8 August 2011 | Draft

Implication of Toroidal Transformation of the Crown of Thorns

Design challenge to enable integrative comprehension of global dynamics

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Introduction

This is an exercise in identifying a set of threads which might be fruitfully woven together into an unusual pattern of significance of relevance to both the challenge of governance and of individuation. It follows from earlier reflections (*Interweaving Thematic Threads and Learning Pathways*, 2010; *Magic Carpets as Psychoactive Systems Diagrams*, 2010).

The relationships between the threads selected here are necessarily less than evident -- or there would be no current challenge to integrative "joined-up" thinking. As one of the threads, the nature of that relationship is understood to require a form of Knight's move thinking -- shifting from linear predictability to a contrasting orientation characteristic of creativity, "new thinking" and a change of paradigm. It is a degree of circularity in the pattern of such moves which is explored here as a key to the coherence of a mode of correlative thinking enabled by various correspondences (*Theories of Correspondences -- and potential equivalences between them in correlative thinking*, 2007). These considerations are explored as a means of addressing dysfunctional patterns of "monkeying" and those of the associated "blame game"(*Monkeying with Global Governance*, 2011).

This exploration follows from arguments previously made with respect to providing a context for the classical Knight's move of chess (and *go*), and the related significance attached to such thinking in its pathological and creative interpretations (*Reframing "monkeying" in terms of Knight's move patterns*, 2011; *Engendering confidence and identity within learning / action cycles*, 2011). In that respect the following argument offers new insights into both "monkeying" and the more integrative pattern associated with the style of governance implied by the archetype of the "Knights of the Roundtable" and their potential interaction.

The argument focuses on the torus as an intermediary between a "flatland" approach to governance (characteristic of most planning) and the current need for a necessarily elusive "global" approach requiring "joined-up thinking". Various forms of symbolic recognition of this toroidal intermediary are noted across cultures.

As one such symbol, specific reference is made to to the Crown of Thorns as offering an indication of the extremely painful consequences of the currently fragmented "crowning" configuration of global strategies. This could be said to be derived from linear ("thorn-like") mindsets, ill-adapted to the non-linear nature of the dynamics characteristic of the global system. Here the possibility is explored of transforming this dysfunctional linearity through forms of curvature more consistent with those dynamics. This gives explicit place to the developmental S-curve seen as characteristic of elusive global "change" and the challenge of navigating the adaptive cycle -- as highlighted by the Resilience Alliance and the work of Thomas Homer-Dixon (*The Upside of Down: ca strophe, creativity, and the*

renewal of civilization, 2006; *The Ingenuity Gap*, 2000). An earlier exploration focused on learnings from patterns in nature (*Enabling Governance through the Dynamics of Nature: exemplified by cognitive implication of vortices and helicoidal flow*, 2010).

New "projects" now tend to be exercises in (psychological) "projection" consequent upon linear thinking -- effectively "trajectories" designed to impact "targets". What seems to be required for sustainability however is to enable the emergence of cyclic thinking (*Psychology of Sustainability: embodying cyclic environmental processes*, 2002; *Consciously Self-reflexive Global Initiatives*, 2007).

The argument is necessarily speculative and presumptuously ambitious -- with the associated limitations on the appropriateness of such an exploration included as another of the threads. As a challenge to creative design possibilities, the question is the range of seemingly disparate elements that can be fruitfully woven together in consideration of a requisite pattern of non-linearity.

The current period is witness to considerable existential uncertainty resulting from global crises engendered by misgovernance of the highest order -- for which responsibility and blame are systematically distributed through finger-pointing, without anybody being called effectively to account. Typical of the times, it is however unclear whether serious concern is more appropriate than humour (*Ungovernability of Sustainable Global Democracy?* 2011; *Responsibility for Global Governance*, 2008).

In this context, the very recent declaration of Rick Perry -- the person who may well be elected as the next "most powerful man on the planet" -- merits careful attention:

> Right now, America is in crisis. We have been besieged by financial debt, terrorism, and a multitude of natural disasters. As a nation, we must come together and call upon Jesus to guide us through unprecedented struggles, and thank him for the blessings of freedom we so richly enjoy... Some problems are beyond our power to solve.... with praying people asking God's forgiveness, wisdom and provision for our state and nation. There is hope for America. It lies in heaven, and we will find it on our knees. (*Rick Perry under fire for planning Christian prayer rally and fast*, *The Guardian*, 5 August 2011)

In the light of such beliefs, Christians might be usefully challenged to imagine what symbolic "upgrade" Jesus would consider appropriate to the original Crown of Thorns.

Thread-weaving design considerations

The following "design considerations" are also usefully understood as "entry points" to an argument that make particular use of circularity. It is assumed that the points will be necessarily alienating to some or readily considered as irrelevant. It is part of the purpose of the argument to design into the framework presented the sense of irrelevance of perspectives that may well be much preferred by others.

