laetus in praesens

Alternative view of segmented documents via Kairos

27 July 2015 | Draft

# Optimizing Web Surfing Pathways for the Overloaded

## Polyhedral insights from the travelling salesman problem of operations research

-- / --

Introduction
Travelling consumer problem in terms of polyhedral combinatorics
Symmetrical polyhedra as a key to memorable organization of thematic sites
Polyhedral implications for "cognitive governance" understood in "global" terms?
Transformation: changing cognitive gear and pattern shape shifting
Web navigation of a global configuration?
Implications of closest packing of polyhedra for wizdome design?
Memorable pattern language for configuring sets of websites?
References

## Introduction

Web users have an increasing concern with how best to manage their web surfing experience given the constraints of time and information overload. The question explored here is a possible means of moving beyond a browser checklist of links ("Favourites") and bookmarks, whether or not these are carefully nested within menus and organized by theme.

One early approach was the webring -- popular in the 1990s and early 2000s. This is a collection of websites linked together in a circular structure, and usually organized around a specific theme, often educational or social. Given the subsequent development of search engines, a current priority is building links between websites and ensuring that these are not vulnerable to abusive manipulation. The challenge of organizing relationships between websites can also be explored in terms of the conversation threading of internet exchanges (*Interweaving Thematic Threads and Learning Pathways: noonautics, magic carpets and wizdomes*, 2010).

A fundamental issue for any web user is how many websites to inspect with what priority and with what frequency, given the constraints of information overload. There are naturally limits to how much time to invest, as well as the desire and the capacity to absorb information, as discussed previously (*Investing Attention Essential to Viable Growth: radical self-reflexive reappropriation of financial skills and insights*, 2014).

As explored here, this condition for "information consumers" bears a curious resemblance to much-studied preoccupations of operations research and theoretical computer science -- known as the travelling salesman problem (TSP) and the travelling purchaser problem. These can be explained as determining the shortest possible route between a set of cities, given the known distances between each pair of cities, with a final return to the city of origin. For example, applications exist enabling any vehicle driver to determine the fastest roundtrip between multiple destinations as a complement to Google maps.

The resemblance is all the more striking in that the web user may be as much a consumer of information at each site as intervening proactively there in some form of advocacy or marketing role. More generally the web user could even be understood as adopting an oversight or surveillance posture with respect to any new information on each of the sites selected. The question explored here is how insights from operations research might inform the organization of the hyperlink pathways along which the web surfer might most efficiently travel within the time accorded to such activity.

The degree of possible relevance to web surfing is illustrated by recent research on application of TSP to the consumer shopping experience (James J. Jong Hyuk Park, et al. *Future Information Technology: FutureTech 2013*). Of much greater interest here is the challenge of integrative comprehension of the pattern of insights garnered and cultivated whilst engaged with the web. The possibility explored is that there is a case for polyhedral organization of the pattern to render it memorable and insightful as a whole -- since the constraints of geographic dispersion no longer apply in cyberspace.

If one is in the habit of visiting 5,10, 20, 50 websites (or more) at various frequencies, the issue is whether there are more fruitful ways to organize such a tour? Given the challenge of information overload, does this provide a more meaningful context to explore the increasing problem of "link fatigue" -- namely exposure to yet another link one is encouraged to investigate? Might such organization

enable habitual patterns to be transcended such as to ensure the emergence of more integrative insights?

## Travelling consumer problem in terms of polyhedral combinatorics

The *Wikipedia* entry provides a useful summary of the travelling salesman problem (TSP) with illustrative animations (see also YouTube animations). It remains a very important problem mathematically. Although very well studied, it continues to tantalize researchers. Because of its importance, its ease of description, its many applications, and its historical inspiration for countless algorithms, it is expected to continue to be an important feature of operations research.

The question is how to adapt the language of the original application to the challenge of more effective organization of web browsing under conditions of information overload. Another valuable summary is that of Rajesh Matai, et al. (*Traveling Salesman Problem: an overview of applications, formulations, and solution approaches*. 2010). This notes the distinction between:

- a symmetric travelling salesman problem (sTSP), namely the case of finding a minimal length closed tour that visits each city once, with the distance between two cities being the same in each opposite direction -- thereby forming an undirected graph (and halving the number of possible solutions). In browsing this would presumably be the case where a return link was provided from a given site to the previous site visited
- asymmetric travelling salesman problem (aTSP), namely the case when paths may not exist in both directions or the distances might be different, forming a directed graph -- for example as a consequence of traffic collisions, one-way streets, and airfares for cities with different departure and arrival fees. In browsing this would presumably be the case where no return link was provided from a given site to the previous site visited and another return route had to be found
- a multi travelling salesman problem (mTSP) considers the case of multiple salesmen based at one node. In browsing this is the potential case when use can be made of multiple artificial agents to extend browsing capacity, such as by predetermining which sites might be worth visiting during a given session as a result of having been updated with new information..

Although no mention is seemingly made of the application of any variant of TSP to the browsing experience, the wide variety of developed applications indicated by Matai (2010) show how general is the mathematical problem and therefore how probable it is that interesting insights could be derived for user navigation of cyberspace:

| **Developed adaptations to solutions of the travelling salesman problem** | |
|---|---|
| ***Applications of TSP*** | ***Applications of mTSP*** |
| Drilling of printed circuit boards | Printing press scheduling problem |
| Overhauling gas turbine engines | School bus routing problem |
| X-Ray crystallography | Crew scheduling problem |
| Computer wiring | Interview scheduling problem |
| The order-picking problem in warehouses | Hot rolling scheduling problem |
| Vehicle routing | Mission planning problem |
| Mask plotting in PCB production | Design of global navigation satellite system surveying networks |

Even with a moderate number of nodes, an optimum solution takes an excessive amount of computational time in practice (as indicated in a *Wikipedia* animation of exact algorithms). Considerable attention is therefore given to heuristic and approximate algorithms as alternatives, of which Matai (2010) indicates the following:

| **Heuristic and approximate algorithms to the traveling salesman problem** | | |
|---|---|---|
| ***Tour construction approaches*** | ***Tour improvement*** | ***Other*** |
| Closest neighbor heuristic | 2-opt and 3-opt heuristics | Ant colony optimization |
| Greedy heuristic | k-opt heuristic | The Held-Karp lower bound |
| Insertion heuristic | Lin-Kernighan algorithm | Heuristic solution approaches for mTSP |
| Christofide heuristic | Tabu search | |
| | Simulated annealing | |
| | Genetic algorithm | |

Presumably, without any such formal articulation, web users may already endeavour (without knowing it) to optimize their daily web sessions using approximation processes of these kinds (and others) -- if only unconsciously. This may be as evident with respect to visiting selected shelves in a supermarket, shops in a mall, or TV channel surfing.

Of particular relevance to the following argument is the extensive exploration of polyhedral theory in relation to exact algorithms for solving the TSP -- as indicated in the selection of references (below). These notably draws on polyhedral combinatorics, a branch of mathematics that studies the problems of counting and describing the faces ofconvex polyhedraand higher-dimensionalconvex polytopes. The question is the nature of any correspondence of potential relevance to the organization of web browsing as framed below by the additional constraint of memorability (K. Marriott, et al, *Memorability of Visual Features in Network Diagrams*, *Visualization and Computer Graphics*, 2012).

## Symmetrical polyhedra as a key to memorable organization of thematic sites

In exploring the possible adaptation of solutions to the travelling salesman problem (TSP) to more effective web browsing, a valuable additional constraint can be framed in terms of memory and memorability -- namely that of the web user (as potential consumer or interventionist on the web sites visited). Memory in those terms is not of concern in conventional approaches to the TSP since a valuable solution can simply be presented as a checklist of sites (cities) to be visited.

