



# laetus in praesens

Alternative view of segmented documents via Kairos

1st July 2012 | Draft

## Middle East Peace Potential through Dynamics in Spherical Geometry

### Engendering connectivity from incommensurable 5-fold and 6-fold conceptual frameworks

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Produced on the occasion of worldwide media focus on the patterned movement of a truncated icosahedron in Kiev, whilst the United Nations demonstrates its long-standing inability to elicit relevant intelligence in more creative response to ongoing massacres in the Middle East.

## Introduction

This is an exploration of the hypothesis that unique belief systems depend for their coherence on distinctive patterns typically embodied in geometrical symbols in two dimensions. On the basis of that **assumption**, the case tentatively explored here is that of the "incommensurability" of the 5-fold [Star of Islam](#) and the 6-fold [Star of David](#) of Judaism -- both symbols appearing on flags of the nations having those distinct faiths.

Efforts at the reconciliation of the perspectives embodied in these faiths are seen here to be as questionable as that of endeavouring to fit together the pieces of a jigsaw puzzle on a flat surface -- when the pieces do not "match" or "fit" together. That approach may be compared to the practice of the impatient in seeking to "force" the pieces together -- even "banging" on them to ensure that they lock together. Even if "successful", the resulting picture is "wrong". This metaphor has both conceptual implications and territorial implications. To employ a chemical metaphor, in the case of Judaism the pieces might be understood as 6-valent, in comparison with "5-valent" pieces of Islam. The guiding assumptions for this exploration are the much-quoted statements of Albert Einstein:

- The significant problems we face can not be solved at the same level of thinking we were at when we created them.
- To repeat the same thing over and over again, and yet to expect a different result, this is a form of insanity.

The approach taken here explores the possibility that the "pieces" only fit together on a three-dimensional surface, namely a sphere. It is the spherical geometry that then merits consideration, together with the challenge of how to get from any "mis-fitting" two-dimensional layout to a three-dimensional form. Of course, two-dimensional layouts are far more readily comprehensible. Hence the focus on them. However the three-dimensional layout has the potential of rendering comprehensible a far more elegant layout which may well exemplify intuitions characteristic of the faiths so dramatically opposed.

The approach follows from various earlier explorations of the potential of mathematics to offer a new perspectives on these issues, including: [Geometry of Thinking for Sustainable Global Governance](#) (2009), [Spherical Configuration of Categories -- to reflect systemic](#)

*patterns of environmental checks and balances* (1994), *Using Disagreements for Superordinate Frame Configuration* (1993), and *Mathematical Theology: future science of confidence in belief* (2011).

In the case of the Middle East, the argument has previously been developed that the opposing cultures are widely recognized as highly competent in number theory and geometry (cf. *Wikipedia List of Jewish mathematicians*; Keith Devlin, *The Mathematical Legacy of Islam*, 2002; Keith Critchlow, *Islamic Patterns: an analytical and cosmological approach*, 1999; Issam El-Said, *Islamic Art and Architecture: the system of geometric design*, 2008; Center for South Asian and Middle Eastern Studies, *Islamic Mathematics*; 2000, G. Donald Allen, *Islamic Mathematics and Mathematicians*, 2000; *Wikipedia, Mathematics in Medieval Islam*).

Carra de Vaux continues to be quoted by Islamic scholars to the effect that : *they (the Muslims) were indisputably the founders of plane and spherical geometry, which did not, strictly speaking, exist among the Greeks (The Philosophers of Islam, 1921).*

**These skills do not however seem to have been brought to bear on the psychosocial dynamics which divide them in relation to territory -- other than with respect to ballistics.** Nor does there appear to have been any interest in suggesting that this possibility merits exploration, as previously discussed (*And When the Bombing Stops? Territorial conflict as a challenge to mathematicians*, 2000).



What follows is essentially the description of the development of an interactive visualization experiment to illustrate possibilities accessible through any web browser using **scalable vector graphics**. The focus is on the experimental design and modification of the visualization, effectively enabling imaginative exploration of possibilities -- both playfully and as offering a **pattern language** through which differences and their implications can be disputed in terms of design aesthetics. The experiment also highlights the potential of a visualization package, the appropriately named **Stella Polyhedron Navigator**, through which the "global" implications of such patterns can be explored, as previously suggested (*Polyhedral Pattern Language: software facilitation of emergence, representation and transformation of psycho-social organization*, 2008).

## Fundamental cognitive patterns assumed to be characteristic of belief systems

The assumption here is that an extremely fundamental cognitive identification is associated with the distinct patterns below -- thereby constituting an expression of quite different faiths. These distinct patterns may imply a yet more fundamental identification which cannot be readily expressed through symbols in two dimensions.

Whilst these symbols are indeed distinctly favoured in the flags of the cultures in conflict as considered here, the nature of their fundamental relationship to some form of cognitive identity could be questioned. The case of the Star of David is more straightforward. The use of the **five-pointed star** by Islamic nations is however subject to reservations -- especially given its use in the **flags of non-Islamic cultures** and for other purposes. The **Red Star** has been widely used as a symbol of both Socialism and Communism. **Use of the six-pointed star by Muslims** -- derives from a period when the Star of David and the **Seal of Solomon** were more harmoniously related in accord with medieval Jewish, Christian and Islamic legends.

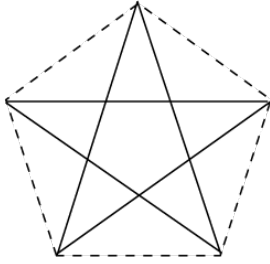
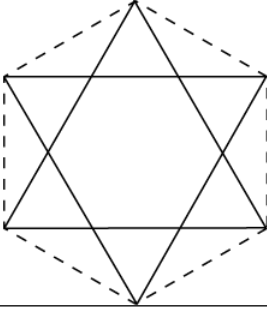
Of relevance to the argument here, the 5-pointed star was a focal symbol of the **Ottoman Empire**. through which the **crescent moon and star** became affiliated with the Muslim world. This type of flag has become the *de facto* Islamic flag, and is used, with variations, by multiple Muslim lands such as Algeria, Azerbaijan, the Comoros, Malaysia, Maldives, Mauritania, Pakistan, Tunisia, Turkey, Turkmenistan, the Turkish Republic of Northern Cyprus, Uzbekistan, and the Western Sahara. As the crescent and star have no religious significance however, some Muslim scholars are against attaching these signs on mosques and minarets or using them to denote Muslim societies. The 5-pointed star as used by Islam is not limited to red in colour, green may also be used, or white against a background such as red or green.

Stars considered symbolic of cultural identity	
Star of Islam	Star of David
	

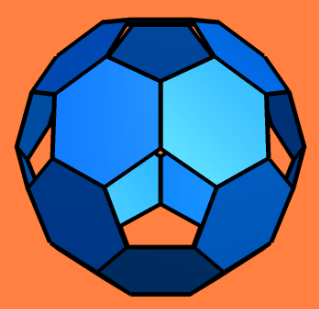
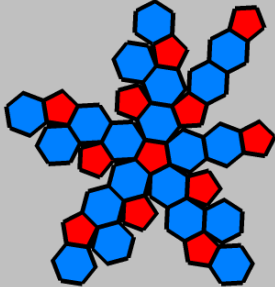
Mathematically these patterns cannot be readily combined. This issue is described in mathematics in terms of tiling, described generically as **tesselation** (cf Craig Kaplan, *The Trouble with Five, Plus Magazine*, 2007). This of course has immediate relevance to the attribution and layout of territory -- with all the implications for politics and violence which have characterized the Middle East (*And When the Bombing Stops? Territorial conflict as a challenge to mathematicians*, 2000).

It is noteworthy that in the quest for a key metaphor for the structural challenges of the immediate future, the tessellation of disparate orders was identified by the sesquicentennial symposium of Boston University (Lance Morrow, *Metaphors of The World, Unite!, Time*, 16 Oct. 1989). Such tilings of zones of order, might also be understood metaphorically as paving stones separated by cracks of varying sizes. For optimists these might be hairline cracks; for pessimists the degree of separation might be such as to prevent movement and to offer little protection from the disorderly 'gunge' rising from beneath.

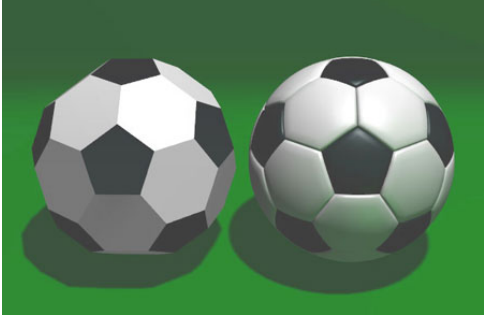
The approach taken here is to treat each of these patterns as "carried" by a polygon. This gives rise to the following, which still cannot be "fitted" together as a pattern in two-dimensions..

Symbolic stars embedded in corresponding polygons	
5-fold Star	6-fold Star
	

A set of hexagons and pentagons can however be uniquely fitted together as a particular three-dimensional polyhedron, namely the truncated icosahedron. This has a total of 32 faces: 20 hexagons and 12 pentagons. It has 60 vertices with 90 edges of the same length. These are of two different types according to their position in the structure: 60 of one type, and 30 of another.

Truncated icosahedron (images produced using <a href="#">Stella Polyhedron Navigator</a> )	
Animation of folded form	Image of unfolded net form
	

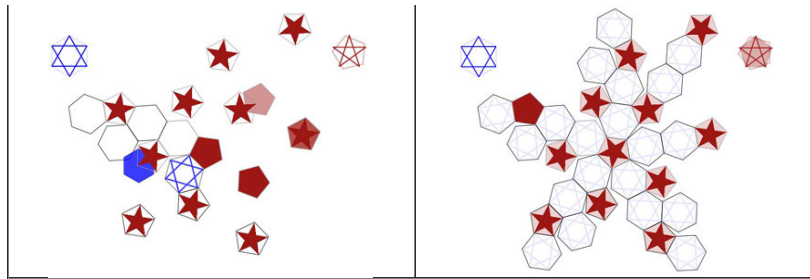
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Truncated icosahedron compared with ball used in association football (reproduced from <a href="#">Wikimedia entry</a> )

At the time of writing, it is the movement of the football which is the focus of media attention in the <a href="#">European Football Championship in Kiev (2012)</a>

## Animation of interaction and interlocking between cognitive patterns

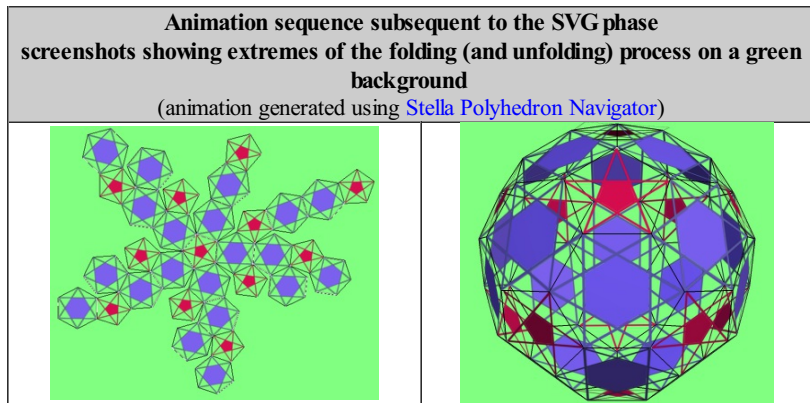
Rather than introducing the visualization with a verbal explanation, this is presented immediately through the following screenshots of phases in the animation accessible [here](#). A commentary is given below.