Formal / Objective

- **Predictability and pattern breaking: Knight's move**: Projection onto a torus of a cyclic pattern of Knight's moves as potentially suggestive of more integrative possibilities of dialogue.
- **Geometry: implications for thinking and identity**: Implications of geometrical forms in the quest for subtler forms of thinking and identity.
- **Curvature: experiential and cognitive**: Cognitive implications of curvature implied by mapping the Knight's move onto a torus (horizon effects, etc) in contrast with those over a flat board

Informal / Subjective

- **Dialogue: towards the dynamics of the archetypal "Round Table"?**: Dynamic patterns inherent in fruitful dialogue, as implied by those of the archetypal Round Table and the cognitive "moves" that might be variously characteristic of the "Knights" seated there.
- **Creativity: from "rock logic" to "water logic"?** The subtle combination of creativity, irrationality, surprise and "cognitive catastrophe" associated with the non-linearity of the Knight's move
- **Aesthetics of harmony**: Aesthetic considerations potentially characteristic of more integrative thinking, notably through embodying the golden ratio into the pattern design

- **Psychocultural**
  - **Mnemonic necessities** : Enabling "re-membering" to enable sustainable comprehension of greater complexity
  - **Symbols: vital psychoactive focus**: Associating an emergent design with symbols of central, integrative significance to various belief systems:
  - **Comprehension**: Rendering explicit, to some degree, the challenges to further comprehension of greater complexity and subtlety
- **Psychoactive implication**
  - **Limitations and constraints on integrative frameworks**: Recognizing the inadequacies of an exercise referring to insights necessarily explored extensively by others of far greater competence
  - **Musical facilitation of integrative comprehension**: Recognition of the role of musical harmony in enabling non-linear thinking
  - **Self-reflexivity essential to appropriate design**

The "design" framework follows from earlier concerns (*Designing Global Self-governance for the Future*, 2010; *Further Constraints on Conceptual Container Design*, 1983; *Organization and Lifestyle Design: characteristics of a nonverbal structural language*, 1978). It is notably reinforced by the work of Christopher Alexander (*Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole*, *International Journal for Unconventional Computing (IJUC)*, 5, 2009).

Predictability and pattern-breaking: the Knight's move

Being predictable: Much is associated with predictability. Formally this can be framed in terms of obedience to "rules" which can be variously based on a sequence of "points". These can be data points, points in an argument, geographical points, etc. Two points may be understood as defining a "line" of some kind -- as in a line of argument. A third point confirms this. A pattern has thereby been set confirming a rule. In chess most pieces move in a line -- often unconstrained by distance. One piece, the Knight, breaks this rule of linearity, after appearing to obey it. After three moves in a line, it changes direction and moves orthogonally -- in the form of an L. It is however constrained to this pattern.

This switch to "*non sequitur*", from the predictability of "*sequitur*", has been considered of significance from a number of perspectives. It has notably been considered indicative of creativity, but has also been considered as emblematic of pathological inconsistency. It could also be considered consistent with any con game through which promises are made, expectations are raised, and hopes dashed -- as is characteristic of politics, as previously discussed ((*Strategic Inflation of Expectations and Inconsequential Drift*, 2009; *Cultivating Global Strategic Fantasies of Choic*e, 2010).

Knight's tour: The possibilities of the Knight's move have been the subject of extensive study, most specifically over the 8x8 squares of a chess board. The mathematical formalization has been extended to boards of many sizes. Of particular interest are the conditions under which the Knight can "visit" all the squares on a board -- whether starting from a particular position and whether ending where it started. This is known as the Knight's tour problem. It is important mathematically as a version of the travelling salesman problem -- namely ensuring the shortest route enabling a salesman to visit all potential clients on any trip. Both problems are defined mathematically in terms of Hamiltonian paths and Hamiltonian cycles. Separately the potential of considering such paths as learning pathways was suggested (*Learning through Hamiltonian cycles and pathways*, 2011)

Knight's move in chess (and *go*): For the purposes of the following argument, the repetition of the classic Knight's move is repeated within a minimal 3x3 pattern of squares -- effectively a subset of the 8x8 pattern of a flat chess board. This gives rise to a "circular" pattern as shown in Fig. 1 and Fig. 2. Note that whilst the pattern constitutes a form of "tour" -- returning to its point of origin -- it omits the central square and therefore cannot be considered as a conventional "Knight's tour".

The first four of the eight moves are shown in Fig. 2. The reverse direction is also possible.

Patterns of Knight's moves

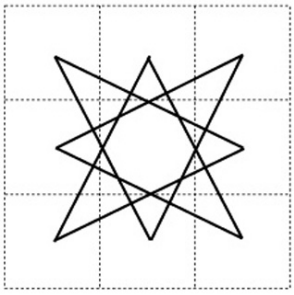
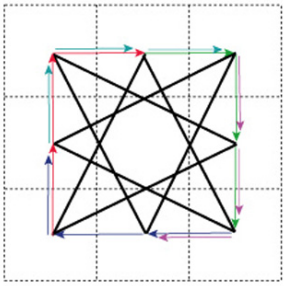
Fig. 1: Circular pattern of the Knight's moves (omitting the steps resulting in each move, shown in Fig. 2)	**Fig. 2: Showing 4 of 8 Knight's moves in the pattern** (as separately detailed in Fig. 3)
	

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Fig. 3a: Separate articulation of 8 of the Knight's moves
(providing an indication of the dynamics of both the "blame-game" and an archetypal "Round Table")