Such a checklist does **not** require any additional constraint of memorability imposed upon it -- irrespective of whether the recipient of the checklist develops such devices for convenience. The "salesman" is neither required to memorize the checklist as a whole nor to comprehend how the solution has been determined to be optimal. For the "salesman" it is just a question of following instructions to navigate a labyrinth -- as could be provided to any intelligent agent, including a robot.

**Memory requirements**: Curiously "memory" is however of great concern in computing any TSP solution due to the constraints imposed by computer memory management on complex calculations, as evident in some papers regarding enhanced design of memory organization (A. L. Ignatyev, et al, *Solving the Travelling Salesman Problem on Shared and Distributed Memory Multiprocessor System*, 2009; Khushboo Aggarwal, et al. *A High Performance Algorithm for Solving Large Scale Travelling Salesman Problem using distributed Memory Architectures*, *Indian Journal of Computer Science and Engineering*, 2011).

The relevance of polyhedral theory to memory organization of computers is a matter of interest (see *Frameworks supporting the polyhedral model within a computer*; *Polyhedral compilation*) As introduced by Philippe Clauss, et al. (*Polyhedral Techniques for Parametric Memory Requirement Estimation*, 2012):

> Memory requirement estimation is an important issue in the development of embedded systems, since memory directly influences performance, cost, and power consumption. It is therefore crucial to have tools that automatically compute accurate estimates of the memory requirements of programs to better control the development process and avoid some catastrophic execution exceptions. Compute-intensive applications often spend most of their execution time in nested loops. The polyhedral model provides a powerful abstraction to reason about analyses and transformations on such loop nests by viewing an instance, or iteration, of each statement as an integer point in a polyhedron. From such a representation and a precise characterization of inter and intra-statement dependences, it is possible to analyze loop nests statically in a completely mathematical setting relying on machinery from linear algebra, integer linear programming and polynomial algebra. The analyses made on integer points in polyhedra correspond to effective quantitative characteristics of the original code such as the variable liveness or the amount of consumed memory.

**Memorability**: The matter of interest here is any degree of convergence between the polyhedral approaches to TSP and the manner in which a viable solution -- as a pattern of learning pathways -- emerges for the web user and can be recognized as especially memorable. This issue touches upon the science of mnemonics, as notably reviewed by Francis Yates (*The Art of Memory*, 1966) with respect to the method of loci -- appropriately understood in terms of "sites". The method, known in classical Greece and Rome, is a general designation for mnemonic techniques that rely upon memorized spatial relationships to establish, order and recollect memorial content. The term is currently most often found in specialized works on psychology, neurobiology and memory.

**Early solutions to the "roving attention" problem?**

Graphical memory devices from the works ofGiordano Bruno
"Memory-seals". Licensed under Public Domain via Wikipedia.

**Memorability and symmetry**: The organization of sets of places -- websites in this case -- may then be usefully explored in terms of the more general issue of the representation of sets, as discussed separately (*Representation, Comprehension and Communication of Sets: the Role of Number*, 1978; *Patterns of Conceptual Integration*, 1984). As noted by Jan B Deregowski: *Psychologically speaking, symmetry is not a relatively superficial characteristic of pattern and object like, say, color but that it is an essential characteristic which affects profoundly both their immediate perception and their memorability* (*Perception of Symmetry: some psychological observations*,

*Symmetry: Culture and Science*, 1992). As summarized by Sarah Francis (*Perceptions of Symmetry and Color in Environmental Logos*, 2014):

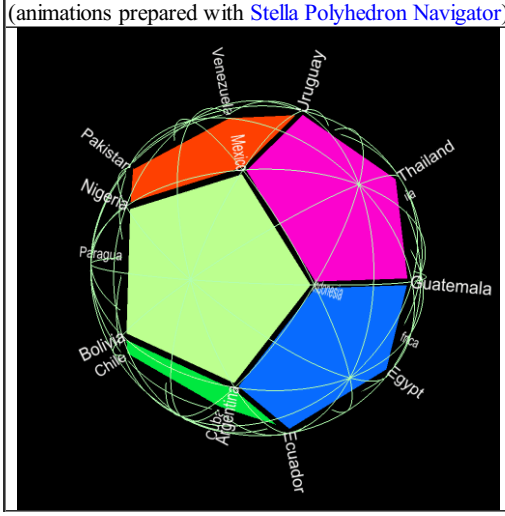
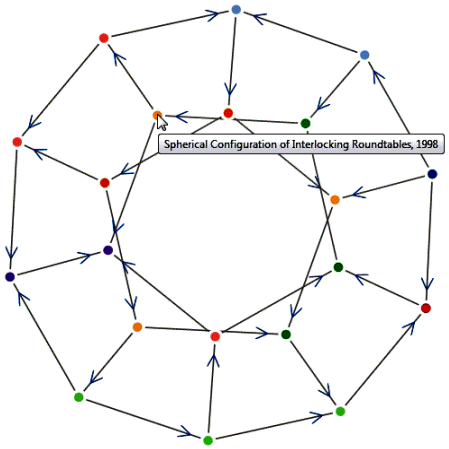
> Since the detection of symmetry results from evolutionary biological processes, it is no surprise that humans detect symmetrical objects faster than asymmetrical ones. Symmetry research has shown that symmetry can be detected preattentively that is, perceived when the stimulus is briefly presented... Researchers have also found that symmetrical patterns are more easily remembered and reproduced than asymmetrical ones. Attneave (1955) conducted three experiments investigating the effect of symmetry on memory and found that in all three studies symmetrical patterns were more easily remembered than a symmetrical ones. Attneave's (1955) study confirmed the Gestalt doctrine that symmetry contributes to 'figural goodness', which is also favorable to memory. Gestalt psychology has long argued that symmetry is the prime determinant of "pattern goodness" (Henderson and Cote, 1998), which also produces positive affect (Clement, 1964; Garner and Clement, 1963).

**Comprehensibility**: The argument can be taken further through the work of George Lakoff and Rafael Nu?±ez (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2001). This brings into focus the interplay between **meaningful comprehension** of a pattern and **analytical explorations** of patterns. A notable "compromise", or "interface" between these extremes is played by polyhedra with varying degrees of symmetry. These offer "sites" at their vertices with which meaning can be associated through mapping -- as in the method of loci. The comprehensible organization of the simpler polyhedra then offers a 3D map of a set of websites -- effectively a form of integration of the preoccupations of the particular web user in favouring those particular sites.

Curiously, but typical of analytical exploration of polyhedra in their most general sense, there are few illustrations of polyhedra or polytopes in presentation of the travelling salesman problem using polyhedral theory. The theory is used to engender checklists of sites, not to render the pattern comprehensible other than through equations. For mathematicians, comprehensibility is associated with the equations and not with visualization of the geometry. The qualification to this remark is that the progression of iterative analysis of a TSP may be occasionally presented as the evolving 2D graph of a network (as illustrated in the *Wikipedia* profile on TSP).

The merit for conventional analysis of removing the constraint of comprehensibility (typically associated with polyhedral symmetry) is that far more complex pathways may be identified. Presumably, presented as a checklist or a minimally ordered graph, this would be satisfactory for some web users in that it offers pointers to ordered exploration of a pattern of websites. The issue of interest here is the extent to which ordering through polyhedral symmetry is correlated with a degree of response to the challenge of information overload. Does the "solution" then reflect the emergence of integrative insight -- as might be required by understandings of "oversight" and "surveillance" of a pattern of domains?

The following images indicate possibilities of associating thematic content with vertices of a dodecahedron (as an example). That on the right associates titles of selected references cited in this document with the vertices, as is evident in the interactive version (based on force-directed graph drawing); arrows and colouring for illustrative purposes only. Neither of the images currently has activated web links to thematic content.

| **Illustrative examples of mapping onto 20 vertices of a dodecahedron** | |
|---|---|
| G20 Group of Developing Countries<br>(animations prepared with Stella Polyhedron Navigator) | Papers referenced in this document (one highlighted)<br>(screen shot of interactive 3D version using d3.js) |
|  |  |

## Polyhedral implications for "cognitive governance" understood in "global" terms?

The argument here is a development of previous preoccupation with the polyhedral organization of institutions and issues in a global context (*Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors*, 2008; *Polyhedral Pattern Language: software facilitation of emergence, representation and transformation of psycho-social organization*, 2008).

