic pattern then provides a framework for discussing simplicty vs complexity, especially in relation to their explicit and implicit forms.

- **Cognitive role of metaphor**:
  - **learning metaphors**: however understood, modes of knowing and associated learning opportunities may be usefully associated with particular metaphors, fables and aphorisms. This suggests an understanding of a ***periodic table of metaphors***, or a ***periodic table of learning stories.*** Such global understanding highlights the merit (as a "cognitive tookit") of an analogue to the multiplication tables fundamental to the process of learning arithmetic
  - **generative metaphor**: given the acknowledged relationship between metaphor, creativity and comprehension in mathematics, those metaphors characteristic of any particular mode of knowing are usefully to be understood as generative metaphors (with what is implied by research on their value to psychosocial organization). Such metaphors, as indicated by Donald Schon, are figurative descriptions of social situations, usually implicit and even semi-conscious but that shape the way problems are tackled (*Generative metaphor and policy-making*, 1995)
  - **kinship network**: in the light of the pattern of familial and kinship metaphors developed within Chinese culture to enable the pattern of *I Ching* hexagrams to be understood as a whole (see Annex 1), such metaphors could offer a valuable tool for providing a sense of coherence and integrity to any pattern of human life and knowing. Of particular interest, in relationship to a periodic pattern and from a symbolic perspective, are the two primary "parental" roles, the immediate "offspring" and the "sibling" relationships. There is even the possibility that extended kinship networks could be encoded by such patterns.
- **Identity**:
  - **imprecision, uncertainty and avoidance of premature closure**: although the nature of atoms may be considered the primordial question of natural science, the matter has not been completely resolved (p. 154); accurate calculations do not constitute explanations (p. 185); emphasis is placed on the imprecision of any definition of exactly what they are and how they might be fruitfully classified, despite the array of disciplines deployed to clarify this (pp. 1-2). This gives legitimacy to the avoidance of such closure in the case of individual identity (*Being What You Want: problematic kataphatic identity vs. potential of apophatic identity?* 2008; Hilary Lawson, *Closure: a story of everything*, 2001).
  - **wave nature of phenomena**: elements are appropriately recognized as pulsing dynamic micorobjects interacting with space-time as spherical standing waves in bound domains, with the nodes of such waves as the fundamental constituents of atoms and molecules, providing physical signifiance to the imaginary terms in a complex wave (p. 120, 154). Analogous understandings can be used to frame the nature and coherence of individual identity (*Liberation of Integration, Universality and Concord -- through pattern, oscillation, harmony and embodiment*, 1980; *Emergence of Cyclical Psycho-social Identity: sustainability as "psyclically" defined*, 2007).
  - **identity metaphors and models**: beyond the inadequacy of simplisitc distinctions of human identity, a periodic pattern offers a variety of ways in which identity might be understood. It might be associated with a particular "element" in the pattern (as discussed in the earlier paper), notably with its associated "isotopes", with a "group" of elements, with the set of elements of a given "period", with elements having common attributes (eg "positively" vs "negatively" charged), or with the periodic pattern as a whole
  - **implicit structure**: it has only recently been recognized the extent to which the properties of the elements derive from their relationship with each other and that exploration of their periodicity is effectively a relational science (p. 77); although no final determination has been made, whether or not it is possible (Kurt Gödel, etc), there is a recognition of the degree to which there is a mathematical structure (notably a topological structure, p. 97) underlying the set of properties of the elements which can therefore be generated by theoretical models (p. 79), although the basic question of how the periodicity is reflected in mathematics remains obscure (p. 266).
  - **interactive representations**: given the variety of orderings of elements now possible for the Periodic Table, in terms of various properties, such facilities could be adapted to give a sense of the various modes of knowing and associated identification with them. Challenging in this respect is the significance potentially to be derived from innovative dynamic representations of orbitals (see Java orbital viewer applet)
- **Individuation and maturation**:
  - **stability**: as speculatively explored in the main paper (*Periodic Pattern of Human Life: the Periodic Table as a metaphor of lifelong learning*, 2009), a form of "stability" is acquired by ageing through the sequence of a periodic table, acquiring "mass" (if not "gravitas"), until processes of degeneracy and instability set in (leading to death). The relative stability of the chemical elements, based on atomic number, provides an intriguing mnemonic framework through which to explore such ageing -- given that the number of naturally occurring stable elements is closely related to years of possible life expectancy -- with uranium being the highest at 92. It offers ways of reframing the experience of people of "old before their years" or being "eternally young".
  - **insight**: this framework also encourages discussion of the implication of "isotopes" of far less stability, with only the shortest of half-lives. These might usefully be undertstood as forms of coherence and creativity which many experience, but which last only relativel briefly. Intriguingly credibility is given to the existence of isotopes with half-lives of