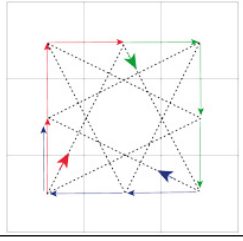
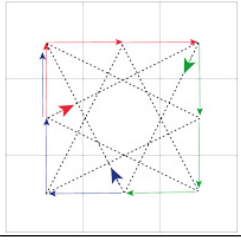
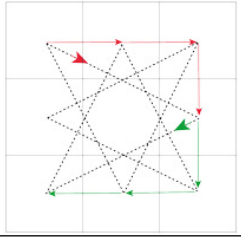
Fig. 3a	Fig. 3b	Fig. 3c
		

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Fig. 3b: Separate articulation of 8 of the Knight's moves

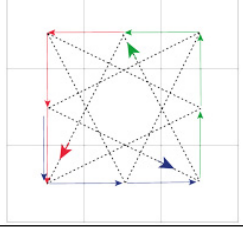
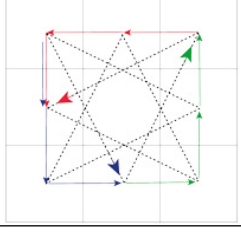
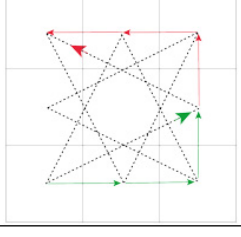
Fig. 3d	Fig. 3e	Fig. 3f
		

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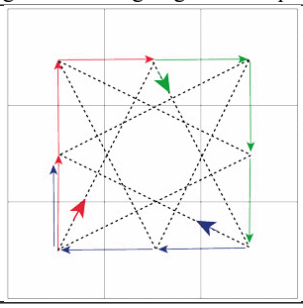
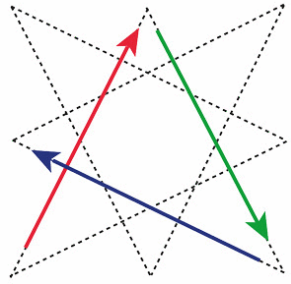
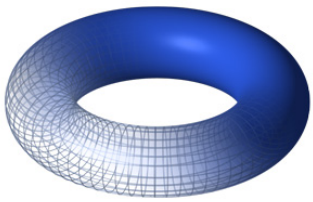
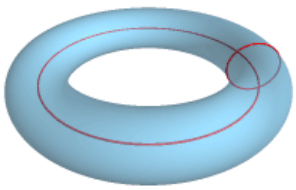
Fig. 4: Animation of 8 of the Knight's moves (based on Fig. 3) (suggestive of the dynamics of both the blame game and Knights of the Round Table)	
Fig. 4a: Including original 3x3 square	Fig. 4b: Omitting original 3x3 square
	

Blame game and patterns of buck-passing: Although their implications for more fruitful dialogue are discussed below, these images are especially suggestive of how a "blame game" works. Each initiative fulfils its functions in a predictable manner as expected -- but adds a creative twist to give it undue advantage, as with an "under-the-counter sideline". However when called to account each is able to imply that another in the pattern is to blame. The pattern is then a useful model of how "buck-passing" works in the process of "monkeying" with the system (*Monkeying with Global Governance*, 2011). The pattern may of course be applied to the set of functions of a single actor as an indicator of the pattern through which it is able to shift its ground when called. Relevant efforts to focus on the nature of that game include:

- Nathanael Fast, *How To Stop the Blame Game*, *Harvard Business Review*, 13 May 2010
- Ben Dattner and Darren Dahl, *The Blame Game: how the hidden rules of credit and blame determine our success or failure*, 2011
- Scott Wetzler and Diane Cole *Is It You or Is It Me?: why couples play the blame game*, 1999

Torus and globality: As noted below, there is a case for considering other geometric forms with which to associate strategic reflection -- beyond the conventional board-game, necessarily flat. This is especially relevant where the widespread preoccupation with "global" organization is necessarily associated with the sphere. In continuing the argument relating to the movement of the Knight, as a feature of strategic thinking, it is therefore appropriate to note both the extent to which this has been explored in relation to the torus (shown in Fig. 5)..

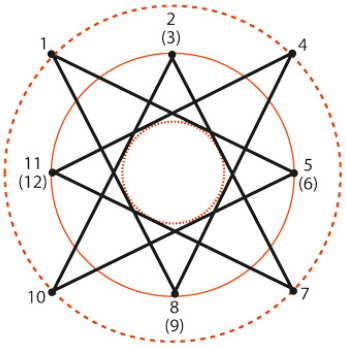
The manner in which the geometry of the torus is itself intimately related to that of the sphere (as discussed in the relevant *Wikipedia* entry) is depicted in the animation of Fig. 6. This shows that as the distance to the axis of revolution decreases, the ring torus becomes a spindle torus and then degenerates (in mathematical terms) into a sphere. This "degeneration" is to a visually simpler form with which the globality of the planet is so readily associated. Lost in this simplification is the relative complexity of the torus with which challenging strategic dynamics may be fruitfully associated -- possibly enabling more effective engagement with them.