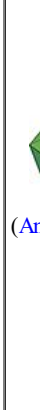
Of particular relevance is the possibility that networks of organizations active in cyberspace might be more fruitfully organized in polyhedral form in order to enhance their collective integrity and viability, as discussed separately (*Polyhedral Empowerment of Networks through Symmetry: psycho-social implications for organization and global governance*, 2008). Similar arguments can be made with respect to the thematic preoccupations of such networks, interrelated as they are through patterns of discourse (*Spherical Configuration of Interlocking Roundtables: Internet enhancement of global self-organization through patterns of dialogue*, 1998).

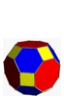
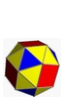
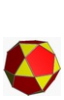
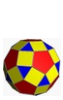
In this light, the focus with respect to the web user, vulnerable to overload, is the form a polyhedral organization of sets of websites might take to enhance the regular browsing experience -- for that user. Given the parallels highlighted above to computer memory organization, the implications of "overload" in both cases merit comparison. Additionally, would the pattern so configured constitute an integrative mnemonic clue to the associated preoccupations? Such "tricks" are characteristic of the skills reported by participants in competitions like the World Memory Championships.

The early use of a webring (mentioned above) offers a clue to more integrative organization. The simpler symmetrical polyhedra are notably characterized by interlocking great circles variously passing through polyhedral nodes, understood here as websites.

| Platonic polyhedra (regular) (reproduced from Wikipedia) | | | | | | |
|---|---|---|---|---|---|---|
| Polyhedron | | Vertices | Edges | Faces | Schläfli symbol | Vertex config. |
| tetrahedron |  | 4 | 6 | 4 | {3, 3} | 3.3.3 |
| hexahedron (cube) |  | 8 | 12 | 6 | {4, 3} | 4.4.4 |
| octahedron |  | 6 | 12 | 8 | {3, 4} | 3.3.3.3 |
| dodecahedron |  | 20 | 30 | 12 | {5, 3} | 5.5.5 |
| icosahedron |  | 12 | 30 | 20 | {3, 5} | 3.3.3.3.3 |

The possibility indicated by the above polyhedra is that, according to the number of websites to be regularly visited, their vertices could be used to map the sites. The edges would then be indicative of possible link pathways between them. More complex patterns, suitable for mapping a larger number of websites, are indicated by the following table from Wikipedia, however especially relevant is another Wikipedia table (*List of uniform polyhedra by vertex figure*).

| Archimedean polyhedra (semi-regular) (reproduced from Wikipedia) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Name (Alternative name) | Schläfli Coxeter | Transparent | Solid | Net | Vertex figure | Faces | | Edges | Vertices |
| truncated tetrahedron | {3,3}  |  (Animation) |  |  | 3.6.6  | 8 | 4 triangles 4 hexagons | 18 | 12 |
| cuboctahedron (rhombitetratetrahedron) | r{4,3} or rr{3,3}  |  (Animation) |  |  | 3.4.3.4  | 14 | 8 triangles 6 squares | 24 | 12 |
| truncated cube | t{4,3}  |  (Animation) |  |  | 3.8.8  | 14 | 8 triangles 6 octagons | 36 | 24 |
| truncated octahedron (truncated tetratetrahedron) | t{3,4} or tr{3,3}  |  (Animation) |  |  | 4.6.6  | 14 | 6 squares 8 hexagons | 36 | 24 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | – ◉ – ◉ | | | | | | | | |
| rhombicuboctahedron (small rhombicuboctahedron) | rr{4,3} ◉ 4 ● – ◉ | (Animation) |  |  |  | 26 | 8 triangles 18 squares | 48 | 24 |
| truncated cuboctahedron (great rhombicuboctahedron) | tr{4,3} ◉ 4 ◉ – ◉ | (Animation) |  |  |  | 26 | 12 squares 8 hexagons 6 octagons | 72 | 48 |
| snub cube (snub cuboctahedron) | sr{4,3} ○ 4 ○ – ○ | (Animation) |  |  |  | 38 | 32 triangles 6 squares | 60 | 24 |
| icosidodecahedron | r{5,3} ● 5 ◉ – ● | (Animation) |  |  |  | 32 | 20 triangles 12 pentagons | 60 | 30 |
| truncated dodecahedron | t{5,3} ◉ 5 ◉ – ● | (Animation) |  |  |  | 32 | 20 triangles 12 decagons | 90 | 60 |
| truncated icosahedron | t{3,5} ◉ – ◉ 5 ● | (Animation) |  |  |  | 32 | 12 pentagons 20 hexagons | 90 | 60 |
| rhombicosidodecahedron (small rhombicosidodecahedron) | rr{5,3} ◉ 5 ● – ◉ | (Animation) |  |  |  | 62 | 20 triangles 30 squares 12 pentagons | 120 | 60 |
| truncated icosidodecahedron (great rhombicosidodecahedron) | tr{5,3} ◉ 5 ◉ – ◉ | (Animation) |  |  |  | 62 | 30 squares 20 hexagons 12 decagons | 180 | 120 |
| snub dodecahedron (snub icosidodecahedron) | sr{5,3} ○ 5 ○ – ○ | (Animation) |  |  |  | 92 | 80 triangles 12 pentagons | 150 | 60 |

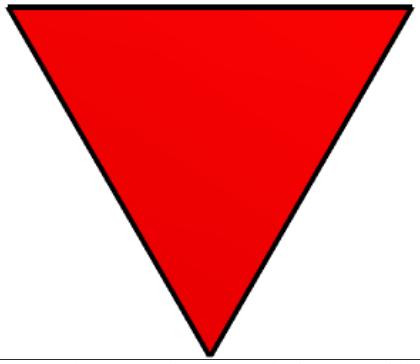
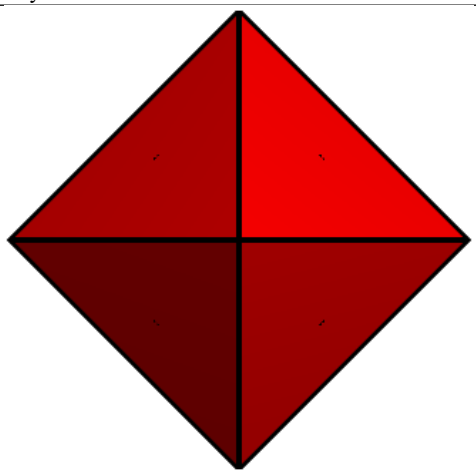
Of further interest is then the order in which these links (indicated by the edges) are traversed by the user within the pattern provided by a given polyhedron. Furthermore, also of interest is the regularity with which given sites are visited within the pattern -- with possible

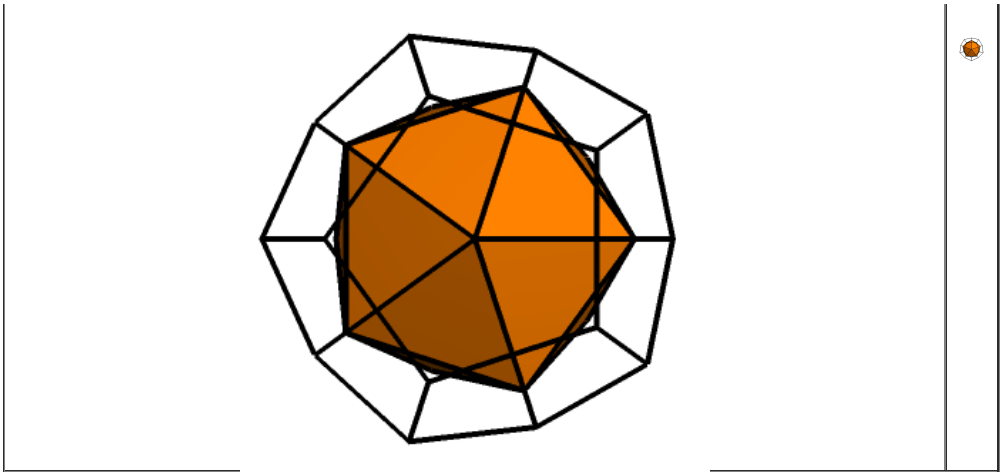
significance to be found in the symmetrical "regularity" of the Platonic polyhedra and the "semi-regularity" of the Archimedean polyhedra. A further issue is the direction in which links are traversed through the pattern -- in terms of chirality.