Screenshots indicating phases in SVG portion of animation sequence [ <a href="#">click for separate SVG animation</a> -- right-click on active animation for options -- improvements welcome NB: Animation works in Firefox and Opera; effects do <b>NOT</b> work properly in Google Chrome, Internet Explorer or Safari]	
Early stage in animation	Late stage in animation

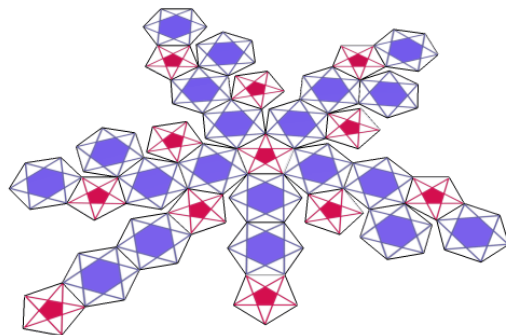
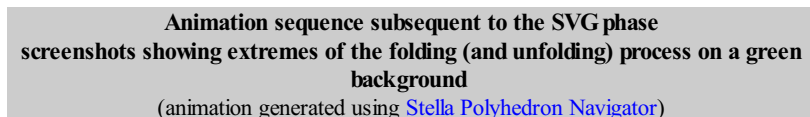


The animation progressively "populates" a space (a "territory") in two dimensions with instances of either a hexagon or a pentagon -- as "vehicles" for the Star of David and the Islamic Star respectively. The polygonal vehicles are also usefully understood as [jigs](#) or templates enabling appropriate positioning -- as in many processes for manufacturing objects which need to be held accurately in place and ensure pattern replicability. The final pattern reinforces the point that the two sets of polygons cannot be "fitted" into any coherent design -- even though some edges between the two different shapes can be established as common, as "joining".

The seemingly arbitrary pattern can, as a whole, be "folded" into a spherical form in three dimensions, as indicated by the following screen shots and the accompanying animation.



The screenshots above, and the animation below, are suggestive of the challenge and possibility of getting the contrasting 5-fold and 6-fold patterns to "fit together" in three dimensions. The subsequent commentary endeavours to clarify some of the implications.



The stages of the animation are suggestive of an intuitive relation between the [Star and Crescent](#) of Islam. Disposed together otherwise, the elements of the unfolded net could well be indicative of either the 5-fold or 6-fold stars.

An association [football is constructed](#) by stitching together 32 panels of waterproofed leather or plastic: 12 regular pentagons and 20 regular hexagons. As noted, the 32-panel configuration is the spherical polyhedron corresponding to the truncated icosahedron; it is spherical because the faces bulge from the pressure of the air inside. The [basketball](#) and [volleyball](#) are **not** constructed in this way. Reference is often made to the football design, when describing carbon [buckyballs](#) or the root structure of [geodesic domes](#), as an illustration of the truncated icosahedron.

## Interactive design of cognitive pattern animation

It should be stressed that the approach taken here is a "proof of concept" with respect to such an animation and the possibilities of its design. The result demonstrated has a variety of inadequacies. These can be variously attributed to:

- technical incompetence in making full use of the [scalable vector graphics](#) (SVG) coding
- design incompetence of optimizing parameters to enhance the elegance of the result
- contrasting views on what viewers might variously consider "elegant"
- political implications consequent on some of the design choices, possibly interpreted as favouring one perspective or the other
- preferences for the "story" to be told by the dynamics of the animation

The advantage of the approach taken in using SVG is that -- with a minimum of skill -- the code can be readily modified to change the animation. **SVG code can be changed using any text editor**. It does not need to be compiled. Whilst modifying the structure of the animation as a whole may require more skill, parameters such as the following can be readily changed (as discussed more systematically below):

- colour attributed to various elements, including filling an area (or not)
- width of lines ("resistance")?
- duration of certain phases and of the animation as a whole

Changing duration means that the rates of one phase relative to another can be modified, offering the possibilities of:

- accentuating a degree of pulsating rhythm
- accentuating a degree of randomness
- changing the pace of certain phases in relation to others

Clearly these changes may serve primarily to suggest:

- more complex changes to the coding to reflect subtler insights
- possibilities of "tuning", especially in the final form
- improvements to the software to facilitate such experimentation
- addition of music and/or commentary

The design context is usefully clarified by the information in the following box insert -- which may be readily ignored, but is useful to enable others of greater competence to improve on the approach and the design.

<b>Technical issues regarding animation production and modification: SVG Phase</b>	
<p>Although SVG code is relatively easy to modify, correctly defining an SVG file is somewhat challenging. It is easier to use some kind of SVG editor to supply the code framework. There are a number of special <a href="#">SVG editors</a> offering many facilities. The above animation however involved a certain amount of "cheating" to bypass levels of complexity calling for higher competence.</p>	
<p>Steps in the process were:</p> <ul style="list-style-type: none"> <li>• Use of <a href="#">Stella Polyhedron Navigator</a> to:               <ul style="list-style-type: none"> <li>◦ produce the unfolded (flat) truncated icosahedron net</li> <li>◦ export of the net to a PNG file (a PDF export was an alternative)</li> </ul> </li> <li>• Use of <a href="#">Adobe Illustrator</a> to:               <ul style="list-style-type: none"> <li>◦ import the PNG file</li> <li>◦ automatically trace the PNG network to form an Adobe object (using the <a href="#">Live Trace</a> facility)</li> <li>◦ adjustment of the Adobe image to compensate for inadequacies in the trace</li> <li>◦ export of the image to an SVG file (<i>excluding</i> possibility of subsequent editing in Adobe, which otherwise adds many lines of useless code)</li> <li>◦ <i>NB: This process avoided use of the animation facilities in Adobe, calling for higher competence</i></li> </ul> </li> <li>• Use of a text editor (with limited competence) to:               <ul style="list-style-type: none"> <li>◦ introduce animation effects</li> <li>◦ introduce phases into the animation</li> <li>◦ adjust timings, colour, etc</li> <li>◦ make revisions following many tests</li> <li>◦ save the SVG file (locally)</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Use of various web browsers to test the SVG animation (locally and on web)               <ul style="list-style-type: none"> <li>◦ Opera (offered clearest feedback on errors)</li> <li>◦ Firefox</li> <li>◦ Chrome (did not show all animation effects)</li> <li>◦ Internet Explorer (did not show all animation effects)</li> <li>◦ Safari (did not show all animation effects)</li> </ul> </li> <li>• Use of <a href="#">Stella Polyhedron Navigator</a> (as noted below) to:               <ul style="list-style-type: none"> <li>◦ reproduce the truncated icosahedron corresponding (as a net) to the final phase of the SVG animation</li> <li>◦ export video of animation of truncated icosahedron</li> </ul> </li> <li>• Final stages naturally involved:               <ul style="list-style-type: none"> <li>◦ uploading SVG file to website</li> <li>◦ uploading video to website</li> <li>◦ links to both SVG and video were provided from this document</li> <li>◦ <i>NB: The possibility of embedding the video into the SVG file was avoided as posing a possible problem for browsers, prior to use of HTML</i></li> </ul> </li> </ul>
<p>Note that the animation as presented is cumbersome and inefficient. There are many refinements that could be introduced -- although some might make it more difficult for the less competent to modify the animation subsequently. More explanatory comments could be embedded in the SVG file. Groups of objects could be defined in more significant ways. It is possible that a subsequent stage in the evolution of the Stella Polyhedron Navigator would enable direct export of polyhedral nets to SVG, avoiding the need to use Adobe Illustrator to trace the pattern -- but provided the export included the contextual coding (binding to namespaces, etc)</p>	
<p><i>NB: Modifying the SVG portion of the animation: The current version of the Opera browser offers facilities to control the SVG animation (right-click) and view the source code. Upgrades envisaged to other browsers</i></p>	

are expected to offer this facility. The code can be easily downloaded for experimentation by any user. *The animation can be magnified to any size screen without loss of resolution -- useful for presentation and discussion*

The note above shows how the [Stella Polyhedron Navigator](#) was used to facilitate production of the first phase of the animation using SVG. There is a case for considering how this application could be used by itself to experiment with useful animations -- avoiding the issues with respect to technical competence in SVG and the inability of some browsers to handle SVG animation.

Technical issues regarding animation production and modification: Stella Phase	
<p>In retrospect SVG was used because of the facility that it offers to <b>control details of disparate features</b> of the animation, especially timing (as noted in discussion of the design options below). The Stella application offers far greater <b>control of the global form</b> and was used for that purpose as shown in the video above.</p> <p>Many design options implemented in SVG could however be explored in Stella in order to avoid the complications of using both applications -- especially since use of Stella is far more intuitive and user-friendly, offering a vast range of polyhedral resources and manipulation possibilities. Key issues are how experimentation is to be done and how any final product is to be rendered available to others. The constraint with SVG is browser compatibility (as noted above). The constraint with Stella is that it is a proprietary application so that its output can only be made available over the web in a non-interactive mode, typically as images or in any standard video format.</p>	<p><a href="#">Stella Polyhedron Navigator</a> can therefore be used to:</p> <ul style="list-style-type: none"> <li>• reproduce the truncated icosahedron corresponding (as a net) to the final phase of the SVG animation</li> <li>• fold the net version into spherical form</li> <li>• load Star of David images onto hexagonal faces</li> <li>• load Islamic Star onto pentagonal faces</li> <li>• adjust images on faces</li> <li>• export video of animation of truncated icosahedron</li> </ul> <p>Of particular interest is the ability of Stella to add "tabs" to unfolded polyhedral nets so that when a net is printed it the net can be cut out and folded into spherical form with the aid of adhesive on the tabs. It thus enables three dimensional forms to be constructed and marked experimentally or permanently.</p>

## Visualizing alternative stories through manipulation of animation design options

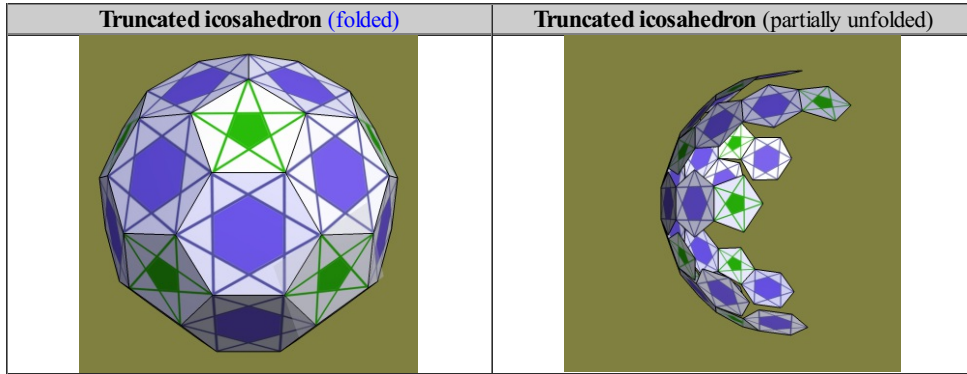
The purpose is to enable people to experiment interactively with various "stories", especially in the two-dimensional territorial occupation phase. Clearly some options privilege one framework and place the other at a disadvantage. The options could be seen like a musical keyboard in which the challenge is to render elegant the relationships between the two by manipulating the many combinations of parameters. The animation offered as a demo is the consequence of one set of design choices. Clearly many others are possible. The animation could itself be improved in a variety of ways with greater expertise..