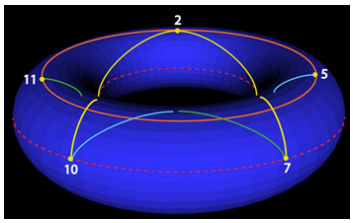
Fig. 5: Perspective view of a torus (reproduced from *Wikipedia* entry)	**Fig. 6: Torus-Sphere transformation** (reproduced from *Wikipedia* entry)
	

Projection of pattern of Knight's moves onto a torus: Returning to the pattern of Knight's moves, it is of great interest to consider them as projected onto a torus as depicted in Fig. 7. This possibility has been previously addressed (John J. Watkins and Rebecca L. Hoenigman, *Knight's Tours on a Torus*, *Mathematics Magazine*, 1997; Kelley Seibel, *The Knight's Tour on the Cylinder and Torus*. *REU Proceedings*, 1994). Seibel notably shows that, mapped onto a torus, a Knight's circuit is possible for any size of non-trivial chess board.

Arc movement: A plan view, with a numbering of the points, is shown in Fig. 7a. This projection implies that -- rather than an L-shaped **linear move** on a flat surface -- **here the Knight is moving in a series of arcs over the surface of the torus** (as in Fig. 7b) . This also implies that some of the points to which it moves are on the underside of the torus as seen in a plan view (from above). These are numbered in Fig. 7a in brackets (as with points 3, 6, 9, and 12).

Fig. 7: Projection of Knight's moves onto torus (preliminary indication of "moves" in excess of 8)	
Fig. 7a: As in Fig. 1 (omitting intermediary steps shown in Fig. 2)	Fig. 7b: Showing the "lines" of the moves as "curves" (omitting many curves for clarity)



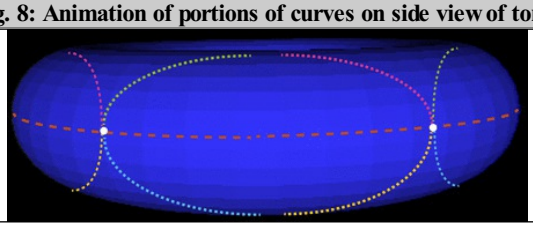


Three types of point: In following the L-shaped track of arcs whereby the Knight gets from point to point, it is now interesting to recognize that there are "horizon effects" whereby the next stage along the arc to which the Knight is moving is no longer necessarily (predictably) visible along a "line of sight" as on the chess board. In fact, looking at the pattern of Fig. 7a as a whole, there are three groups of points:

- **upper-side of the torus** (2, 5, 8, 11) -- these might be said to be visible to each other (and only to each other), being at the same level. None are however directly connected to one another.
- **under-side of the torus** (3, 6, 9, 12) -- these might might be said to be visible to each other (and only to each other), being at the same level. None are however directly connected to one another.
- **outer circumference of the torus** (1, 4, 7, 10) -- these are invisible to each other and to any other point. None are however directly connected to one another.

It might also be said that "traffic" to points directly accessible involves a form of "catastrophe" -- as implied by the curvature of the geometry, understood as a non-linear discontinuity with cognitive implications. This sense can be usefully related to that implied by the jargon phrase of being "thrown a curved ball" or of throwing one. As discussed below, it is in this sense that the connectivity of the sections of this argument could be considered as non-linear.

Fig. 8: Animation of portions of curves on side view of torus



Emergent toroidal form: The above "mapping" exercise onto a torus can be understood as obscuring the extent to which the pattern of the dynamics of the Knights moves effectively engender, define and sustain the torus. Cognitively, are the moves defined by the torus or do they define a toroidal form.

Of particular interest, potentially, is the manner in which the S-curves through the hole of the torus are effectively tangential to the circumference of an "absent" sphere "nestled" there. This may point to another subtler understanding of globality.

Geometry: implications for thinking and identity

The previous remarks focus on formal, mechanistic issues. The question that merits attention is their implication for psychological engagement with the geometry. Especially intriguing is the integrative significance to be attached to "globality" -- and how that is experienced and intimately associated, if only through metaphor, with particular geometric forms and patterns.

This has been extensively discussed separately -- notably with respect to the geometric progression from points, to lines (of argument), to fields (of study), to more complex forms -- including those implying a degree of (fruitful) paradox, as with the Mobius strip and the Klein bottle (*Metaphorical Geometry in Quest of Globality*, 2009; *Engaging with Globality -- through cognitive lines, circlets, crowns or holes*, 2009). The implications in support of a sense of identity have also been explored (*Geometry, Topology and Dynamics of Identity*, 2009).

Especially significant to this exploration is the work of:

- Christopher Alexander (*Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole*, *International Journal for Unconventional Computing (IJUC)*, 2009) as discussed in *Harmony-Comprehension and Wholeness-Engendering eliciting psychosocial transformational principles from design* (2010)
- Ron Atkin (*Multidimensional Man; can man live in 3-dimensional space?* 1981; *Combinatorial Connectivities in Social Systems; an application of simplicial complex structures to the study of large organizations*, 1977) as summarized in *Comprehension: Social organization determined by incommunicability of insights* (1995)
- Stafford Beer (*Beyond Dispute: the invention of team syntegrity*, 1994) as discussed in *Spherical Configuration of Interlocking Roundtables: Internet enhancement of global self-organization through patterns of dialogue* (1998)
- R. Buckminster Fuller (*Synergetics: explorations in the geometry of thinking* 1975) as discussed in *Geometry of Thinking for Sustainable Global Governance*(2009)