There are of course numerous other polyhedra with various properties which may trigger insights into their potential relevance for patterning web browsing. Some may prove of greater relevance to enhancing integrative comprehension of the thematic content of the pattern. Personal preferences may come into play with respect to memorability -- even including unusual polyhedra with lower degrees of symmetry (as indicated below). Patterns of greater abstraction may be of even greater relevance to exploration of contexts for some form of cognitive fusion as argued separately (*Enactivating a Cognitive Fusion Reactor: Imaginal Transformation of Energy Resourcing (ITER-8)*, 2006).

## Transformation: changing cognitive gear and pattern shape shifting

Geometry and topology have studied extensively the transformations between patterns associated with polyhedra. These are best known and understood with respect to the Platonic polyhedra, as represented by the following morphing animations (reproduced from *Embodying Global Hegemony through a Sustaining Pattern of Discourse: cognitive challenge of dominion over all one surveys*, 2015). Of relevance to enhanced web browsing is then the potential capacity to shift from one configuration to another -- and what this may imply in terms of engagement with thematic content.

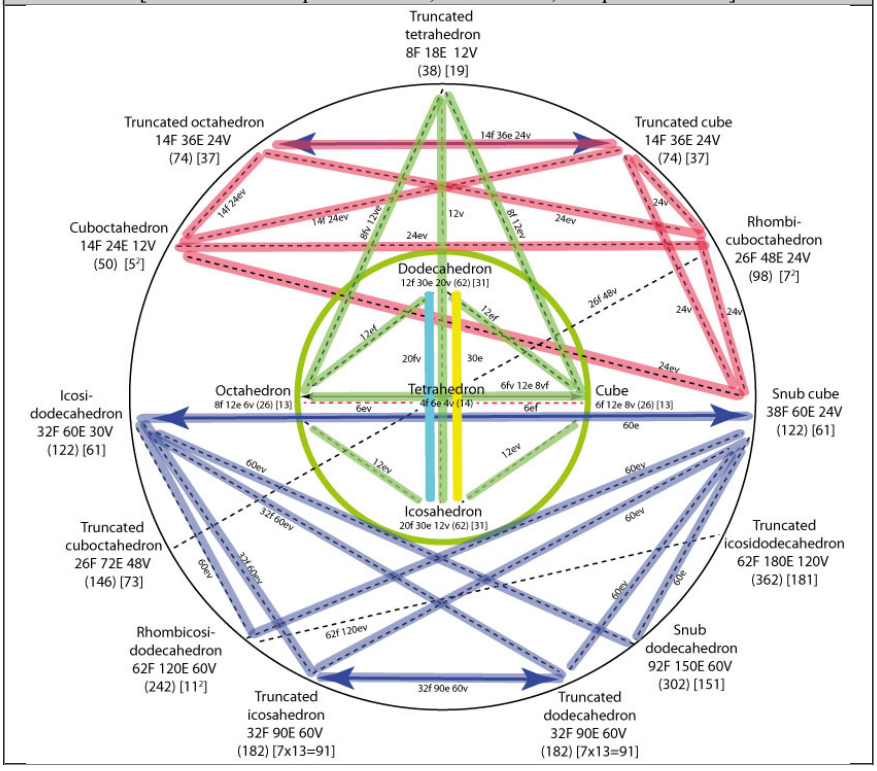
| **Animation of morphing between Platonic polyhedra and their duals**<br>(all animations prepared using Stella Polyhedron Navigator) |
| --- |
| Variants of Tetrahedron-to-Tetrahedron morphing<br>potentially indicative of a "tetrahedral conversation" and the challenge to it? |
|  |
| Variants of Cube-to-Octahedron morphing<br>potentially indicative of a "cubic conversation" and the challenge to it? |
|  |
| Variants of Icosahedron-to-Dodecahedron morphing<br>potentially indicative of a "dodecahedral conversation" and the challenge to it? |
| |



Shifting from one configuration to another in this way can be understood metaphorically as a "change of gear". Of interest is to extend the metaphor to the design of a "conceptual gear box" through which to navigate cyberspace, as previously suggested (*The Future of Comprehension: conceptual birdcages and functional basket-weaving*, 1980). Understood otherwise, the issue is how to transcend a particular focus in order to engage with a more general pattern when appropriate. The metaphor is well-recognized in terms of the zooming facilities available on web maps. It is curious that no such facilities have been developed systematically for thematic content.

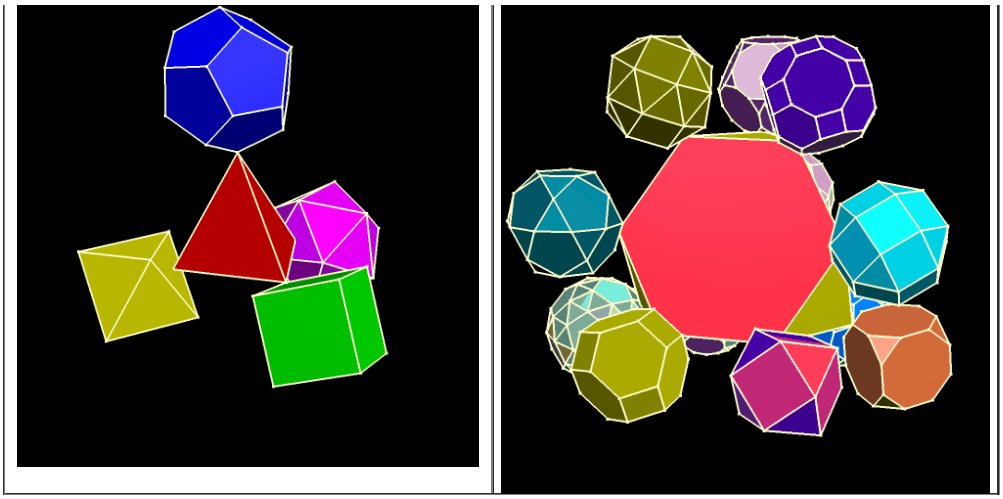
The following schematic is of interest as a means of indicating correspondences between the Platonic and Archimedean polyhedra which could serve as the basis for transformations between corresponding patterns of web tours. The schematic is reproduced from an earlier study where it is discussed in detail (*Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life? Number 37 as indicative of fruitful pathways of transformation?* 2015). The vertex count in each case indicates the range of websites which could be variously mapped by such polyhedra -- namely from 4 (on the tetrahedron) to 120 (on the truncated icosidodecahedron).