Interactive design options in composing a story		
Option	5-fold Islamic Star	6-fold Star of David
Generating symbol	change visibility (variable) change "pulse" rate	change visibility (variable) change "pulse" rate
Polygonal "vehicle" container: line width	thin to invisible thick	thin to invisible thick
Polygonal "vehicle" container: colour	neutral (black) alternative colour alternating colour ("blinking")	neutral (black) alternative colour alternating colour ("blinking")
Polygonal "vehicle" container: phasing visibility	permanent periodic unpredictable/random	permanent periodic unpredictable/random
Symbol	outline (optional width) filled	outline (optional width) filled
Symbol colour	conventional red alternative colour alternating colour ("blinking")	conventional blue alternative colour alternating colour ("blinking")
Symbol: phasing visibility	permanent periodic unpredictable/random	permanent periodic unpredictable/random
Space prior to "occupation"	polygon (absent/present) symbol (absent/present)	polygon (absent/present) symbol (absent/present)
Space "occupation" phase	do before 6-fold do concurrently do after 6-fold	do before 5-fold do concurrently do after 5-fold
Space "occupation" rate	slowly (encroachment?) faster (invasion/rockets?)	slowly (encroachment?) faster (invasion/rockets?)
Consequence of site "occupation"	polygon (empty/colour filled) symbol (absent/visible)	polygon (empty/colour filled) symbol (absent/visible)
On completion of all "occupation" (A)	make symbols (dimly) visible blink symbols (randomly) symbols (occasionally) visible empty polygons (or colour fill)	make symbols (dimly) visible blink symbols (randomly) symbols (occasionally) visible empty polygons (and colour fill)
On completion of all "occupation" (B) -- success scenario	highlight (blink) connectivity lines through 5-fold and 6-fold outline symbols	highlight (blink) connectivity lines through 5-fold and 6-fold outline symbols
On completion of all "occupation" (B) -- success scenario	highlight (blink) connectivity lines through <b>clusters</b> of 5-fold and 6-fold outline symbols	highlight (blink) connectivity lines through <b>clusters</b> of 5-fold and 6-fold outline symbols

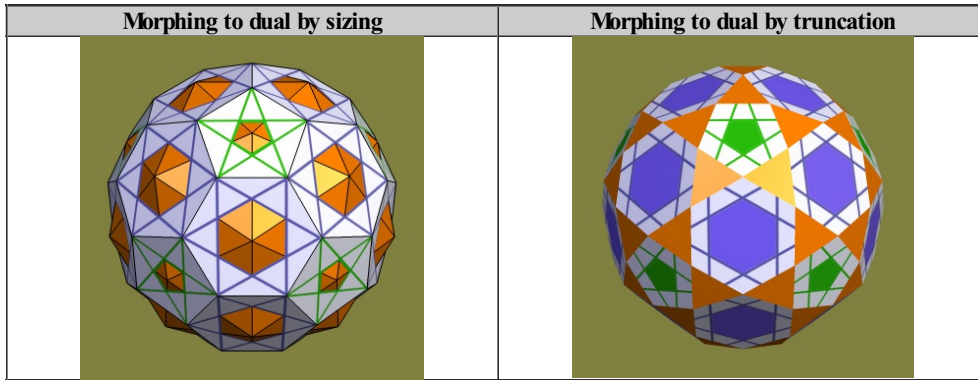
On completion of all "occupation" (D) -- failure scenario	move 5-fold polygons back to origin, emptying site locations	move 6-fold polygons back to origin, emptying site locations
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## Polyhedral catalysts of global imagination

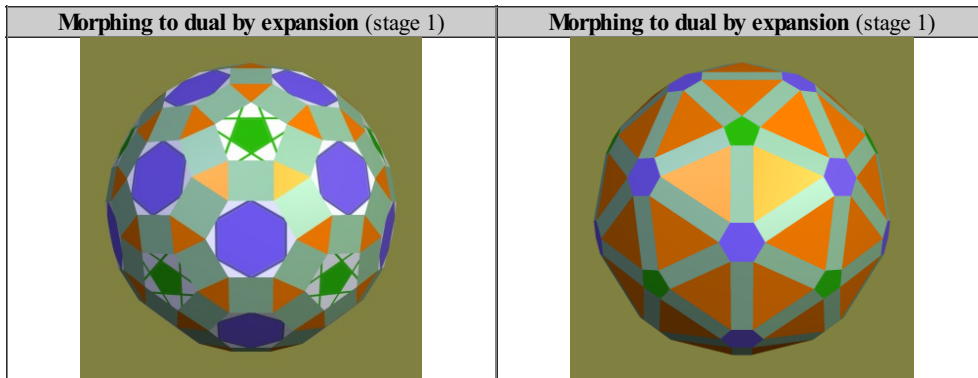
The following images display the truncated icosahedron from the animation above. The 5-pointed star and 6-pointed star are however displayed in white backgrounds with an alternative colouring scheme. The remaining images show various geometrical transformations of the form using the features of the Stella Polyhedron Navigator. The purpose is to encourage imaginative reflection through alternative windows on the relationship between the 5-fold and the 6-fold -- as it may apply in particular to the Middle East. As with the various [map projections](#) of the globe, the images are included to provoke the question as to what they might variously offer, if anything, as a new way of seeing the relationships there.





The [polyhedral dual](#) of the truncated icosahedron is the [pentakis dodecahedron](#) into which it may be transformed by various morphing processes illustrated by the following images. Especially intriguing is the distinctive emergence of the triangle. If considered representative of Christianity, as the pentagon and hexagon are associated here with Islam and Judaism respectively, these highlight the potential of a polyhedral pattern language to explore the cognitive challenges of Jerusalem and the holiness for which it is esteemed -- irrespective of the political implications. However the dynamics of the transformation through morphing -- constrained by the geometry -- suggest that the "language" needs to be understood **dynamically**, rather than with respect to any particular **static** extreme. This argument relates to that with respect to resonance, as explored below.

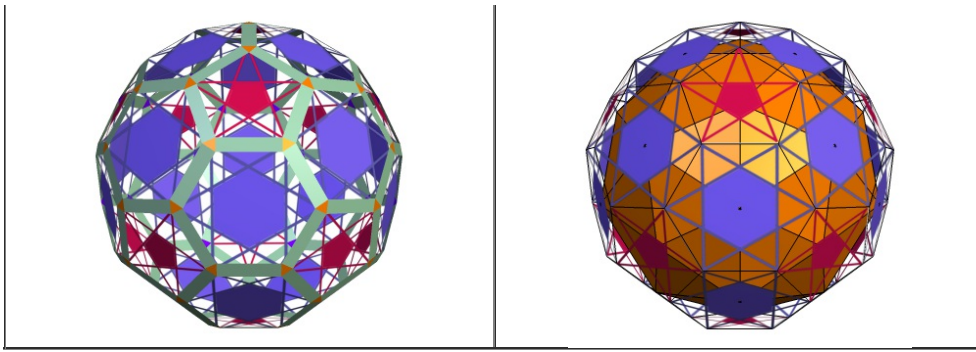


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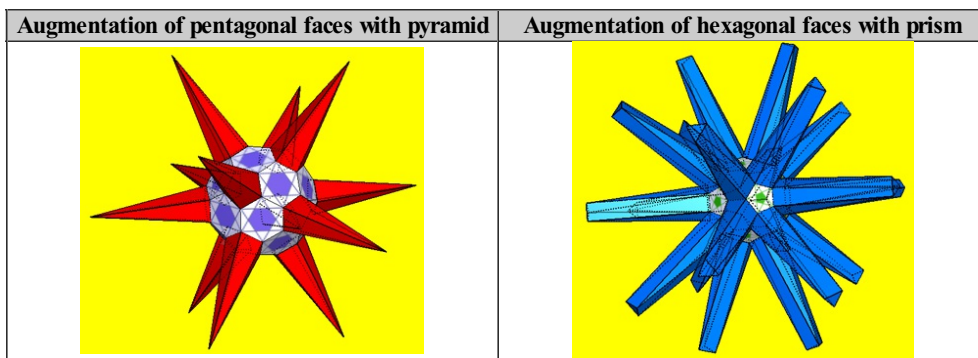
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Morphing of transparent variant (example 1)	Morphing of transparent variant (example 2)
	

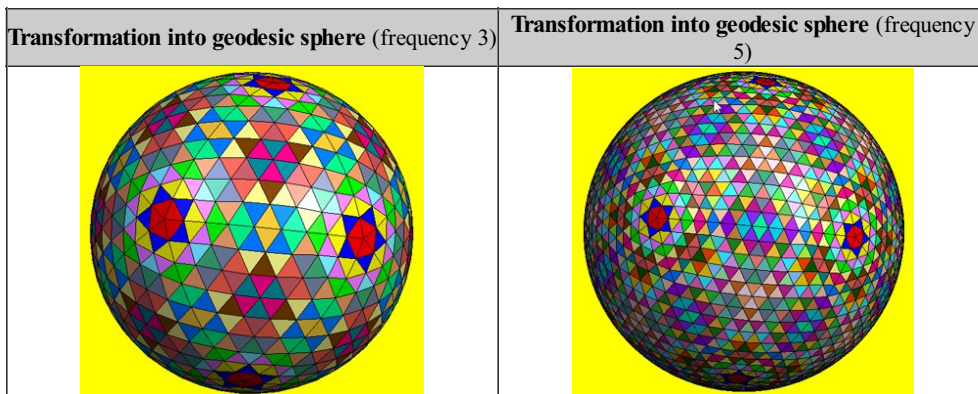


The implications of any morphing process can also be explored from the perspective that the truncated icosahedron is a form of "compromise" between an embedded dodecahedron with 20 vertices (touching the 20 hexagonal faces) and an embedded icosahedron with 12 vertices (touching the 12 pentagonal faces). With one or other emerging dynamically to a greater degree through morphing, this recalls the importance attached by R. Buckminster Fuller to a related form, the [cuboctahedron](#), which he described as being a [vector equilibrium](#) because of the "pumping" process through which it could be transformed (*Vector Equilibrium and its Transformation Pathways*, 1980).

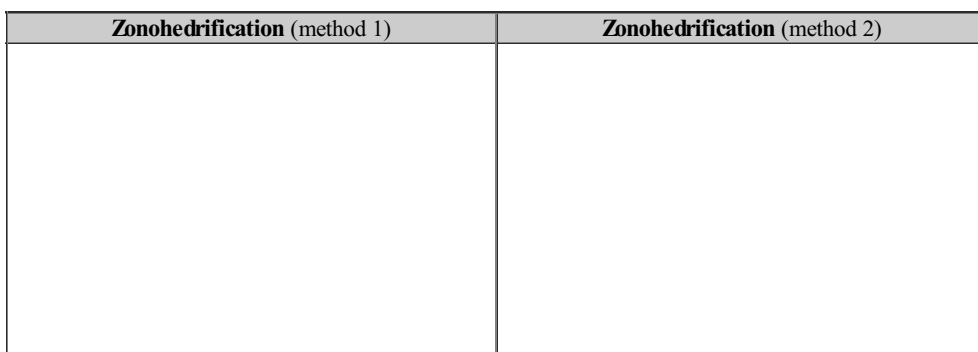
Selected faces of the truncated icosahedron can also be "augmented" as illustrated by the following pair of images.

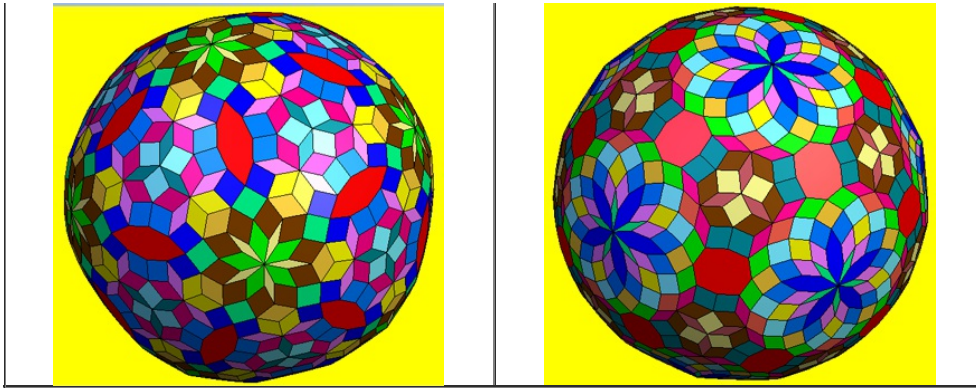


The truncated icosahedron may be transformed into a variety of [geodesic spheres](#) -- according to the frequency, namely the number of subdivisions along each edge of that original polyhedron.

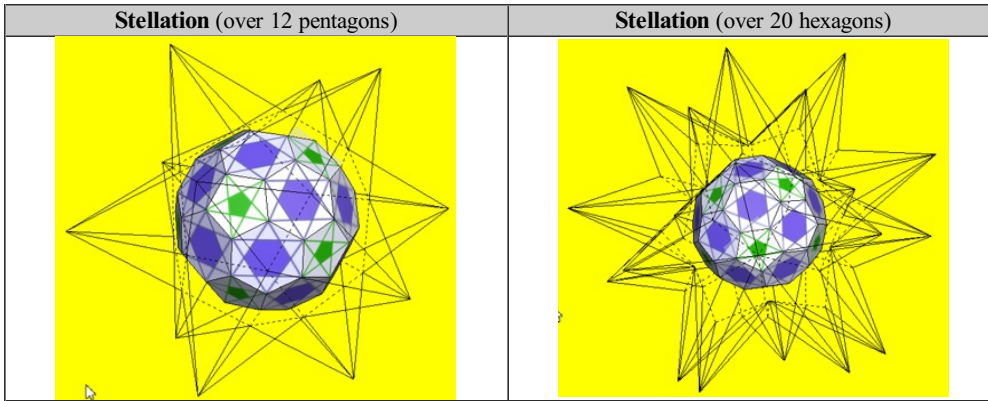


The original truncated icosahedron may also be transformed into a [zonohedron](#) according to various rules, as illustrated by the following.