- Arthur M. Young (*The Geometry of Meaning*, 1984) as discussed in *Characteristics of phases in 12-phase learning / action cycles* (1998) and *Typology of 12 complementary strategies essential to sustainable development* (1998) and *Typology of 12 complementary dialogue modes essential to sustainable dialogue* (1998)
- Michael Schiltz (*Form and Medium: a mathematical reconstruction*, *Image [&] Narrative*, 6, 2003) as discussed in *Comprehension of Requisite Variety for Sustainable Psychosocial Dynamics: transforming a matrix classification onto intertwined tori* (2006)

A very helpful summary of relevant possibilities is provided by Eileen Clegg and Bonnie DeVarco (*What is the Shape of Thought? The Intersection of Nature, Geometry, and Communication*, *Shape of Thought Approach*, 26 July 2010). More comprehensive are the works by John D. Barrow (*Cosmic Imagery: key images in the history of science*, 2008) and Michel Random (*L'Art Visionnaire*, 1991)

This argument has necessarily worked with the simplest geometric forms, seeking to relate a flat surface with a sphere through a torus. Each is understood as offering a kind of template over which the dynamics of daily life are variously organized and played out. Board games, as template of choice for competitive sport and (collective) strategy of every kind, are however a challenge to relate comprehensibly to any concern with globality and its governance. The dynamics of games illustrate the manner in which one can be placed at a disadvantage, or gain advantage, by any ability for non-linearity, unpredictability or creativity.

Some of the cognitive surprises in changing from a flat template to one inspired by the sphere have been delightfully explored in a special form of fiction by mathematicians (Edwin A Abbott, *Flatland: a romance of many dimensions*, 1884; Dionys Burger, *Sphereland: a fantasy about curved spaces and an expanding universe*, 1965; A.K. Dewdney, *The Planiverse*, 1984; Ian Stewart, *Flatterland*, 2001; Rudy Rucker, Spaceland, 2002). These are all designed to give a sense of the multi-dimensionality that people lose when trapped in a space of lower dimensionality. Game developers have now envisaged a "torusland":

> Flatland need not be entirely "flat"! It could be Sphereland, for instance, where our (non-Euclidean) Square is restricted to the surface of a sphere. Or Rubberland (as one might call it), which can be stretched and distorted easily in three dimensions. Or Torusland where such well-known characters as Pacman reside (*Flatland: the videogame*, *gamedev.net*, January 2010).

The merits of such geometry is that it is typically within a cognitive "grasp" as compared for the far richer patterns otherwise available from mathematics. It is the nature of this comprehensive grasp which can be rendered more consciously explicit through accessible geometry. Excessive complexity effectively creates a horizon effect -- forcing significance "over the horizon" beyond immediate ken and out of range of behaviours dependent on familiarity.

The objective geometry as a template then gives a degree of expression to subjective experience and its organization. This might include the experiential:

- sense of centre
- sense of focus
- sense of place
- sense of "making" a point -- or "getting" or "taking" a point
- sense of placement and organization -- putting things in their place, or giving them a place
- sense of proximity and distance
- sense of orientation

Curvature: experiential and cognitive

Cognitive curvature: There are seemingly no references to "cognitive curvature" as such -- other than one insightful Twitter comment to the effect that "*cognitive curvature produced by the manipulation of the stories on the brain, opens the narrative space hosting the stories themselves*". Learning curves and the learning curve effect offer valuable insights into the experience and efficiency of learning -- but primarily in terms of external observation rather than from the actual subjective experience of the learner.

Curvature in economics: From an economic perspective, the term experience curve has been widely applied to the reduction in a corporation's unit cost of manufacturing for each doubling of the volume that it produced. As indicated by Lex Borghans, et al. (*The Economics and Psychology of Personality Traits*, *The Journal of Human Resources*, 2008, XLIII, 4), a risk preference parameter (also referred to as "risk aversion" or "risk tolerance") is a measure of the curvature of the utility function -- often estimated through Euler equations, as reviewed by Martin Browning, et al. (*Micro Data and General Equilibrium Models*, 1999). In the case of strategy, Reinier Geel recognizes the challenge of "*having to follow the curvature of the influencing manager that is out of synchronisation*" (*Strategic Management: the radical revolutionary strategic management matrix for predators*, 2011)

Kinaesthetic insight: Presumably similar arguments could apply with respect to intellectual productivity. Especially interesting is how curvature is sensed. Insights are potentially to be derived from kinaesthetic learning as originally related to the bodily-kinaesthetic intelligence, identified by Howard Gardner (*Multiple Intelligences: the theory in practice*, 1993). Many sports require considerable skill in curve sensing.

Related issues are developed from the perspective of cognitive psychology by George Lakoff and Mark Johnson (*Philosophy In The Flesh: the embodied mind and its challenge to Western Thought*, 1999). A wider review of the implications has recently been made by Kenneth C. Bausch (*Body Wisdom: interplay of body and ego*, 2010).

Notions of "curvature" are of course central to the experience of (female) attractiveness and the erotic experience. The term "well-rounded" is applicable both to human physical and experiential qualities. Given its fundamental significance, this calls for exploration from

a topological perspective (*Reframing the Dynamics of Engaging with Otherness: triadic correspondences between Topology, Kama Sutra and I Ching*, 2011).