**Map highlighting distinctive relationships pathways between spherically symmetrical polyhedra (regular and semi-regular)**
F=faces, E=edges, V=vertices (Total of these in parenthesis)
[Total reduced to prime number, other than 2, in square brackets]




Another sense of the possible zooming operation of a conceptual gearbox is offered by the following animations (also reproduced from *Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life? Number 37 as indicative of fruitful pathways of transformation?* 2015)

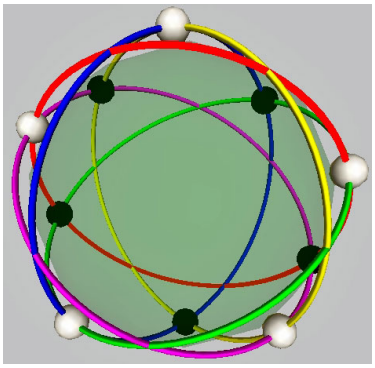
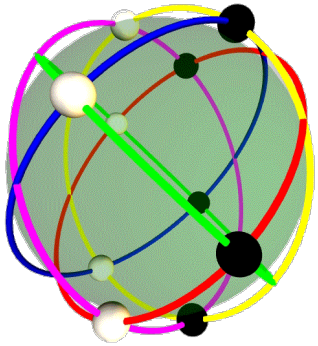
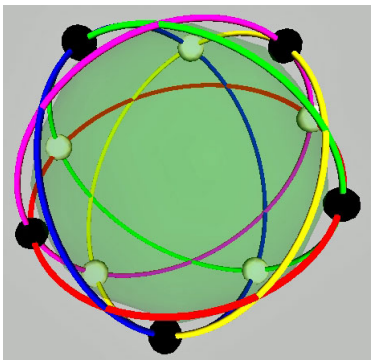
| **Configuration in three dimensions of Platonic and Archimedean polyhedra** | |
|---|---|
| Indication of 4 Platonic polyhedra configured around tetrahedron | Indication of 12 Archimedean polyhedra configured around truncated tetrahedron |
| | |



## Web navigation of a global configuration?

Especially intriguing with respect to any sense of "global" navigation of cyberspace are the insights originally offered by the mathematics of the Pentagramma Mirificum with regard to navigation of the terrestrial globe, as discussed separately (*Global Psychosocial Implication in the Pentagramma Mirificum: clues from spherical geometry to "getting around" and circumnavigating imaginatively*, 2015). The question is whether this strangely fundamental structure offers clues as yet to be explored -- of relevance to web browsing

**Screen shots of the complementary variants of *Pentagramma Mirificum* on opposite sides of a sphere**
(one pattern with 5 white balls, the other pattern with 5 black balls; 5 circles link both patterns together)
[options for viewing interactive 3D animations: classic browser-based WRL format; or via standalone FreeWRL viewer]

| White pattern in front(vertical); black pattern (inverted) seen **through** sphere | Side view of linkage between variants showing great circle continuity **around** sphere | Black pattern in front (inverted); white pattern (vertical) seen **through** sphere |
|---|---|---|
|  |  |  |

## Implications of closest packing of polyhedra for wizdome design

The arguments above help to reframe an earlier exploration of "wizdomes" (*Transforming Static Websites into Mobile "Wizdomes": enabling change through intertwining dynamic and configurative metaphors*, 2007). That discussion was introduced by the following considerations:

> There is the interesting possibility that "site" may come to be understood as a static outmoded metaphor for the manner in which people and collectives find it appropriate to engage with the universe of knowledge. Site implies a particular location, especially the location with which the web user has some involvement and which may be deliberately constructed as an articulation of individual or collective identity. From there one can travel to other locations which others have configured to represent their's.
>
> However, whilst the "site" may reflect considerable effort in articulating a static identity -- whether or not it has interactive facilities analogous to those that might be expected in a person's house -- it says nothing about the dynamics of how a person moves and how identity may be associated with that. There may be links to other sites -- like travel books in a home library -- but the dynamics and style of that movement are only partially represented. Even more interesting is the question of "who" moves. There is a sense that an abstract entity, a "visitor", travels to other sites as an observer, a consumer, a tourist -- along the information highway. Possibly some form of link may be brought back -- like a photograph or memento. Arrangements may be

> made to "keep in touch" through an exchange of addresses. As the person responsible for a site, one may in turn make arrangements to receive such visitors.
>
> The question asked in what follows is whether more fruitful understanding of these processes would emerge from changing metaphor.

In a section in that document on *"Wizdomes" -- beyond the drop-down menu* the point is made that:

> Curiously the primary metaphor for the organization of knowledge on a website is the culinary "menu" implying a taste-based process of choice. Behind the array of drop down menus is an "architecture" through which hierarchies of categories are structured. Humanity has not been able to generate a particularly fruitful organization of such categories...
>
> The concern here is therefore with the possibility of shifting from an architectural metaphor for the organization of knowledge to one in which knowledge elements are configured in new ways -- possibly consistent with new paradigms in architecture. Of these one of the most interesting is possibly the "dome" as exemplified by the work of R. Buckminster Fuller on geodesic domes (*Synergetics; explorations in the geometry of thinking*, 1975-9). The question is how might higher orders of organization of knowledge -- wisdom -- be experienced if their elements were configured spherically rather than in drop down hierarchically-structured menus. (*Spherical Configuration of Categories -- to reflect systemic patterns of environmental checks and balances*, 1994)
>
> But the particular emphasis here is on the personalized construction of such "knowledge domes" -- and the cognitive relationship to them. It could well be argued that the achievement of "gurus"in any domain is their spherical configuration of knowledge into an integrative whole -- a sort of "wizdome". It is the complex dynamics of this emergent wizdome that constitutes the "strange attractor" with which their followers resonate in some way (*Human Values as Strange Attractors: coevolution of classes of governance principles*, 1993). Its quality of being a "sacred space" may derive from such architecture -- as with the impressive patterns of the design of domed mosques.

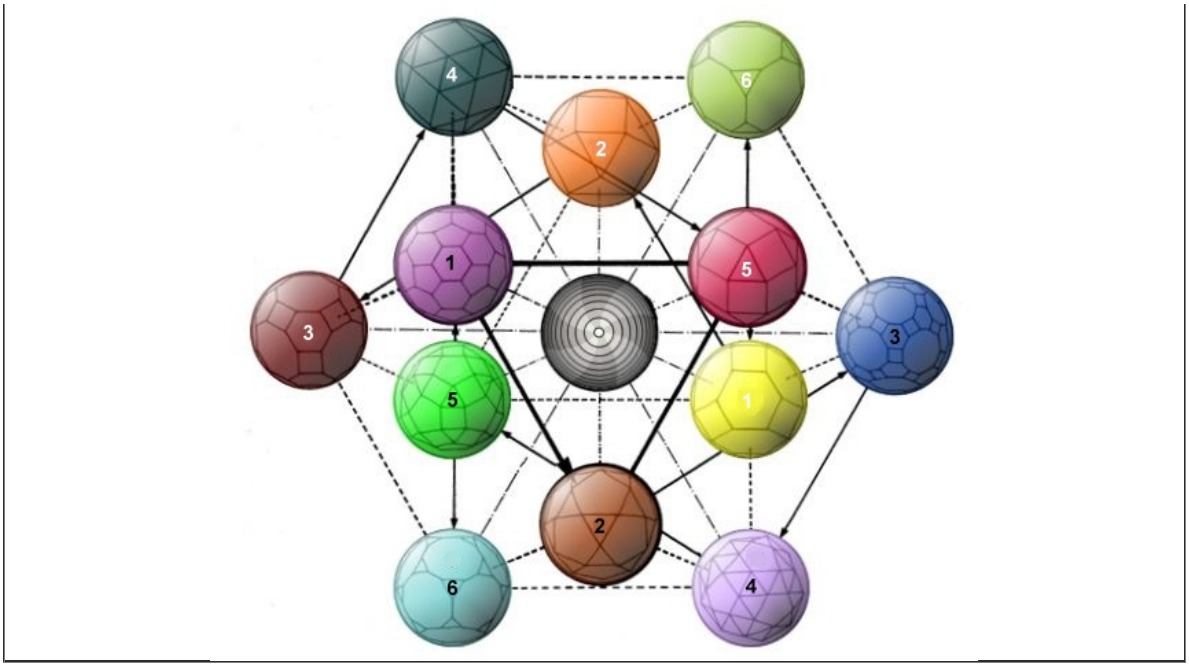
In the subsequent specialtive section on *"Whizzing around" -- engaging with the songlines of the noosphere*, the suggestion was made that:

> Combining these two suggested shifts in metaphor -- to the spherical and to the dynamic -- the question for the individual is whether what is required is to design such a "wizdome" from the elements of knowledge accumulated on any current website. Can such knowledge elements be configured spherically in a fruitful manner for that individual? Can a site be "endomed" or "domified"? What kinds of insights and expertise are required to bring about any such "enwrapping" of knowledge -- beyond what the problematic aspects of cocooning? What is to be "encompassed" and how is this to be distinguished from any "encyclopedic" ambition (cf *Cyclopean Vision vs Poly-sensual Engagement*, 2006)?
>
> Additionally however, rather than a static dome, can such a wizdome be designed as a vehicle? Or, more intriguing, **is it possible that its viability as a structure is specifically dependent on its movement as a dynamic structure** -- as much a "whizdome" as a "wizdome"?