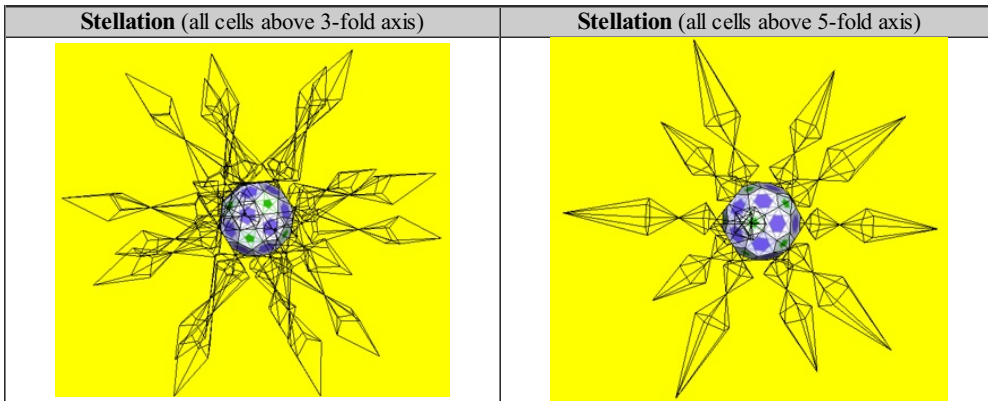




The original truncated icosahedron may be subjected to a [stellation](#) process, namely producing a new polyhedron with faces that lie in the same planes as the faces of that original. The process consists of extending elements such as edges or face planes, usually in a symmetrical way, until they meet each other again. There are 1117 stellations identified by Stella Polyhedron Navigator of which particular examples are given below.

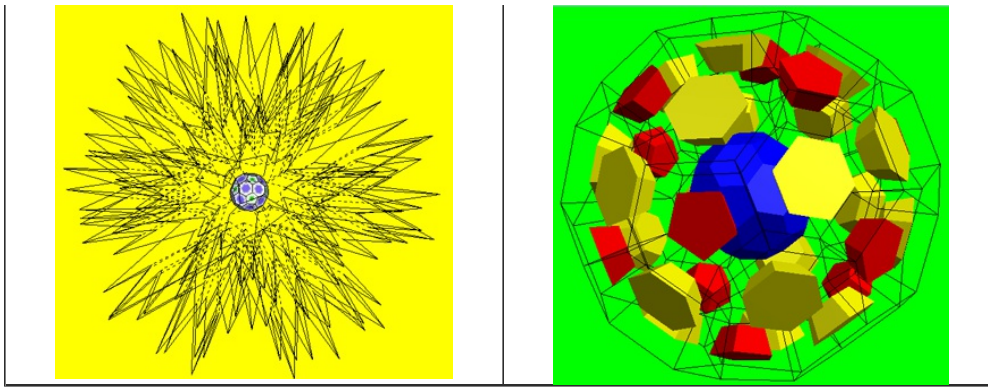


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In addition to the final product of the stellation process (shown below), various transformations of the original truncated icosahedron are possible within Stella Polyhedron Navigator in order to give a "glimpse" of their structure projected in four dimensions. One of these is presented below.

Final valid stellation (of 1117)	Elaboration of four-dimensional prism



## Design, pattern language and geometry

The visualization gives focus to the possibilities of design in relation to [pattern language](#) and geometry. Design and pattern language have been a primary focus of [Christopher Alexander](#) in the quest for the qualities characteristic of an attractive place to be (*A Pattern Language*, 1977). This suggests the extension of "pattern language" to non-material domains, as experimentally elaborated separately (*5-fold Pattern Language*, 1984) as part of larger exercise (*Patterns of Conceptual Integration*, 1984) including a review of *Patterns of N-foldness: Comparison of integrated multi-set concept schemes as forms of presentation* (1980).

.In the light of Alexander's more recent work (*The Nature of Order: an essay on the art of building and the nature of the universe*, 2003-4) he has stressed the geometrical aspects of order in a quest for a computational approach to "wholeness-extending" design (*Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole*, *International Journal for Unconventional Computing (IJUC)*, 2009). Significantly he concludes that paper with the statement:

I hope the idea of harmony-seeking computation may then sit alongside other methods as a new tool in an armoury of well-founded computational techniques to be used when appropriate. It is likely to be appropriate whenever a computational task is defined more by issues of adaptation, health, wholeness, and wellness, with reference to the position some system in some still larger whole, or perhaps even by a desire for beauty, or life, or elegance. All these might one day play a key role in very general kinds of computation. Science, architecture, biology, ecology, physics, cosmology - and computation - may all be the better for it.

This approach is discussed separately (*Harmony-Comprehension and Wholeness-Engendering: eliciting psychosocial transformational principles from design*, 2010).

The question in relation to the design of any animation is what is the design which most fruitfully integrates disparate elements, whether visually or in terms of the rhythm and pace of its "story". The suggestion here is that a "design platform", such as offered by SVG, would enable many to experiment with different stories and to focus dispute on the implications of those stories as designs which could potentially embody greater elegance -- or recognize different preferences in that respect.

As implied by the quest of Alexander, are there designs in three-dimensions which might embody a greater sense of harmony?

## Dialogue implications of design and geometry

One approach to the pentagonal and hexagonal spaces is to see them as different dialogue arenas, whether in terms of theme or style. Common sides in any configuration of those spaces -- pentagon meets hexagon -- could be interpreted as a common discussion thread, or as an issue confronted from two sides.

In complex situations like the Middle East there are many fora and many discussion threads. There is little attention to how these constitute any kind of system -- or to how different dialogue arenas might be related. The issue has been partly explored separately (*Interweaving Thematic Threads and Learning Pathways: nonnautics, magic carpets and widdomes*, 2010), notably in relation to configuration through the web in three-dimensions (*Spherical Configuration of Interlocking Roundtables: Internet enhancement of global self-organization through patterns of dialogue*, 1998), and notably in relation to governance (*Enabling a 12-fold Pattern of Systemic Dialogue for Governance*, 2011).

With respect to a pattern of dialogue arenas envisaged as "two-dimensional", the question is how to envisage the "hooking capacity" which would enable the pattern as a whole to be "folded" into "three-dimensions" -- whatever that may be understood in terms of greater integration, "clicking into place", or enabling emergence and recognition of understandings of a subtler order. Such understandings are of course acclaimed in principle in the case of faiths obliged to offer two-dimensional reflections of an understanding which cannot be fully embodied within such constraints.

A pattern of dialogue arenas in "two-dimensions" may simply not be viable, however they are shuffled into different configurations -- as suggested by the animation. To what degree it may "work" in "three-dimensions" remains to be determined, and therefore merits attention.

Reference is made to the football design, as an illustration of the truncated icosahedron when describing carbon [buckyballs](#) or the root structure of [geodesic domes](#), as separately discussed (*Understanding Sustainable Dialogue: the secret within Bucky's Ball?* 1996).

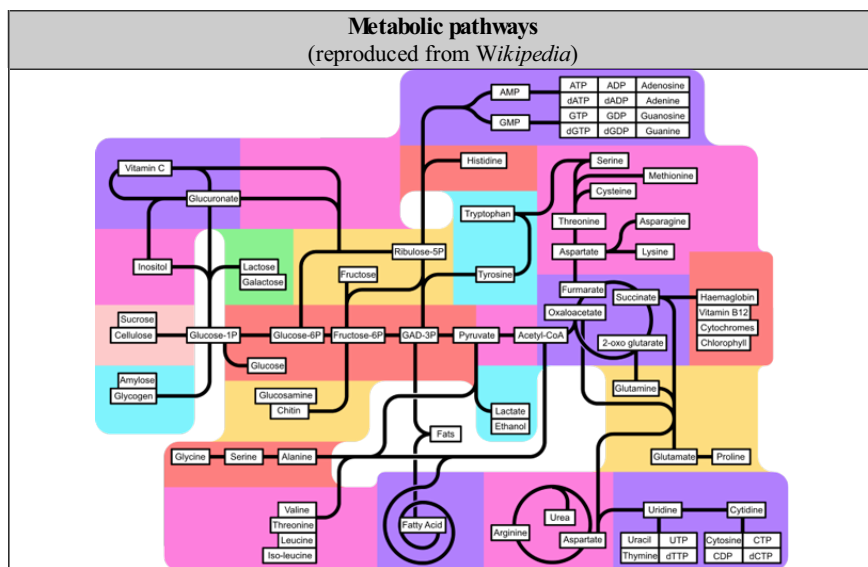
## Systems perspective on three-dimensional cognitive configuration and communication

There is widespread familiarity with systems diagrams, wiring diagrams and circuit boards as represented in two dimensions. They embody insights into connectivity vital to the operation of many systems. The two-dimensionality of the representations may of course obscure the fact that cross-over points do not necessarily imply connectivity.

The key question raised by the Middle East dynamic is how much connectivity is required to channel dynamics which otherwise take chaotic form? What form does this connectivity need to take? How complex does the pattern need to be to hold the complexity of the situation -- specifically to reconcile patterns of organization based on 6-fold and 5-fold (cognitive) organization?

It is appropriate to note that the effectiveness of memory operation in supercomputers is dependent on wiring best understood in terms of three dimensional configurations -- notably patterned on polyhedral forms. Why should it be so readily assumed that the complex dynamics of the Middle East do not require consideration of corresponding complexity?

In the light of a biological systems perspective, requisite integration involves a pattern of [metabolic pathways](#). It is possible to see the chaos of the Middle East in terms of sub-systems of such a pattern -- between which adequate connectivity has not been provided to ensure viability. "Viability" means "life" -- as enabled by appropriate feedback loops through which the system is cybernetically controlled.



What gets treated as an irrelevant externality by this approach, as discussed separately (*Reintegration of a Remaindered World: cognitive recycling of objects of systemic neglect*, 2011)? What gets dangerously excluded from any such pattern of checks and balances? Given the concern with change in relation to globalization, it is interesting to note that "metabolism" derives etymologically from "change" and "throw" (in the sense of ballistics, but also of ball in its various senses).

Especially fascinating from this "metabolic" perspective is the sense of a requisite "meta-pattern", as notably articulated by Gregory Bateson (*Mind and Nature: a necessary unity*, 1979):

The pattern which connects is a meta-pattern. It is a pattern of patterns. It is that meta-pattern which defines the vast generalization that, indeed, it is patterns which connect.

And it is from this perspective that he warned in a much-cited phrase: *Break the pattern which connects the items of learning and you necessarily destroy all quality*. How is this to be understood in relation to the symmetry breaking mentioned above -- where it is the higher degrees of symmetry which are essential to the pattern, its memorability, and its communicability over time? How does this relate to governance of sustainability? Of some relevance, from a systems perspective, is the increasing recognizing of the role of trillions of bacteria in the metabolism and viability of the human body -- outnumbering human cells by 10 to 1 (Jennifer Ackerman, *How Bacteria in Our Bodies Protect Our Health*, *Scientific American*, June 2012). Otherwise titled "*The Ultimate Social Network*", the article asks with respect to "*Your Inner Ecosystem*", who's in control?

The question of what patterns need to be connected to enable integrative emergence can be discussed in terms of symbols, sets of principles or values, or other cognitive fundamentals. In the case of faiths, this may be understood in terms of theology -- narrowly understood. However "theology" may be generalized to include any beliefs in fundamental, central subtleties, as separately discussed (*Mathematical Theology: future science of confidence in belief*, 2011; *Patterns of N-foldness: Comparison of integrated multi-set concept schemes as forms of presentation*, 1980).

Also of relevance to the contrasting "orientations" which become evident in three dimensions, is the variety of patterns required to constitute an integrative whole -- an alternative perspective on the cybernetic [Law of Requisite Variety](#) -- as it might apply to knowing within human psychosocial systems. Whereas much has been made metaphorically for the "lateral thinking" which is strangely consistent with "two-dimensional" creativity, and to that extent comprehensible to a degree, more mysterious is what might be implied by

"voluminous thinking" consequent on eliciting "three-dimensional" insight (*From Lateral Thinking to Voluminous Thinking: unexplored options for subterranean habitats in dense urban areas*, 2007).