Language and communication: Also with its implications for cognitive curvature are the arguments of Emile van der Zee in discussing *Why We Can Talk about Bulging Barrels and Spinning Spirals: curvature representation in the Lexical Interface* (In: Urpo Nikanne and Emile van der Zee, *Cognitive Interfaces: constraints on linking cognitive information*, 2000, pp. 143-4):

> Every language contains words or expressions that refer to path curvature, object curvature and surface curvature. For example, in English it is possible to say that *Andy zigzagged down the hill*, that *Penny's dress has pleats in it*, and that *Muttley's tail is curved*. These examples show that such different words as verbs, nouns and adjectives may refer to particular curvature distinctions.... And, is it possible by looking at the meaning of curvature words to say something about the properties of the spatial to conceptual interface; a cognitive interface whose existence explains how it is possible to talk about what we see.

In his Figure 7.2 (p. 148) he presents a valuable set of images, distinguished as follows:

> A helicopter perspective on possible Figure movements along differently curved paths. TRANS (translational movement), NTRANS (non-translational movement), PATHR (orientation change of the path axis with respect to an external axis) and NPATHR (no such change) account for differences in extrinsic path-curvature. FIGR (rotation of a Figure part around an axis represented on that figure) and NFIGR (no such rotation) account for differences in intrinsic path-curvature.

With respect to such a "helicopter perspective", it is appropriate to recall the efforts of the designer of the Bell helicopter, Arthur Young (mentioned above), to generalize from experience of control of the vehicle, even envisaging the possibility of a "psychopter" (*The Geometry of Meaning*, 1984).

Cognitive implications of shape: Numerous studies have quantitatively represented the shape feature. These have mainly focused on an object under a particular cognitive condition for the quantitative representation (Shoki Kawanishi, et al. *Analysis of the Shape Feature in Consideration of the Influence of Cognitive Conditions*. IASDR Proceedings, 2009). The latter report notes that human impression is influenced by diverse cognitive conditions. To quantify shape features, the authors consider the relationship between an object and cognitive conditions. They concluded:

- complexity and beauty are not influenced by shadow position in changing proportion and curvature distribution.
- beauty is influenced by shadow position in changing curvature distribution.
- in the shape features without influence of the condition, complexity is represented by the curvature entropy as attribute elements, and beauty is represented by the degree of the golden ratio as attribute elements.
- in the shape feature with influence of the condition, beauty is represented by the distance between the centre of gravity and the shadow as state elements

In the case of virtual reality, the credibility of its "reality" calls for special attention to the experience of curvature. In one such summary, this is noted by Michael Hohl (*This is Not Here : connectedness, remote experiences and immersive telematic art*, 2006):

> Back in the spring of 1966, [Stewart] Brand sat on the roof terrace of his house overlooking San Francisco and became aware of what he perceived to be the curvature of the earth. This gave him the idea that from an even further elevated point, the whole earth would become visible. This insight made him realise that seeing the earth as a whole would have a great impact on peoples direct perception of the world. Brand recognised that although space flight had been around for almost ten years still, there was no image available showing the entire earth.

Experiencing arcs and S-curves: It is then interesting to recognize the observational "experience" during any of the Knight's three-arc moves, with its three stages of three distinct types -- using the move 1-6 (resulting from 1-2-4-6):

- same orientation (continuous arc):
  - rising to the upper-side of the torus, as from 1 (outer circumference) to 2 (on the central ring on the upper-side)
  - descending to the outer circumference, as from 2 (on the central ring) to 4 (outer circumference)
- distinct orientation
  - arcing under the under-side of the torus, as from 4 (outer circumference) to 6 (on the central ring on the under-side)

Here "rising" implies climbing up from 1 to the horizon on which 2 is located (and where it finally become visible). Then "descending" implies a "down hill" movement from 2 to 4 on the outer circumference with the latter likely to be visible at an early stage. The final move is then a "twist" under the torus from 4 to 6.

It is interesting to note that the resultant direct "move" for the Knight, as from 1 to 6 in the above example (through the central hole of the torus), involves a shorter downward arc linking into a longer upward arc -- namely an S-curve. It is the set of such S-curves for all 8 Knight's moves which defines the inner surface of the torus (recognizing that the moves may also be reversed giving a further 8). Clearly **in a plan view each of these S-curves appears to be a straight line**.

S-curve theory: Many natural processes, including those of complex system learning curves, exhibit a progression from small beginnings that accelerates and approaches a climax over time. When a detailed description is lacking, a sigmoid curve is often used, namely one produced by a mathematical function having an "S" shape of which a common form is the logistic function. S-Curve theory is used in project management as a tool to help learners and team members grasp the importance of monitoring the growth, progress and

performance of ongoing projects. A such it makes it possible to represent the utilization of resources over the proposed time of the project (Ciel S. Cantoria, *Understanding the S-Curve Theory for Project Management Monitoring*, *Bright Hub*, 24 June 2011). At first, as funds are put into development, progress is frustratingly slow. Then, as research uncovers the key pieces of information necessary to make advances, the pace surges. Finally, progress slows down again, and each successive innovation requires a greater outlay of resources.