Application of the songline metaphor to the global noosphere indeed merits consideration, as previously argued (*From Information Highways to Songlines of the Noosphere: global configuration of hypertext pathways as a prerequisite for meaningful collective transformation*, 1996). However, with respect to any understanding of a wizdome, it may also be considered in terms of the personal noosphere of the individual web user -- as implied by the argument above -- namely the songlines of one's own noosphere.

As indicated above, it is in this sense that how polyhedra may relate, to one another to enable various pathways, is potentially of great interest. One set of clues is offered by the schematic produced by Keith Critchlow (*Order in Space: a design source book*, 1969) and reproduced below. This offers a sense of how 12 semi-regular polyhedra pack most effectively around the 13th -- understood in terms of the geometry of sphere packing.

<table>
<tr><th colspan="2">Archimedean polyhedra<br>(reproduced from Towards Polyhedral Global Governance complexifying oversimplistic strategic metaphors, 2008, and from Union of Intelligible Associations: remembering dynamic identity through a dodecameral mind, 2005)</th></tr>
<tr><td>Successive truncations of octahedron<br>2, 3, 4-fold symmetry</td><td>Successive truncations of icosahedron<br>2, 3, 5-fold symmetry</td></tr>
<tr><td>1. truncated octahedron (14 polygons: 4 / 6 sided)<br>2. cuboctahedron / vector equilibrium (14: 3 / 4)<br>3. truncated cuboctahedron (26: 4 / 6 / 8)<br>4. snub cube (38: 3 / 4)<br>5. rhombicuboctahedron (26: 3 / 4)<br>6. truncated cube / hexahedron (14: 3 / 8)</td><td>1. truncated icosahedron (32 polygons: 5 / 6 sided)<br>2. icosidodecahedron (32: 3 / 5)<br>3. truncated icosidodecahedron (62: 4 / 5 / 10)<br>4. snub dodecahedron (92: 3 / 5)<br>5. rhombicosidodecahedron (62: 3 / 4 / 5)<br>6. truncated dodecahedron (32: 3 / 10)</td></tr>
<tr><td colspan="2">truncated tetrahedron (8 polygons: 3 / 6 sided)</td></tr>
<tr><td colspan="2">Arrangement of the 12 Archimedean polyhedra in their most regular pattern, a cuboctahedron, around a truncated tetrahedron</td></tr>
<tr><td colspan="2"></td></tr>
</table>




Of some relevance, Crichlow's more recent work has focused on flowers (*The Hidden Geometry of Flowers: living rhythms, form and number*, 2011). This theme is curiously relevant to the challenge of designing web sites as (strange) attractors for pollinating insects, readily compared with web surfers faced with a field of wild flowers of competing design. The convergence of threads of the above argument is remarkably illustrated by a recent paper by Gabriele Martino (*Solving a Traveling Salesman Problem with a Flower Structure*, *Journal of Applied Mathematics and Physics*, 2014) stating: *A cycle on the polyhedron that pass once on each city is a flower with the corolla as origin (center) and the coordinate axis of cities petals.* Of related interest is a *Gallery of Polyhedral Flower Arrangements: engendering sustainable psycho-social systems through metaphor* (2014).

## Memorable pattern language for configuring sets of websites?

**Developing a pattern language to configure sets of websites**: The much-cited work on pattern language by Christopher Alexander (*A Pattern Language*, 1977) is especially remarkable in this context because of its unexpected seminal role with respect to computer program design (James O. Coplien and Douglas Schmidt, *Pattern Languages of Program Design*, 1995).

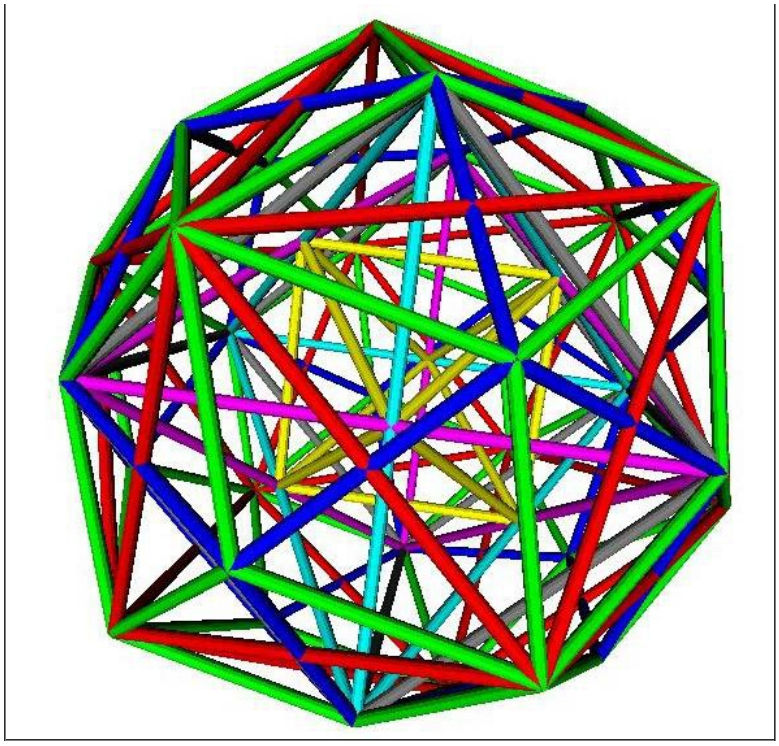
Whilst the focus with respect to the information sciences is on software design, there is clearly the possibility of adapting such a language to configurations of multiple websites in a knowledge society -- and the patterns whereby they may be fruitfully navigated.

The argument could be considered somewhat ironic in that Alexander's previous work had been focused on the design of a place where one could "feel at home". This sense of "home" is the "quality without a name" which he identified as the quality of "a place to be" (*The Timeless Way of Building*, 1979). It is in this sense that for a web user the configuration of web sites habitually visited frames that sense of home -- in knowledge space.

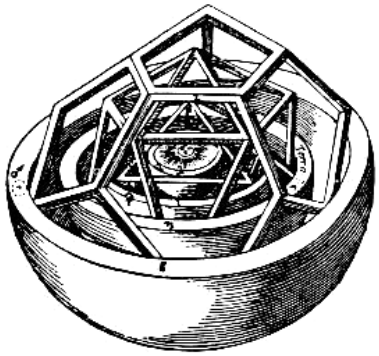
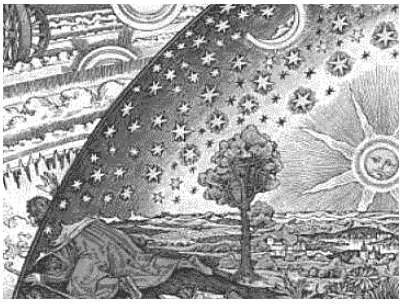
One early exercise in response to the psychosocial challenge was the adaptation of the 254 interlinked patterns as originally elaborated by Alexander (*A Pattern Language*, 1977) into a similar pattern of psychosocial analogues (*5-fold Pattern Language*, 1984). Alexander's more recent work has stressed a geometrical perspective (*Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole*. *International Journal for Unconventional Computing (IJUC)*, 2009), as discussed separately (*Harmony-Comprehension and Wholeness-Engendering: eliciting psychosocial transformational principles from design*, 2010).