## Triangulation, connectivity and "stitching": enabling coherent global system dynamics

The role of triangulation has been reviewed separately with respect to [triadic logic](#), [triadic dialectics](#), [triadic strategic applications](#), [triadic conceptualization](#) and

[triadic education and learning](#) (*Triangulation of Incommensurable Concepts for Global Configuration*, 2011). This follows from earlier exploration of *Spherical Configuration of Categories to Reflect Systemic Patterns of Environmental Checks and Balances* (1994).

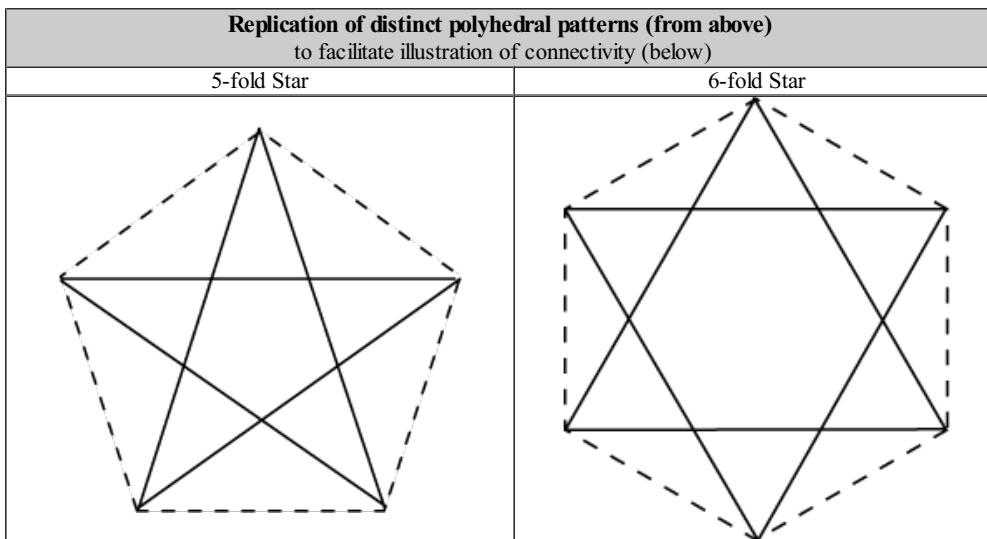
That discussion notably referred to the focus of [R. Buckminster Fuller](#) who argued extensively for the fundamental importance of triangulation as the basis for the stability of structures, notably with respect to his application of spherical triangulation to geodesic domes (*Synergetics: explorations in the geometry of thinking*, 1975). He demonstrates the need for [omnitriangulation](#) as a fundamental requirement of system integrity:

Not until we have *three* noncommonly polarized, great-circle bands providing omnitriangulation as in a spherical octahedron, do we have the great circles acting structurally to self-interstabilize their respective spherical positionings

It is possible therefore that the integrity of psychosocial systems, and the connectivity of the "patterns which connect" of [Gregory Bateson](#), involve an "omnitriangulated" **emotional** engagement.

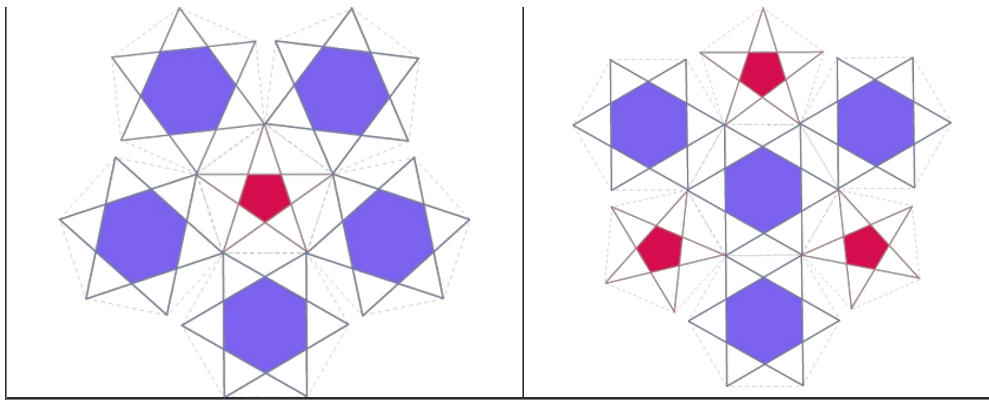
Fuller's insights have been applied separately in the explorations of the structural requirements for the possible polyhedral organization of governance (*Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors*, 2008; *Configuring Global Governance Groups: experimental visualization of possible integrative relationships*, 2008; *Geometry of Thinking for Sustainable Global Governance: cognitive Implication of Synergetics*, 2009).

The argument has also been developed in relation to a degree of intuitive understanding of geometry upon which strategic discourse relies through metaphor (*Experience of Cognitive Implication in Fundamental Geometry: unexamined metaphoric framing of strategic discourse*, 2012). This noted the case made from the perspective of cognitive psychology by [George Lakoff](#) and [Rafael Nuñez](#) (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2001).



The following images show how the lines defining the stars connect together in the truncated icosahedral pattern -- also indicating where necessary connectivity is not (quite) possible in two-dimensions. Note that the function of the external polygons as "vehicles" or templates is now of secondary significance -- although those forms are now engendered within each star form, and coloured accordingly..

Illustration of the two patterns of connectivity between neighbouring star configurations	
Connectivity of 6-fold Stars around a 5-fold Star	Connectivity of 5-fold Stars around a 6-fold Star



The "lines" in the image above also call for reflection on how they might function in systemic terms, especially with respect to the emergent dynamics of any such system:

- according to a **biological metaphor**, they would constitute segments in vital metabolic pathways (shown above) -- only completed when connected in three dimensions.
- according to an **electrical circuit metaphor**, they would constitute elements of a **wiring diagram** or **circuit board** -- again only enabling functionality when the connections are established in three dimensions (as in the case of supercomputer memory organization).
- according to a **brain metaphor**, the links could be understood as **neural synapses**, namely a structure that permits a neuron to pass an electrical or chemical signal to another cell (susceptible of simulation as in the ambitious **Blue Brain Project**)
- according to the (dialogue) **communication metaphor**, these elements can be understood as **discourse threads** -- only enabling integrative discourse when appropriately connected in three dimensions.
- according to a **legal metaphor** in which the linking elements could be understood in terms of vital relationships between elements of a (global) agreement, notably as a guarantee of appropriate checks and balances, and reporting obligations.
- according to a **learning metaphor** in which the links were indicative of **learning pathways** through which knowledge is built progressively, notably ensuring integrative linkage between the elements of knowledge acquired.
- according to an **architectural metaphor** in which structural elements may have distinct functions in distributing tension or compression (or both), as is especially obvious in the case of the **geodesic domes** whose polyhedral structure and dynamics may well be compared to that of a football

Other suggestive metaphors, readily recognized as having greater cognitive relevance, include:

- a **concept map**, namely a diagram showing relationships among concepts, as a graphical tool for organizing and representing knowledge.
- an **argument map**, namely a visual representation of the structure of an argument in informal logic -- typically including the components of an argument such as a main contention, premises, co-premises, objections, rebuttals and lemmas
- a **cognitive map** (known also as mental maps, **mind maps**, cognitive models, or **mental models**) indicative of a pattern of psychological transformations by which information can be acquired regarding the metaphorical spatial environment.
- a **conceptual graph**, namely a formalism for knowledge representation.
- a **topic map**, namely as a standard for the representation and interchange of knowledge

Potentially more intriguing are the implications suggested by:

- reflecting and refracting optical systems whose functioning depends on the mutual angle of orientation of their elements. As a metaphor associated with "vision", these merit careful attention with regard to the capacity to present and focus an image -- as understood with respect to strategy development and eliciting consensus. The cognitive implications of mirroring of notably attracted extensive study.
- aerial arrays dependent for their functioning on the orientation of their elements with respect to one another
- associated with the optical metaphor is the facetting required to enhance the brilliance of precious stones -- with which human values are so intimately related (*Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue*, 2003)

The pentagon and the hexagon, as shown above, are intimately related in geometrical terms to a form of triangulation from which the 5-fold and 6-fold stars emerge. The question which might fruitfully be asked in relation to the earlier arguments concerning the cognitive significance of polyhedra and triangulation, is whether there are systemic implications suggested by the metaphors mentioned immediately above.

The patterns suggest the significance of connectivity being enabled or disabled in dynamic systemic terms -- readily understood in the operation of a model train set in which traffic is allowed or disallowed. If such connectivity were to be understood as sequentially phased, as is characteristic of electrical systems, is there then a sense of effects analogous to those of the rotation operations of motors and dynamos? How might cognitive "cycles" be identified as characteristic of the global organization of the pentagon-hexagon patterns in a truncated icosahedron (cf. *Cognitive Cycles Vital to Sustainable Self-Governance*, 2009)?

## Resonance, fullerenes and the Middle East?

As noted above, reference is often made to football design, when describing carbon **buckyballs** or the root structure of **geodesic domes**,

as an illustration of the truncated icosahedron. "Buckyballs", in honour of R. Buckminster Fuller, was the early term for a "new" form of carbon -- only recently recognized by chemists in 1985. The general term for such cage-like polyhedra molecular structures is now [fullerenes](#) of which the most important is  $C_{60}$  -- as with the number of vertices in a truncated icosahedron, to whose polyhedral structure it corresponds (see especially (cf [Fullerenes](#), *Kirk-Othmer Encyclopedia of Chemical Technology*). It has been inferred that fullerenes  $C_n$  can be constructed for  $n=20$  and for all even  $n= 24$ . They have  $n$  vertices (i.e. C-atoms),  $3n/2$  edges, 12 pentagonal and  $(n-20)/2$  hexagonal faces. Other forms include  $C_{70}$ ,  $C_{76}$ ,  $C_{82}$  and  $C_{84}$  -- with predictions extending to  $C_{3996}$ . The smallest "fullerene" is said to be  $C_{20}$ , which has no hexagonal faces, and with its atoms positioned at the vertices of a pentagonal [dodecahedron](#).

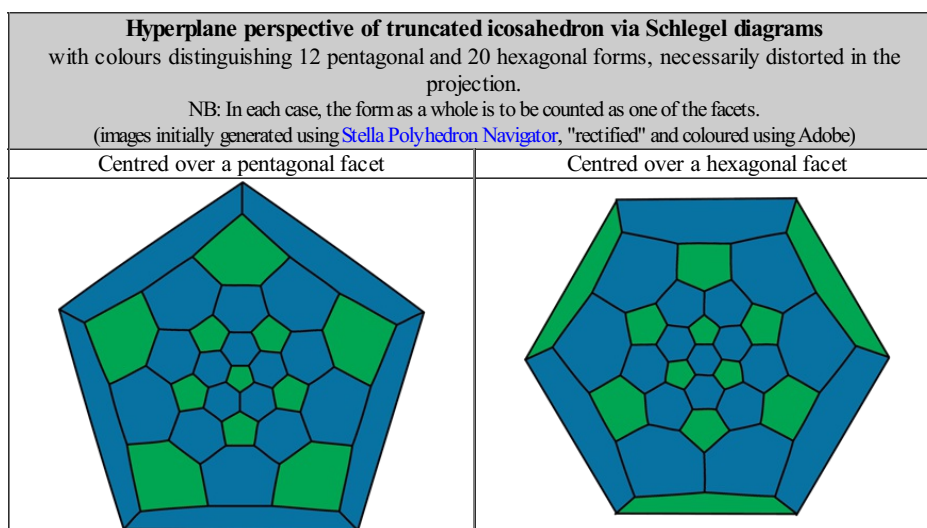
Of the greatest importance to the structural viability of the simplest molecules essential to life is the phenomenon of [resonance](#) whereby the possible bonding between the carbon atoms in the structure takes a dynamic alternating form. It is this dynamic form which is understood as being energetically the most efficient and economic -- giving rise to structures known as [resonance hybrids](#). The structure is then understood to be represented by several contributing structures (also called resonance structures or canonical forms).

The recognition of the fullerenes resulted in early investigation of the nature of resonance within  $C_{60}$ . For example Harald Friepertinger (*The Cycle Index of the Symmetry Group of the Fullerene  $C_{60}$* , 1996), in a section entitled *The resonance structure of the fullerene  $C_{60}$* , notes that it was already known that the fullerene  $C_{60}$  had 12500 resonance structures (D.J. Klein, T.G. Schmalz, G.E. Hite, and W.A. Seitz. Resonance in  $C_{60}$ , Buckminsterfullerene. *Journal American Chemical Society*, 108, 1986, pp. 1301 - 1302). Friepertinger produces a valuable [tabular summary](#) indicating those which are essentially different.