A theory regarding the universal nature of S-curves has recently been developed by Adrian Bejan and Sylvie Lorente (*The constructal law origin of the logistics S curve*. *Journal of Applied Physics*, 2011; 110: 024901). This constructal law is based on the principle that flow systems evolve their designs over time to facilitate flow access, reducing and distributing friction or other forms of resistance. As summarized (*Seeing the S-curve in everything*, *ScienceDaily*, 20 July 2011):

> From economic trends, population growth, the spread of cancer, or the adoption of new technology, certain patterns inevitably seem to emerge. A new technology, for example, begins with slow acceptance, followed by explosive growth, only to level off before "hitting the wall." When plotted on graph, this pattern of growth takes the shape of an "S." While this S-curve has long been recognized by economists and scientists, a Duke University professor believes that a theory he developed explains the reason for the prevalence of this particular pattern, and thus provides a scientific basis for its appearance throughout nature and the human-made world.

Curvature of time: More speculatively, as noted by Cynthia Sue Larson (*Time shifting*, *Reality Shifters News*, 2006):

> Time shifting is a relatively new term which describes the ancient phenomenon observed by shamans, yogis, and other spiritual adepts capable of accessing, experiencing and influencing events in the past, present, and future. Time shifts are reality shifts in which the dimension of reality we know as "time" undergoes some kind of observable transformation. Time shifts appear to us in such a way that we can observe them in the form of time loops, time travel, time slowing down, and retrocausality.

Physicists also speculate on the matter (*Curvature of Time? Physics Forums*, 3 January 2007):

> We can all see what curvature of space looks like, just by throwing a ball and watching it follow the natural geodesic. But what does curvature of time look like? How do we experience it? We typically experience the passage of time in what seems to be a forward linear manner. The forward part seems to be due to how our nervous system works, thus giving a chronological bias towards causality in our perception. But if we can see how gravity curves space, then how do we perceive how it affects time?

To the extent that the physics of space-time and astronautics offers models for reflection on "noonautics", there is merit in considering the cognitive implication of any such movement within the noosphere (*Towards an Astrophysics of the Knowledge Universe? from astronautics to noonautics*, 2006; *Noonautics: four modes of travelling and navigating the knowledge universe?* 2006). How might the "mythical quest" of each Knight then be understood?

Dialogue: towards the dynamics of the archetypal "Round Table"?

It is curious that the quality of dialogue with regard to the challenges of global governance appears to be itself fundamentally inadequate to engaging effectively with those challenges. This is despite the many interdisciplinary and inter-sectoral symposia, and claims of a multiplicity of facilitators, models and processes -- franchised and otherwise. Each is presented as the "best thing since sliced bread", but with no capacity to engage with advocates of other processes. Dialogue then deteriorates into binary assertion and denial -- with outcomes significantly reframed by "spin".

This strategic challenge situation is reflected both in academic debate and in that between worldviews, as separately discussed (*Guidelines for Critical Dialogue between Worldviews*, 2006). Given the inherent complexity, the question is how to imagine the dynamic patterns inherent in fruitful dialogue, as implied by those of the archetypal Round Table and the cognitive "moves" that might be variously characteristic of the "Knights" seated there and the game-playing in which they might indulge (*Imagining the Real Challenge and Realizing the Imaginal Pathway of Sustainable Transformation*, 2007). How is each Knight able to "catch" and pass on the "curve balls" characteristically thrown by others? How does such a Round Table "work"?

The challenge may be explored in terms of the aesthetics of play (*Enacting Transformative Integral Thinking through Playful Elegance*, 2010). As specific cognitive biases of individual Knight's, various frameworks for understanding their "curvature skills" have been identified (*Systems of Categories Distinguishing Cultural Biases*, 1993).

Creativity: from "rock logic" to "water logic"?

Much is made of the vital importance to the future of creativity and "new thinking", as by Edward de Bono (*New Thinking for the New Millennium*, 2000) and Thomas Homer-Dixon (*The Ingenuity Gap*. 2000). For de Bono this can be understood as a shift from the locked linearity characterized by "I Am Right, You Are Wrong" (*From This to the New Renaissance: From Rock Logic to Water Logic*, 1990).

The subtle combination of creativity, irrationality, surprise and "cognitive catastrophe" are associated with the non-linearity of the Knight's move. Change is to be understood and achieved by the possibility of "getting off a train" of linear thought -- otherwise expressed as "out-of-the-box".

The relation of "visual thinking" to creativity has been a theme of interest since the study of Rudolf Arnheim (*Visual Thinking*, 1969),

most recently the theme of Thomas G. West (*In The Minds Eye*, 1997), Various texts on the nature and extent of such thinking are notably introduced with variants of this quote from Albert Einstein:

> Words and language, whether written or spoken, do not seem to play any part in my thought processes. The psychological entities that serve as building blocks for my thought are certain signs or images, more or less clear, that I can reproduce at will.

The issue is of special significance with the current extent of information overload, as conventionally presented in text form. A creative image is held to be "worth a thousand words" as an aid to integrative comprehension.

An argument can be made in favour of a quest for subtler "elven" forms of creativity, potentially more appropriate to the elusive balance sought as "sustainability" (*Walking Elven Pathways: enactivating the pattern that connects*, 2006; *Climbing Elven Stairways: DNA as a macroscopic metaphor of polarized psychodynamics*, 2007). This argument is curiously reinforced by the subtle unexpected connectivity -- nicknamed moonshine mathematics -- resulting in insight into symmetry in mathematics (*Potential Psychosocial Significance of Monstrous Moonshine: an exceptional form of symmetry as a Rosetta stone for cognitive frameworks*, 2007).