**Cognitive gearbox understood as nested polyhedral configurations**: More striking in envisaging the design of a cognitive gearbox is the nesting of 5 regular polyhedra (within a semi-regular polyhedron) in the following image (reproduced from *Embodying Global Hegemony through a Sustaining Pattern of Discourse: cognitive challenge of dominion over all one surveys*, 2015).

| **Rhombic Triacontahedron (green) as a nesting framework (combining the images above)**<br>with Dodecahedron (blue), Icosahedron (red), Cube (grey), Octahedron (yellow), with Tetrahedron (cyan) and Tetrahedron (magenta)<br>(accessible in virtual reality viewers -- VRML97 version or X3D version) |
| --- |
| |



In the previous discussion of that structure, comparison was made with the classical model of the solar system as depicted by Johannes Kepler.

| **Nested polyhedral model of solar system of Johannes Kepler**<br>Reproduced from *Wikipedia* entry on *Mysterium Cosmographicum* (1596) | **Imagined Renaissance understanding of the relationship between cognitive spheres** |
|---|---|
|  |  |

**Implications for knowledge cybernetics**: In the discussion of the above nesting of the Platonic polyhedral patterns, emphasis was placed on the dynamics between the forms rather than on the configurations understood statically (as depicted). Given the cybernetic work, as framed by the icosahedron with respect to discourse, by Stafford Beer (*Beyond Dispute: the invention of team syntegrity*, 1994, it is especially interesting to consider the recent work of Maurice Yolles and Gerhard Fink (*A General Theory of Generic Modelling and Paradigm Shifts: cybernetic orders*, *Kybernetes*, 2015).

This might be understood as raising the question as to whether the tetrahedral dynamic (in the morphing animations above) could be interpreted in terms of first order cybernetics, that of the cube-octahedron with second order cybernetics, that of the dodecahedron-icosahedron with third order cybernetics, and that of the rhombic triacontahedron-icosidodecahedron with that of the fourth order.

To what extent is the nesting of polyhedral patterns then to be understood as supportive of degrees of self-reflexivity and the interplay between specific, fundamental and generic insights -- a particular comprehension of changing conceptual gear according to circumstances?

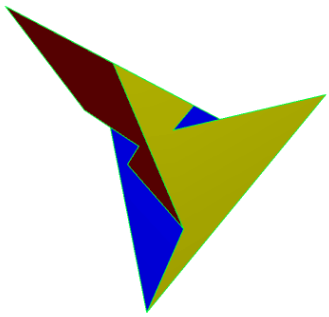
In a web-based society, this question is of particular relevance to societal learning, given the following comment by Steve Wadell (*Societal Learning: creating big-systems change*, *Systems Thinker*, 2001/2002):

> Given their ambitious goals, societal learning initiatives must go well beyond simply coordinating organizations and resources -- often referred to as **single-loop learning or first-order change** because it occurs within current structures and assumptions. Societal learning requires a shift in mental models and the development of new structures and processes, known as **doubleloop learning or second-order change**.
>
> Like organizational learning, societal learning deals with exploring the deep, underlying structures that drive behavior, surfacing

> the basic assumptions we hold that limit our options, and developing innovative approaches to persistent problems... This kind of shift in thinking can spur complex synergies and powerful innovations....Such collaborations can even produce the more rarefied **triple-loop learning**, which involves rethinking the way we actually think about an issue... In systems thinking terms, the challenge of those involved in societal learning is to understand and address numerous large and complex feedback loops. In development and change management terms, the challenge is to transform learning at a project and intellectual level into broad, sustainable systemwide change.

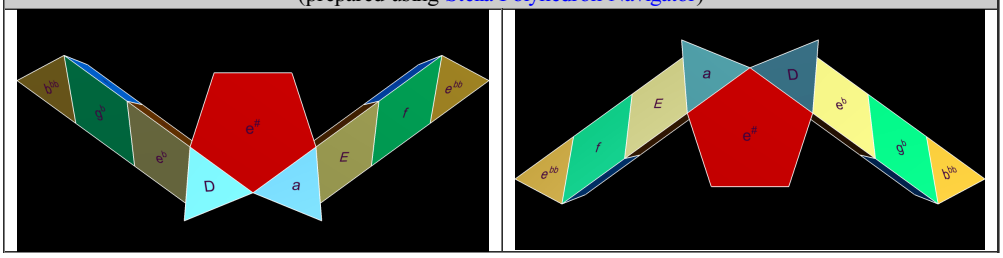
**Memorable assymetric polyhedra**: A striking example is provided by the Szilassi polyhedron (one of the Stewart toroids), as discussed separately (*Polyhedral Configuration of Questions*, 2014) -- topologically a torus, with seven hexagonal faces, **each face sharing an edge with each other face**, perhaps to be recognized as an extreme form of touching.

**Animations of Szilassi polyhedron**
of 7 faces (4 types), 21 edges (12 types), 14 vertices (7 types) [totalling 42=2x3x7; product=$2x3^2 7^3$]
(paired shapes coloured identically; prepared using Stella Polyhedron Navigator)

| Folding together of the two complement nets | Rotation of polyhedron |
|---|---|
|  |  |

A striking example is illustrated below, in the light of previous discussion of the claimed fundamental role of the number 37 (*Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life? Number 37 as indicative of fruitful pathways of transformation?* 2015).

**Animations of a polyhedron named "Wings" with tone values attributed to the faces** (in two orientations)
Polyhedron with 37 faces (of 19 types), 65 edges (5x13 of 33 types), 30 vertices (of 15 types)
(prepared using Stella Polyhedron Navigator)




**Future presentation of search results**: The results of any web search are now typically presented as a laundry list of items of interest, supposedly ranked by algorithms to highlight those of greatest interest -- notably in the light of filter bubbles applied in terms of the assumed preferences of a particular user.

At some stage search results could be presented (optionally) as polyhedral patterns, notably with the evolution of interactive visualization applications such as Data Driven Documents (d3.js). These could be organized such as to enable a form of zooming between simpler and more complex polyhedra, as suggested above.

Of interest is the distinction that might be made between simpler (as meaning more fundamental) and simpler (as meaning more specific) -- and between complex (as encompassing greater variety) and complex (as implying more integrative).

An interesting challenge for website managers would be how to strive for particular positioning within such patterns -- by extension of current efforts at search engine optimization (SEO), namely the process of affecting the visibility of a website or a web page in a search engine's unpaid results.

**Viral marketing and polyhedral viruses**: In a world threatened by viral diseases -- a viral pandemic like influenza -- it is remarkable to note the as yet unexplored degree of correspondence between the icosahedral ordering of both psychosocial mega-problems and of the micro-problems constituted by viruses (Roberto Marabin: *Icosahedral Viruses: 3D Reconstruction*, 2014). As discussed separarely, wicked problems (so-called because their complexity and intractability) might then be usefully recognized as the pandemics of knowledge society and cyberspace (*Encycling wickidity in the light of polyhedral viruses and their mutation*, 2015). The metaphor has long been valued in the case of computer viruses and viral marketing.