There is now a very extensive mathematical and chemical literature on the nature of the connectivity within the truncated icosahedral form, and especially  $C_{60}$ . This research engenders visualizations which are potentially of great relevance to exploring structural configurations of psychosocial significance. A number are noted in the [references](#) (below), but an especially helpful example is that of Heping Zhang and Dong Ye (*Cyclical Edge-connectivity and Resonance of Fullerenes*, 2007).

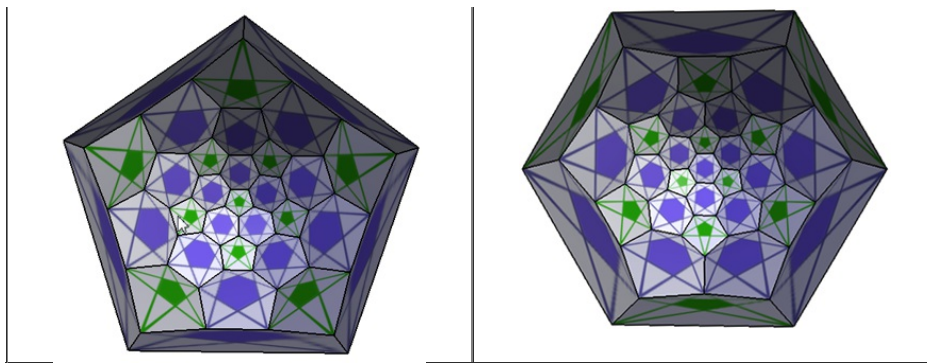
The key question for this argument -- given the truncated icosahedral pattern explored above -- is whether "resonance" in some form, and "cyclical edge-connectivity", have implications for the viability of structures reconciling the differences between the "hexagonal" and "pentagonal" mindsets assumed here to be fundamental to the dynamics in the Middle East. The challenge might well be framed as one of reframing the pattern of edges to form a larger whole (cf *Beyond Edge-bound Comprehension and Modal Impotence: combining q-holes through a pattern language*, 1981). Simply stated, **should the challenges of the Middle East be understood as a problem of resonance -- calling for the quality of thinking applied to resonant structures?** Understood in this way, the very recent recognition of the existence of fullerenes -- as a "new" ordering pattern for one of the commonest elements vital to living structures -- suggests that **current thinking with respect to the Middle East may as yet be insensitive to the viability of more complex possibilities.**

Of particular interest to this approach is the use of a [Schlegel diagram](#) by those exploring resonance within the truncated icosahedron as the polyedral form of the basic fullerene  $C_{60}$ . Such diagrams provide a tool for studying combinatorial and topological properties of [polytopes](#). For three-dimensional polyhedra, they offer a projection into a plane. In the case of a four-dimensional [polychoron](#), this is projected into *3-space*, and is therefore commonly used as a means of visualizing four-dimensional polytopes. Such a diagram is constructed by a [perspective projection](#) viewed from a point outside of the polyhedron, above the center of a selected facet. All vertices and edges of the polytope are projected onto a [hyperplane](#) of that facet.



Experimentally the 5-star and 6-star star images can be presented from the hyperplane perspective (above) as indicated below -- with particular lighting effects.

<b>Hyperplane perspective of truncated icosahedron using star images</b> (addition of images to Schlegel diagram perspective using <a href="#">Stella Polyhedron Navigator</a> )	
Centred over a pentagonal facet	Centred over a hexagonal facet



The standard Schlegel diagram can be used to make evident a unique property of the resonance structure of  $C_{60}$  -- of potential relevance to the above argument -- namely that it avoids double bonds within pentagons and maximizes the number of alternating single-double bonds within hexagons. Thus all hexagons have three conjugated double bonds while all pentagons are empty (as shown in the images below). With respect to connectivity across the pattern, one blogger discusses the possibility that bonding is in some way delocalized across the sphere and offers two alternative bonding patterns (*C60 Double Bonding Networks*, 2010).

<b>Schlegel diagrams of truncated icosahedron distinguishing single and double bonds</b> (orientations corresponding to those above)	
Centred over a pentagonal facet	Centred over a hexagonal facet

Also of great potential interest is recent recognition of what might be understood as the stages in a process of "self-organization" within fullerene polyhedral cages (Stan Schein and Michelle Sands-Kidner, *A Geometric Principle May Guide Self-Assembly of Fullerene Cages from Clathrin Triskelia and from Carbon Atoms*. *Biophys J.* 94, 2008). Such organizational leads merit the most careful attention with respect to the Middle East.

In relation to the animation above, it is clear that the pattern of pentagons and hexagons could be drawn dynamically into a Schlegel configuration and then unfolded out of the hyperplane into spherical form. Whilst this is relatively straightforward using SVG, it is appropriate to note that the four-dimensional features of the Stella Polyhedron Navigator also permit a significant number of perspectival transformations, some of which specifically approximate the perspective from a hyperplane in the form of a Schlegel diagram. Other software exists to generate such diagrams (notably *Polymake*), but possibly less intuitively than the Stella application.

The relevance of a "hyperplane perspective" in this context raises the question as to how appropriate connectivity -- namely global "peace" and "sustainability" -- are best to be represented and comprehended, if not "envisaged". With respect to complex system dynamics for which mathematicians make use of a [complex plane](#), a geometric representation of the complex numbers on a "real axis" and an orthogonal "imaginary axis". Its implications have been considered as a means of exploring the strategic relationship between problematique, resolutique, irresolutique and imaginatique (*Imagining the Real Challenge and Realizing the Imaginal Pathway of Sustainable Transformation*, 2007). The question would then be the relation between a hyperplane and a complex plane with respect to comprehension of "peace" and "sustainability".

**Tiling:** In contrast to the point made with respect to the "jigsaw" impossibility of appropriate tiling of two-dimensional space with respect to Middle East differences, the "hyperplane perspective" indicates how the polygonal "tiles", subject to perspectival distortion, may indeed be configured in alternative tessellations -- seemingly in two dimensions.

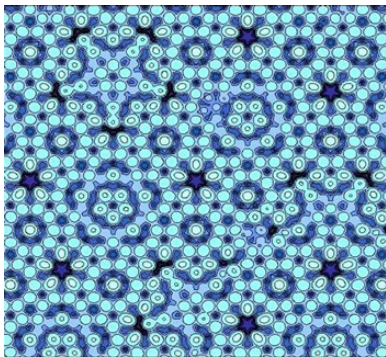
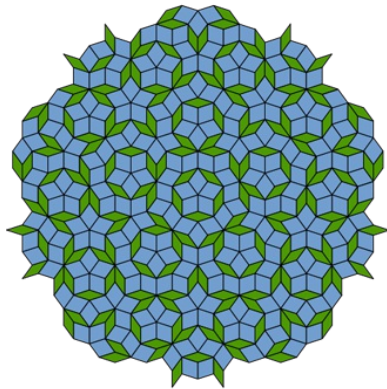
If the tiling metaphor is of relevance to the challenge of reconciling seemingly incompatible frameworks, a further lead is offered by the recent discovery of the unexpected order characteristic of aperiodic tiling patterns of [quasicrystals](#) whose 5-fold symmetry bears an unusual degree of resemblance to what have been recognized as "Islamic patterns" (as noted above). Their discovery revealed a new principle for packing of atoms and molecules -- again suggestive of a mindset necessary to discover new ways of "packing" incompatible perspectives. As with reference to a hyperplane perspective on polyhedra, mathematically, quasicrystals have been shown to be derivable by treating them as projections of a higher-dimensional lattice. There are two known types of quasicrystal:

- polygonal (dihedral) quasicrystals, have an axis of eight, ten, or 12-fold local symmetry (octagonal, decagonal, or dodecagonal quasicrystals, respectively). They are periodic along this axis and quasiperiodic in planes normal to it.
- icosahedral quasicrystals, are aperiodic in all directions.

Again the nature of the mathematical study of quasicrystals helps to frame the question of the nature of the "higher dimensionality" -- to

which the mathematicians of both Islamic and Jewish faith would respectively subscribe in principle -- and what are the possibilities for projections from that context into two dimensions? Of particular interest in the history of quasicrystals is the manner in which their discoverer, the Israeli chemist [Dan Shechtman](#) (Nobel Prize, 2011), was repeatedly [deprecated for his investigations](#) by a previous double Nobel laureate in that same domain. An example for which the alleged *eppur si muove* remains a symbol?

What explorations of other unusual patterns of order are similarly deprecated by those conventionally esteemed as the highest authorities in the matter? With respect to the psychosocial domain, such denial has been honoured by its own acronym TINA ([There Is No Alternative](#)). According to TINA, economic liberalism is the only valid remaining ideology. There is no scope for "new thinking" or the kind of paradigm shift signalled by quasicrystal discovery.

Images relevant to discussion of "tiling" according to the pattern of aperiodic quasicrystals (reproduced from <a href="#">Wikipedia entry</a> )	
Example of quasicrystal pattern	Example of Penrose tiling pattern
	

## Peace as a meta-pattern of resonance: psychosocial, implicit and emergent?

There is of course a very extensive literature on peace -- from a multitude of perspectives. In the case of the Middle East -- as in other arenas of continuing violence -- concern is repeatedly expressed with respect to the "peace process", its breakdown, and "getting it back on track". The argument above is merely a further development of a possibility previously envisaged ([Computer-aided Visualization of Psycho-social Structures: peace as an evolving balance of conceptual and organizational relationships](#), 1971).

"**Voluminous thinking**"? The emphasis here is on the contrast between the peace so desperately sought within a two-dimensional framework and that implied by a three-dimensional framework. The feasibility of peace in the former case were caricatured by comparison with the "forcing" the pieces into a jigsaw puzzle -- despite the fact that they do not interlock. Banging them in is not a viable solution -- irrespective of the quality of the "lateral thinking" applied to that process. The question explored here is with regard to a three-dimensional framework. Might the pieces fit together when understood in that way? The point was made above by reference to "voluminous thinking", illustrated by the example of housing shortage ([From Lateral Thinking to Voluminous Thinking: unexplored options for subterranean habitats in dense urban areas](#), 2007).

A major concern is that **two-dimensional peace** is intuitively comprehensible and appealing, notably in the light of its implications regarding resolution of disputes over flat territory. However it is evident that this is perspective is inadequate -- as when the jigsaw puzzle pieces do not "fit" together. **Three-dimensional peace** indicates that it might imply counter-intuitive comprehension -- as suggested in some measure by the Biblical peace which "[passeth all understanding](#)". Hence the value of animations. The symmetry group mathematics and its visual renderings suggest that there may forms of peace which are an even greater challenge to comprehension ([Dynamics of Symmetry Group Theorizing comprehension of psycho-social implication](#), 2008).

Ironically the three-dimensional approach lends itself more readily to comprehensible visualization as illustrated by the association football with which people interact worldwide. A "peace football" could be made simply by appropriately applying the symbols of the two conflicting Middle Eastern cultures to each of its 32 facets. This could well enable reflection on [Transcending Simplistic Binary Contractual Relationships: what is hindering their exploration?](#) (2012).

**Peace as a meta-pattern:** The nature of cognitive incommensurables, as reflected in geometrical patterns, may require the exploration of far more paradoxical forms. For example, the Stella Polyhedron Navigator facilitates visual exploration of patterns in four-dimensions to some degree. The epistemological implications were explored to a degree in the later work of Gregory Bateson ([Angels Fear: towards an epistemology of the sacred](#), 1987). In this sense peace is best understood, not so much as a pattern in three dimensions, but as a **meta-pattern** to which Bateson refers (above). Such consideration of meta-pattern is evoked in Quaker reflection ([Approaches to Peace and Non-Violence for Modern Quakers](#), 2005).

**Emergent insight:** Curiously the process of constructing a football from flat pieces results in a form of "**emergence**" of a new pattern when these are folded and stitched into spherical form. The step-by-step logic of this process can be readily followed, although it is only through the making process (understood as [poiesis](#)) that its significance is best realized -- as the emergence of a new form of order. It might be said to defy explanation and justification within a two-dimensional framework. Relevant arguments regarding recognition of the bigger picture implied by peace are offered by [Silvia Hartmann](#) ([Meta Patterns and Genius: a simple game to increase a child's intelligence](#), 2005; [The Problem With Peace: why peace is NOT the answer](#), 2009). Does the surprising recent recognition of a new form of carbon -- previously assumed to be one of the commonest and best known elements -- imply the possible emergence of a new form of peace?