microseconds (p. 7, 9), whereas "flashes of cpoherence" of such duration currently are held to be of little credibility -- except for those who experience them. Drug induced experiences might be seen as a form of artificial synthesis of such coherence, analogous to the synthesis of artificial elements -- experiences usefully described by "getting one's head together", etc. The possibility of acceding to, and sustaining, forms of stability (effectively exploring the chain of "islands of stability" of the chemical elements) is also of interest. The main paper notes the total number of currently confirmed elements as 111, with unconfirmed claims made with regard to elements up to 122 (see *Timeline of chemical elements discoveries*). The oldest person in history, whose age has been verified, is Jeanne Calment (1875-1997) -- 122 years. Given that any such islands of stability are associated with a particular "magic number" configuration, also worth exploring are understandings of complex sets of categories in Buddhist logic (cf (*Navigating Alternative Conceptual Realities: clues to the dynamics of enacting new paradigms through movement*, 2002).
    - elements with stable nuclides 81, 263 stable isotopes 8 -- *Tao Te Ching*
- **nuclear dynamics and effects**: with regard to the elements, those heavier are primarily governed by their physical properties associated with the nucleus and its mass, rather than with the chemical properties associated with the outer "superficial" electron shell; such nuclei are characterized by their dynamics, pulsations and constantly changing shape (p. 11-26). This provides a way of discussing people of maturity -- deemed signifiant not because of their momentary behaviour but because of their "weighty" contributions (possibly gravitas). It also offers a means of discussing how they function as "attractors" -- distinct from the superficial attraction of youth.
- **relativistic effects**: nucleons within the nucleus may move at 25% speed of light (p. 15-16). In the case of the heavier elements, such as gold, the core electrons must move at about 60% the speed of light to counterbalance the increased electrostatic attraction of the nucleus. Under such conditions relativistic quantum mechanics needs to be invoked to handle such relativistic effects (p. 189-190); the effects of relativity then upset the ordered filling of the electron shells (p. 6). This provides a framework for discussing recognition, with age, of the limitations of mechanistic ("linear") thinking and of the need to respond to complexity -- as required for sutblter forms of mathematics (cf William Byers, *How Mathematicians Think: using ambiguity, contradiction, and paradox to create mathematics*, 2007). At one stage is the challenge of ambiguity and paradix recognized -- toether with a capacity to handle it creatively?
- **symmetry**: an interesting bridge between the subtleties of mathematics and of personal experience through appreciation of symmetry. Understanding of periodicity amongst the chemical elements is discussed in terms of the "magic numbers" forming an exceptionally stable cluster, in a manner similar to discussion of "sacred geometry". Such "magic numbers" are also recognized in the number of nucleons required to complete atomic-nuclear shells. From a mathematical perspective, these form part of the preoccupation of symmetry group theory, notably used in the study of the periodicity of chemical elements. Symmetry in four and higher dimensions is significant in such explorations (p. 217, 230), as is the classification of elements in the light of quantum mechanics (p. 265). Mass is now understood to be associated with symmetry violation (p. 218). With the increasing influence of relativistic effects, double group symmetry becomes a factor (p. 25, 190). Especially interesting is the cognitive appreciation of very high ordrs of complexity, if only through the representation of strange objects like the Gosset 421 polytope (an 8-dimensional semiregular uniform polytope composed of 17,280 7-simplex and 2,160 7-orthoplex facets) or the E8 (the exceptional simple Lie group of dimension 248), one of the complex structures ever studied (*Mathematicians Map E_8*, 2007). The challenge that attraction to such symmetry implies has been separately explored (*Dynamics of Symmetry Group Theorizing: comprehension of psycho-social implication*, 2008). The question is the extent to which it is possible to embody such higher degrees of symmetry, whether individually (as implied by some understandings of meditative disciplines) or collectively through the web (*Sacralization of Hyperlink Geometry*, 1997).
- **connectivity**: it is the nature of the connectivity and correspondences across any "table" that effectively causes it to "curl" into other diemsnions -- affecting associated iunderstandings of identity, as previosuly discussed (*Comprehension of Requisite Variety for Sustainable Psychosocial Dynamics: transforming a matrix classification onto intertwined tori*, 2006)

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