Inspired by biomimicry, this suggests the value of exploring virology as offering a form of "pattern language" with regard to antigens and antibodies as these might apply to the operation of possible "viral antigens" that could be developed to constrain wicked problems (Reidun Twarock, *Mathematical Virology: a novel approach to the structure and assembly of viruses*, *Phil. Trans. R. Soc. A*, 2006). Could the optimization of proactive web searches be organized in terms of polyhedral frameworks in order to match wicked problem networks understood as achieving their robustness through polyhedral organization?

---

## References


Hernán Abeledo, Ricardo Fukasawa, Artur Pessoa and Eduardo Uchoa. The time dependent traveling salesman problem: polyhedra and algorithm. *Mathematical Programming Computation*. 5, 2013, pp. 27-55 [text]

Christopher Alexander:

- A Pattern Language. Oxford University Press, 1977 [summary]
- The Nature of Order: an essay on the art of building and the nature of the universe. Center for Environmental Structure, 2003-4. [summary]
- New Concepts in Complexity Theory: an overview of the four books of the Nature of Order with emphasis on the scientific problems which are raised. 2003 [text]
- Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole. *International Journal for Unconventional Computing (IJUC)*, 5, 2009 [text]

Norbert Ascheuer, Matteo Fischetti and Martin Grötschel. A polyhedral study of the asymmetric travelling salesman problem with time windows. *Networks*, 36, 2000, pp. 69--79 [abstract]

Fred Attneave. Symmetry, Information, and Memory for Patterns. *The American Journal of Psychology*, 68, 1955, 2, pp. 209-222 [abstract]

Stafford Beer. Beyond Dispute: the invention of team syntegrity. Wiley, 1994

Ivan Brezina Jr. Zuzana Cickova. Solving the Travelling Salesman Problem Using the Ant Colony Optimization. *Management Information Systems*, 6, 2011, 4, pp. 10-14 [text]

T. Christof, M. Oswald and D. O. Theis. Symmetric and Graphical Traveling Salesman Polyhedra. Universität Heidelberg [contents]

Philippe Clauss, Diego Garbervetsky, Vincent Loechner and Sven Verdoolaege. Polyhedral Techniques for Parametric Memory Requirement Estimation. In: Energy-Aware Memory Management for Embedded Multimedia Systems: a computer-aided design approach, CRC Press, 2012, pp. 117-149 [text]

James O. Coplien and Douglas Schmidt. Pattern Languages of Program Design. Addison-Wesley Professional, 1995

Gérard Cornuéjols, Jean Fonlupt and Denis Naddef. The traveling salesman problem on a graph and some related integer polyhedra. *Mathematical Programming*, 33, 1985, 1, pp 1-27 [abstract]

Keith Critchlow:

- The Hidden Geometry of Flowers: living rhythms, form and number. Floris Books, 2011
- Order in Space: a design source book. Thames and Hudson, 1969

Jan B. Deregowski. Perception of Symmetry: some psychological observations. *Symmetry: Culture and Science*, 3, 1992, 3, pp. 263-279 [text]

Marco Dorigo and Luca Maria Gambardella. Ant colonies for the traveling salesman problem. *BioSystems*, 43, 1997, 2, pp. 73-81 [text]

Irina Dumitrescu. Polyhedral Results for the Pickup and Delivery Travelling Salesman Problem. CIRRELT-2008-07, March 2008 [text]

Sandor P. Fekete. Geometry and the Travelling Saleman Problem. University Waterloo, 1992 [text]

Matteo Fischetti and Paolo Toth. A Polyhedral Approach to the Asymmetric Traveling Salesman Problem. *Management Science*, 43, 1997, 11, pp. 1520-1536 [abstract]

Gregory Gutin and Abraham Punnen (Ed.). The Traveling Salesman Problem. *Discrete Optimizatiion*, 3, 2006, 1, pp. 1-94 [special issue]

Eric Horvitz, Susan Dumais and Paul Koch. Models of Memorability: learning, experiments, and applications. Microsft Research [text]

Natasha Jonoska, Anne Taormina and Reidun Twarock. DNA cages with icosahedral symmetry in bionanotechnology. 2008 [text]

Michael J. Kahana. Associative Symmetry and Memory Theory. *Memory and Cognition*, 30, 2002, 6, pp. 823-840 [text]

Phillip Isola, Devi Parikh, Antonio Torralba and Aude Oliva. Understanding the Intrinsic Memorability of Images 2011 [text]

George Lakoff and Rafael Núñez. Where Mathematics Comes From: how the embodied mind brings mathematics into being. Basic Books, 2001

Jon Lee and John F. Raffensperger. Using AMPL for Teaching the TSP. *INFORMS Transactions on Education*, 7, 2006, 1, pp. 37-69 [text]

Mathieu Lihoreau, Lars Chittka and Nigel E. Raine. Trade-off between travel distance and prioritization of high-reward sites in traplining

bumblebees. *Functional Ecology*, 2011 [summary]

Vicky Mak and Natashia Boland:

- Polyhedral results and exact algorithms for the asymmetric travelling salesman problem with replenishment arcs. *Discrete Applied Mathematics*, 155, 2007, 16, pp. 2093-2110 [abstract]
- Facets of the polytope of the asymmetric travelling salesman problem with replenishment arcs. *Discrete Applied Mathematics*, 3, 2006, 1, pp. 33-49 [abstract]

K. Marriott, H. Purchase, M. Wybrow and C. Goncu. Memorability of Visual Features in Network Diagrams. *Visualization and Computer Graphics*, 18, 2012, 12, pp. 2477-2485 [abstract]

Gabriele Martino. Solving a Traveling Salesman Problem with a Flower Structure. *Journal of Applied Mathematics and Physics*, 2, 2014, 7 [text]

Rajesh Matai, Surya Prakash Singh and Murari Lal Mittal. Traveling Salesman Problem: an overview of applications, formulations, and solution approaches. 2010 [text]

Tessa Morrison. The Mnemonics of the Cretan Labyrinth. *Symmetry: Art and Science*, 2, 2002, 1-4, pp. 203-218 [text]

D. Naddef and G. Rinaldi. The Graphical Relaxation: a new framework for the symmetric travelling salesman polytope. *Mathematical Programming*, 58, 1993, pp. 53-87

Manfred W. Padberg and M. R. Rao. The travelling salesman problem and a class of polyhedra of diameter two. *Mathematical Programming*, 7, 1974, 1, pp 32-45 [abstract]

James J. Jong Hyuk Park, Ivan Stojmenovic, Min Choi, and Fatos Xhafa. Future Information Technology: FutureTech 2013. Springer Science and Business Media, 2013

Rudi Pendavingh. Integer Programming and Polyhedral Combinatorics. 2013 [text]

Reidun Twarock. Mathematical Virology: a novel approach to the structure and assembly of viruses, *Phil. Trans. R. Soc. A*, 364, 2006, pp. 3357-3373 [abstract]

Steve Waddell:

- Societal Learning and Change: how governments, business and civil society are creating solutions to complex multi-stakeholder problems. Greenleaf, 2005
- Global Action Networks: creating our future together. Palgrave Macmillan, 2011
- Societal Learning: creating big-systems change.*Systems Thinker*, 2001/2002 [text]

Francis Yates. The Art of Memory. Routledge and Kegan Paul, 1966 [summary]

Maurice Yolles. Organisations as Complex Systems: an introduction to knowledge cybernetics. Information Age Publishing, 2006

Maurice Yolles and Gerhard Fink:

- A General Theory of Generic Modelling and Paradigm Shifts (part 1): the fundamentals. *Kybernetes*, 44, 2015, 2, pp. 283-298 [abstract]
- A General Theory of Generic Modelling and Paradigm Shifts (part 2): cybernetic orders. *Kybernetes*, 44, 2015, 2, pp. 299-310 [abstract]
- A General Theory of Generic Modelling and Paradigm Shifts (part 3): the extension. *Kybernetes*, 44, 2015, 2, pp. 311-328 [abstract]

---