**Indications of counter-intuitive comprehension:** It is useful to note the variety of metaphorical and other pointers to such a counter-intuitive form -- of which there is evidently a degree of intuitive comprehension:

- understandably, the pieces of a football need to be "curved up" into three dimensions (as suggested by the animation)
- any "curving up" requires a degree of recognition of a virtual centre -- an empty centre around which the curving takes place
- success may be described in terms of "integrating" previously disparate pieces, recognized as "fitting" and an "appropriate fit"
- the consequence may be seen as an exemplification of "faith" or "belief", which may be held to be essential to recognition of that emergence and sustaining that counter-intuitive form
- more precisely this may be recognized in terms of the "interlocking" or "interweaving" which ensures (three-dimensional) connectivity previously lacking -- as in the construction of a geodesic dome from disparate elements
- failure may be described in terms of the classic phrase "the centre cannot hold" in the poem by [William Butler Yeats](#) (*The Second Coming*, 1919). It may also be described in terms of something having been "broken", possibly in the form of faith or trust, consequent upon its abuse (cf *Abuse of Faith in Governance*, 2009). This is curiously marked by characterizations of the statements of politicians as "empty" -- as with respect to the outcome of the recent Earth Summit.
- efforts may be made, by those who otherwise deprecate the intangible nature of "faith", to remedy such disintegration by "confidence-building" measures -- as currently evident with respect to the financial system (and bankers), the political system (and politicians), science (and scientists), and the challenge to Iran (with respect to the intentions of its nuclear programme).
- in the case of the global financial system, such confidence-building is recognized as vital to ensuring necessary circulation of resources. Emergent "confidelity" is however ambivalently related to abuse of "confidence" to which systems dependent on confidence are vulnerable (*Primary Global Reserve Currency: the Con? Cognitive implications of a prefix for sustainable confidelity*, 2011).
- such "circulation" may be understood more generally in terms of intangible "values" (*Circulation of the Light: essential metaphor of global sustainability?* 2010). In terms of the three-dimensional forms indicated above this circulation may be especially associated with [great circle](#) pathways whose interlocking is fundamental to the integrity of the system. The counter-intuitive subtle nature of these "pathways", and how they may be "walked", might even be associated with a mythical "elven" quality (*Walking Elven Pathways: enactivating the pattern that connects*, 2006).
- the capacity enabled by transformation from a "flatland" mindset into a form articulated in three-dimensions may be recognized in legend as the special interweaving required to construct and empower a "magic carpet" (*Magic Carpets as Psychoactive System Diagrams*, 2010) or the more recent possibility of constructing a web-enabled "wizdome" (*Transforming Static Websites into Mobile "Wizdomes": enabling change through intertwining dynamic and configurative metaphors*, 2007).

With respect to the "weaving" metaphor of connectivity, it is appropriate to note the beading diagrams for the fullerenes as explored by Bih-Yaw Jin (*C60 beading procedure. The Beaded Molecules: the wonderful world of beaded fullerenes*, October 2011). It is also appropriate to note the importance of carpet design to the principles of order identified in the work of Christopher Alexander (*A Foreshadowing of 21st Century Art: the color and geometry of very early Turkish carpets*, 1993).

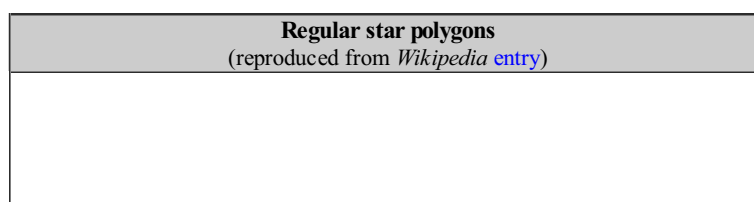
**Design as its own metaphor:** It is intriguing that the challenge of "peace", especially in the Middle East, could be framed metaphorically in terms of design and the tools which currently enable it. The production of the above animation, using one of the common design applications, enables a two-dimensional pattern to be manipulated into three dimensions. The key to this process is the common instruction to "select all", thereby grouping all the elements on the drawing board. With respect to "peace", the sense of how to "select all" remains elusive, usefully understood as the challenge of "seeing things whole" or the ability to distinguish the "wood from the trees". The tendency is simply to select a sub-set of elements, treating the remainder as [externalities](#), as separately discussed (*Reintegration of a Remaindered World: cognitive recycling of objects of systemic neglect*, 2011).

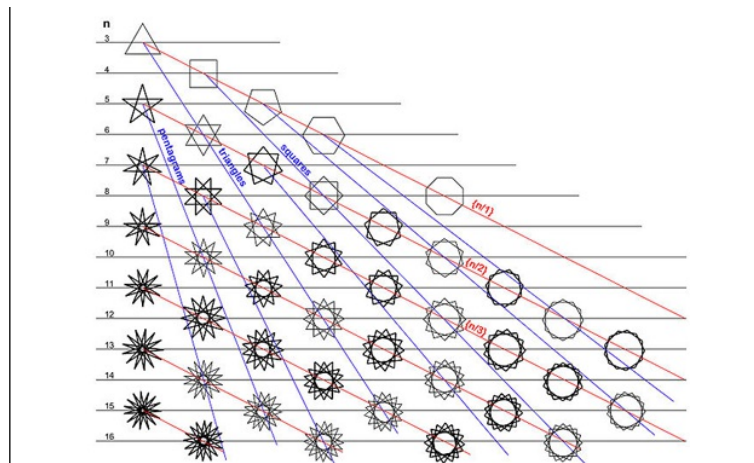
Echoing the quest of physicists for a Theory of Everything in a universe characterized by subtle understanding of curvature, peace might then be understood in terms of enabling curvature into a "global configuration of everything". The challenge might be expressed metaphorically in terms of (en)activating the "great circle" pathways.

## Incommensurable cognitive patterns and their symbolism

The (questionable) assumption of this argument is that effective choice by a group of a primary symbol -- as on its flag -- reflects the distinctive identity of that group and its engagement with life. The focus here is on the 5-fold in relation to the 6-fold. Similar arguments -- and analogous animations -- could be explored with respect to other psychosocial relations that are intractable and incommensurable in "two-dimensions", perhaps 3 vs 4, 4 vs 5, 6 vs 8, etc.

**Recognizing contrasting systems of belief:** The 5 and 6-pointed stars on which this exploration is based are instances of a set of [star polygons](#) which have not as yet been formally defined. Only the most regular ones have been studied in any depth. They can be represented as a sequence of [stellations](#) of a convex regular core polygon. *Wikipedia* offers convenient images of [stellation models](#) of associated polyhedra (see stellations of: [octahedron](#), [dodecahedron](#), [icosahedron](#), [cuboctahedron](#) and [icosidodecahedron](#)). The image below makes clear that the above argument relates only to first two instances in the second diagonal line. Other stars in the diagram are potentially variously significant to other belief systems.





**Embodiment of vital insights by the "other":** In an intractable conflict it is interesting to explore how their "reconciliation" in a three-dimensional configuration would seem to highlight other number-based patterns in an unexpected manner. In the truncated icosahedron, it is the "Islamic" 5-fold which effectively represents the pattern of "12 tribes" so important to the traditions of Israel. There are 12 pentagons in which the Islamic Star has been embedded. Is this characteristic of the sense of problematic "otherness" -- as psychologists would tend to claim? What might the pattern of 20 hexagonal -- with the embedded Star of David -- be recognized as representing as vital within Islamic tradition? Rather than explore this possibility, the question it raises is how the numbers associated with truncated icosahedron (faces, edges, vertices, etc) might interweave elements of both traditions.

**Failure of systemic connectivity:** The widespread importance of 12-fold pattern, notably characteristic of jury systems and the like, is noted separately (*Checklist of 12-fold Principles, Plans, Symbols and Concepts*, 2011). Their operational implications have also been explored (*Enabling a 12-fold Pattern of Systemic Dialogue for Governance*, 2011; *Eliciting a 12-fold Pattern of Generic Operational Insights: Recognition of memory constraints on collective strategic comprehension*, 2011). The disconnect between the 5-fold and 6-fold in the Middle East is an indication of a failure of systemic connectivity and its requisite underlying conceptual organization.

**Polyocular strategic vision:** Whether in that particular case, or in the case of other concept schemes which could be mapped into a set of polygons in two dimensions, each polygon can be understood as having a particular systemic function. In effect each operates as a distinctive "eye" -- an organ of strategic "vision". Configured in three dimensions, it is more understandable that each relates to a subsystem with which a form of "subunderstanding" is associated -- as with dependence on the right or left eye when both are required for depth perception through stereoscopic vision.

A 12-fold vision, implicit in the justification of jury size, is then to be understood as enabling "polyocular vision" as articulated by Magoroh Maruyama (Polyocular Vision or Subunderstanding? *Organization Studies*, 2004). Assumptions relating to a singular vision, or a single strategic sense, can then be usefully challenged (*Cyclopean Vision vs Poly-sensual Engagement*, 2006). How does each of the 12 both understand and not understand -- as implied by "blindspots" and "horizon effects" within the vision metaphor?

**Polyhedra as systems -- of belief:** Also intriguing is the sense in which configurations of pentagons around hexagons, or hexagons around pentagons (as presented above) -- implying extended zones of discourse and systemic preoccupation -- may overlap or interlock within the three-dimensional structure. It is appropriate to note the arguments of R. Buckminster Fuller by which polyhedra, especially the regular polyhedra, are to be understood as systems. This reinforces the subsystemic status of individual polygons, or clusters of polygons, through which any three-dimensional structure is configured.

The case of the Middle East offers a further possibility for exploration -- given the role of Christianity, most notably in relation to Jerusalem. This suggests the need to identify polyhedra which combine polygons representative of Islam and Israel with those distinctively representative of Christianity -- perhaps a triangle.

It follows that there is the intriguing possibility of not only associating the expression of faith with a polygon -- ironically to be caricatured as "putting a face" on each. Going further, there is the possibility of associating the integrity of that faith as a "closed system" with a regular polyhedron, potentially as follows::

- **tetrahedron** (4 triangular faces)
- **hexahedron** / cube (6 square faces)
- **octahedron** (8 triangular faces)
- **dodecahedron** (12 pentagonal faces)
- **icosahedron** (20 triangular faces)

These could then each be understood as the vehicle or container for the respective faith -- when unchallenged by any "other". The mutual challenge of the main faiths is then a form of "geometrical challenge" between the regular polyhedra. In the case of the Middle East, primarily between the dodecahedron and the icosahedron, say. This symbolic mapping could be readily deprecated as simplistic but this would preclude more complex understandings arising from the possibilities in which polyhedra morph into each other dynamically to express particular insights under certain conditions.

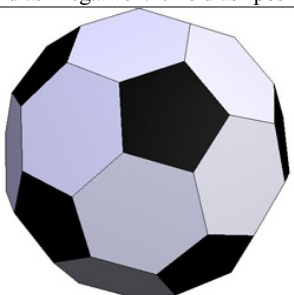
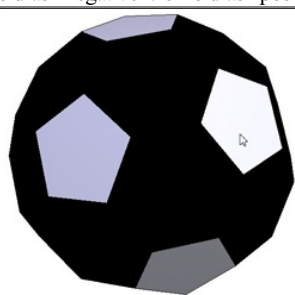
If polygons and polyhedra can be usefully understood as representative of faiths -- if only as mnemonic mappings -- this then raises the possibility of a useful relationship between any periodic table of polytopes (polygons and polyhedra) and any periodic table of religions, as separately discussed (*Tuning a Periodic Table of Religions, Epistemologies and Spirituality -- including the sciences and other belief*

systems, 2007). Within such a framework an atheistic belief system might be appropriately, or provocatively associated with the [nobles gases](#).

**"Resonance"?: Given the role of resonance in enabling the biology of living structures, is there the intriguing possibility that a process akin to resonance may play a vital role with respect to the viability and sustainability of psychosocial systems -- and to "belief"?** Is there some intuition that the currently much-challenged "bond" vital to "confidence" in any psychosocial system -- "my word is my bond" -- has characteristics which could benefit from the insightful consideration of the dynamics of bonds in chemical molecules? How might "confidence-building" then be understood?

Of relevance to this possibility is the extensively articulated description of one process of the [Arab Spring](#) by [Gaston Gordillo](#) (*Resonance and the Egyptian Revolution, Space and Politics*, 6 February 2011), variously indicating:

- What has coalesced as a powerful, unstoppable force on the streets of Egypt is resonance: the assertive collective empathy created by multitudes fighting for the control of space... Resonance is what gives life to this human rhizome and the source of its power.
- Everybody feels the resonance reverberating from Egypt and is trying to make sense of it, to name it. But the words seem inadequate, partial, incomplete: enthusiasm, energy, passion, anger, contagion, electrifying, domino effect. These terms name features of resonance but miss its salience as a physical, affective, political force made up of living bodies.
- Resonance has been conceptually invisible for so long because it involves the most immanent, physical, taken-for-granted dimensions of social life: bodies and space, modulated by the same temporal pulsation.
- ... a defining feature of resonance is that it does not stand still. It is mobile and expands, affecting more and more bodies. This is why so many reporters use metaphors of contagion to explain its expansive force.
- Resonance shakes bodies, even foreign bodies, and makes them act out of empathy. And because it reverberates and is contagious, resonance can travel long distances, spreading outwards from its original node.
- Resonance, in short, forces us to look at wider, complex, ever shifting and fluid topographies of unrest that connect and affect distant and seemingly disconnected geographies.
- What is most unfathomable about resonance is its power, a power that has fueled all the revolutions of human history. Resonance can erode and destroy the most powerful of states, especially when it affects the bodies of those with orders to shoot.
- Resonance... is not a metaphor. The power of affectation that these bodies create is as material as the forms of resonance that are studied in physics and travel through air, water, and solids. Understanding political resonances indeed requires developing a physics of politics. Unlike standard physics, which seeks to find predictability in motion, a physics of political resonance involves the shifting patterns of movement by rhizomes of striving bodies coming together and spreading in unpredictable ways. Resonance unfolds in the realm of political contingency yet through well-defined patterns.

<b>Football illustration of questionable attribution of "positive" and "negative"</b> <b>-- of relevance to Middle East perceptions</b> (given that the standard football is manufactured with a variety of panel colourings and markings)	
5-fold as "negative" / 6-fold as "positive"	6-fold as "negative" / 5-fold as "positive"
	

## Conclusion

The irony of the presentation of this argument, on the occasion of worldwide media focus on the patterned movement of a truncated icosahedron in Kiev, was noted above -- as with the United Nations demonstration of its long-standing inability to elicit relevant intelligence in more creative response to ongoing massacres, most notably in the Middle East. The priority given to the application of the insights of mathematicians of different faiths to the development of ballistics was also noted, in contrast to the possibilities of taking more seriously the challenges of violent difference of belief (*Mathematical Theology: future science of confidence in belief*, 2011). This priority is of course also evident in relation to other security-related opportunities, most notably with respect to [cyberwarfare](#).

The irony is all the greater in that technology has advanced to the point at which footballs (and the boots of players) are in process of being enhanced with computer chips to facilitate game pattern analysis. This is to be compared with the total confusion within the international community regarding the emergence of the global financial crisis and how it might best be managed -- seemingly in the absence of any comprehensible simulation of both remedial strategic possibilities and the implications of global strategic incompetence of a very high order.

**Pattern-learning via game-playing:** There is a case for recognizing the extent to which games are played as an unconscious psychosocial device for learning the patterns which are yet to be appropriately acquired in the handling of psychosocial relations. This point has been made with respect to the organization of popularly preferred styles of music, by contrasting the classical patterns implicit in current institutional organization with the modern music yet to acquire relevant organizational expression ([Jacques Attali](#), *Noise: the political economy of music*, 1977). This suggests that the acquisition of insights from biomimicry and technomimicry could be extended

to "ludomimicry" -- following the critical arguments of Roger Caillois (*The Definition of Play and the Classification of Games*, 2006) regarding those of Johan Huizinga (*Homo Ludens: a study of the play-element in culture*, 1950), as discussed by Andrew Brown (*Agon, Alea, Mimicry and Ilinx, Embodied Knowers*).

What is a global civilization endeavouring to learn by kicking around a truncated icosahedron -- whilst others are being kicked to death? How does such a game relate to the possibility outlined by James P. Carse (*Finite and Infinite Games: a vision of life as play and possibility*, 1987)? Curiously, although it has long been known that there can be no hexagon-pentagon polyhedron with less than 20 vertices, a classical theorem exists to show that polyhedra of any size can be created -- composed solely of hexagons and pentagons -- provided that there are exactly 12 pentagons (*Thayer Watkins, Euler's Theorem Concerning Polyhedra Composed of Pentagons and Hexagons: there must be exactly 12 pentagons*).

What pattern dynamics is society exploring, especially in the Middle East, given that the relevant equation from that theorem excludes the possibility of a polyhedron with 22 vertices -- namely the number of players in association football, acclaimed as the world's most popular sport? If the argument above relates the Islamic faith with the 12 pentagons, what then is the implication of the relationship of Israel to the 20 hexagons in the truncated icosahedron? Why then should the number of hexagons in a hexagon-pentagon polyhedron be constrained to half the number of vertices minus 10 (namely with the number of vertices equal to twice the number of hexagons plus 20)?

**Mathematical theology:** What might the neglected discipline of mathematical theology contribute to reframing the relation of Judaism to Islam? Given the identification of Christianity with Jerusalem, how might a "third polygon" pattern be incorporated into a spherical geometry reframing of the cognitive challenge? Especially intriguing is the sense in which the "holiness" so powerfully associated with Jerusalem by the three faiths may imply an exceptionally elegant geometrical representation in which the three are configured together. This is consistent with the more general argument above that the distinctions between the faiths so important to the peoples of the world (including "science") may be strangely related to distinct cognitive patterning preferences -- represented to a degree by polygonal and polyhedral geometry and the possibilities of its configuration, possible in higher dimensions.

**Global connectivity:** Global civilization is currently characterized by erosion of confidence in various forms of authority with concerns widely expressed regarding the need for "confidence-building" -- but with little sense as to what that means. What is the nature of the connectivity required and how can its emergence be facilitated? It is in this sense that the failure of mathematicians is so extraordinary -- given their insights into "connectivity" of the highest order, and their inability to "connect" with those concerned with "confidence" and "belief".

The disconnect, an "empathy deficit", might be compared to the challenge for those on the autism spectrum -- a recognized characteristic of mathematical excellence -- in comprehending the emotional dimension on which faith is based and which ensures cognitive engagement with a form of "psychoactive curvature" (Zippora Arzi-Gonczarowski, *Self, Empathy, Manipulativity: Mathematical Connections between Higher Order Perception, Emotions, and Social Cognition*, 2001). It is this "empathy curvature" which is evoked by spherical geometry and its "global" implications in a faith-based society (cf *Wikipedia* entry on *Empathizing-Systemizing theory*).

**Quest for a bigger "tent":** It is ironic that one of the most significant contributors to the development of spherical geometry, fundamental to the argument above, was Omar Khayyam -- from a country whose bombardment is currently envisaged with the use of ballistic skills to whose development he could be said to have contributed. Curiously Islam and Judaism share in the fundamental metaphorical significance they attach to the protective role of a tent and to its construction -- a metaphor inherited by Christianity to some degree. Omar Khayyam valued the inspiration of his tent-making tradition, as discussed separately (*Global Brane Comprehension Enabling a Higher Dimensional Big Tent? Strategic implication in encompassing nothing and coming to naught*, 2011). Given the counter-intuitive challenge of "stitching" a football, as noted above, it is appropriate to note his self-reflexive use of the metaphor: *Khayyám, who stitched the tents of science*.

What "higher dimensionality", offering connectivity and coherence, would facilitate the evident incoherence and rivalry between faiths at a "lower dimensionality"? (cf. Stephen Prothero, *God is Not One: the eight rival religions that run the world and why their differences matter*, 2011; Thomas Sowell, *A Conflict of visions: ideological origins of political struggles*, 1987). How might such tent-making now be understood (*Towards higher dimensional "tent-making"?* 2011). Renowned for his skills in both mathematics and poetry, Omar Khayyam exemplifies the insight where characterized as "empathy curvature".

Strangely the nature of the "nothing" which a tent usefully encompasses is echoed both in the football, so enthusiastically kicked around globally, and in enthusiasm for the spirit of the game and its outcome, as might be detected in one of the quatrains of his *Rubáiyát*:

*And if the Wine you drink, the Lip you press,  
End in the Nothing all Things end in -- Yes --  
Then fancy while Thou art, Thou art but what  
Thou shalt be -- Nothing -- Thou shalt not be less.*

At a time when the culture of Omar Khayyam is being associated with Nazism (*Israeli foreign minister Lieberman compares Iran to Nazi Germany, The Guardian*, 28 June 2012), the mysteriously dangerous relationship between "thoughtlessness" and "nothingness" merits further consideration, as can be variously argued (cf. *Unthought as Cognitive Foundation of Global Civilization*, 2012; *Exploring the Hidden Mysteries of Oxfam's Doughnut*, 2012; *Swastika as Dynamic Pattern Underlying Psychosocial Power Processes*, 2012).

The poetic poignancy of "nothing" for the individual is now echoed in the collective experience of collapse of confidence, historic deficits, and the failure of decision-making -- so recently exemplified by the Earth Summit. Reframing of remedies in terms of global missile-enhanced "security" then merely conflates the emptiness of political rhetoric, and the annihilation it engenders, with the nothingness of that insight -- as conventionally understood. It is the cognitive implication of the spherical geometry capable of

encompassing and containing "nothing" that then merits attention. Football offers a good reminder.

**Facilitating imagination:** The argument has been presented using relatively sophisticated computer applications -- although readily accessible. In highlighting the need for a computer-assisted pattern language, the point was made that the Stella Polyhedron Navigator offers numerous facilities vital to education of spherical imagination, as previously argued (*Polyhedral Pattern Language: software facilitation of emergence, representation and transformation of psycho-social organization*, 2008). With the rapid development of mobile applications on smartphones, it is not difficult to predict that such facilities will soon enhance social networking interactivity in surprising ways (*Polyhedral Empowerment of Networks through Symmetry: psycho-social implications for organization and global governance*, 2008).

Whilst such software also enables paper construction of polyhedra for educational purposes, it was also stressed that many "counter-intuitive" aspects of the argument regarding requisite connectivity can be developed using a marker pen and an ordinary football.

**Replicating restrictive territoriality through copyright:** It is appropriate to conclude with a reference to the regrettable tendency for the most remarkable creativity with respect to relevant spherical innovation to be marked by a counter-productive preoccupation with intellectual copyright -- suggesting a tendency to replicate in higher dimensions the problematic patterns so evident with regard to two-dimensional territory. Noteworthy examples relate to the legacy of R. Buckminster Fuller and Stafford Beer (*Beyond Dispute: the invention of team synteegrity*, 1994) and the tragic legal dispute between Stan Tenen and Dan Winter (*Future Coping Strategies: beyond the constraints of proprietary metaphors*, 1992). The only merit this offers is the motivation to explore alternatives by ignoring those based on such misappropriation of the cultural heritage of a global knowledge-based civilization.

#### Afterthought: Relevance of Goldberg Polyhedra as duals of Geodesic Icosahedra

The argument above, focused as it is on the truncated icosahedron, merits amendment and development in the light of the so-called **Goldberg polyhedra**. These are a convex polyhedra made from hexagons and pentagons. They are dual polyhedra of **geodesic spheres**. Icosahedral symmetry ensures that the pentagons are always **regular** and that there are always 12 of them.

Their variety can be explored interactively on the website **Dual Geodesic Icosahedra**. (Dana Mackenzie, *Goldberg Variations: new shapes for molecular cages - flat hexagons and pentagons come together in new twist on old polyhedra*, *Science News*, 14 February 2014). They have aroused interest as being of relevance to new understanding of viral capsids, potentially including the coronavirus (Guang Hu, *Extended Goldberg polyhedra*, *MATCH Communications in Mathematical and in Computer Chemistry*, 59, 2008, 3).

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