Aesthetics of harmony

Curiously, as noted by the *Wikipedia* entry, it is through poetry that the Knight's tour was first recognized:

> The earliest known references to the Knight's Tour problem date back to the 9th century AD. The pattern of a knight's tour on a half-board has been presented in verse form (as a literary constraint) in the highly stylized Sanskrit poem *Kavyalankara* written by the 9th century Indian poet Rudrata, which discusses the art of poetry, especially with relation to theater (Natyashastra). As was often the practice in ornate Sanskrit poetry, the syllabic patterns of this poem elucidate a completely different motif, in this case an open knight's tour on a half-chessboard.

Aesthetic considerations, potentially characteristic of some forms of more integrative thinking, have long been recognized in terms of their embodiment of the golden ratio into pattern design. Surprisingly the geometry of the Knight's move has been has been related to the golden ratio by Gabries Bosia through a 1:2:v5 triangle, as illustrated in a diagram on which an extensive comment is provided by Dan Thomasson (*From Knight Moves to the Golden Ratio and 3:4:5 Right Triangles*, 2005).

As discussed separately (*Associating qualities of harmony and wholeness with geometry*, 2010), Christopher Alexander strongly emphasizes beauty as a driver to "wholeness-extending" -- qualifying conventional understandings of the "attractor" of complexity theory as follows:

> Rather, I suspect that there are, deep in geometry of space, reasons why ring-like structures with this kind of ratio are likely to occur. In current jargon, rings of this particular ratio might be viewed as attractors in some phase space. However, the discovery of geometric attractors in the solutions to systems of dynamic equations is, in my view, only one particular manifestation of the far more general harmony-seeking computations that occur naturally in three-dimensional space. (*Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole. International Journal for Unconventional Computing (IJUC)*,2009)

Appropriate to thei argument, attractors are not "linear" -- minimally they are "curvilinear".

Curiously, despite widespread use of "harmony" in relation to governance, little use is made of the aesthetic principles of harmony as separately discussed (*Aesthetics of Governance in the Year 2490*, 1990; *Aesthetics and Informatics: Art of Information for Policy-making and Community-building*, 1999; *Aesthetic paradox of unitary comprehension*, 1995; *Poetry-making and Policy-making: arranging a marriage between Beauty and the Beast*, 1993).

Given the role of self-reflexivity and paradox articulated by Douglas Hofstadter (*I Am a Strange Loop*, 2007), these may well have implications for psychosocial organization ((*Sustaining a Community of Strange Loops: comprehension and engagement through aesthetic ring transformation*, 2010)

Mnemonic necessities

The vital need to sustain the capacity to "re-member" configurations and patterns of significance of requisite complexity has been separately discussed -- in the face of increasing information overload and the erosion of collective memory (*Societal Learning and the Erosion of Collective Memory*, 1980; *Emerging Memetic Singularity in the Global Knowledge Society*, 2009; *Memetic and Information Diseases in a Knowledge Society: speculations towards the development of cures and preventive measures*, 2008).

The issue may be framed in terms of the organization of sets and the quest for mnemonic devices (*Representation, Comprehension and Communication of Sets: the Role of Number*, 1978; *In Quest of Mnemonic Catalysts -- for comprehension of complex psychosocial dynamics*, 2007; *Patterns Essential to Individual and Global Health?* 2010; *Patterns of N-foldness*, 1980).

The question raised above is whether the circular pattern of Knight's moves provides the basis for such a mnemonic device -- especially with respect to the eternal challenge of psychosocial dynamics and game-playing. How are their disruptive and integrative patterns to be recognized -- preferably in a singular pattern of requisite complexity, as noted above(*Imagining the Real Challenge and Realizing the Imaginal Pathway of Sustainable Transformation*, 2007).

Suggestive comparisons: In this connection it is then intriguing to explore the suggestive contrasts between various possible visual representations of the relationship between "conceptual flatland", the implications of integrative globality, and the torus. The critique of the prevailing flatland mentality has been developed separately (*Irresponsible Dependence on a Flat Earth Mentality -- in response to global governance challenges*, 2008).

Fig 9: Schematics indicating dominant role of "flatland" mindset in comprehension of the globe	
Fig. 9a: Academic mortar board (as occasionally presented)	Fig. 9b: Binary strategic games of governance (with a "lifebuoy" for turbulent seas of chaos)
	

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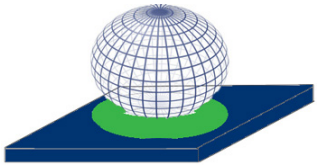
Fig. 10: Schematics of "ideal" understandings of governance	
Fig. 10a: Schematic of ideal temple (combining flat, torus and sphere)	Fig. 10b: Schematic of optimistic ideal of governance (with ever ready "lifebuoy")
	

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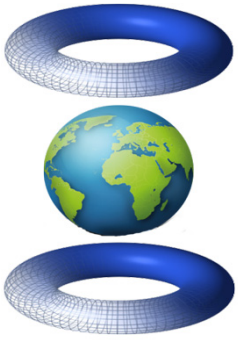
Fig. 11: Symbolic and speculative representations of governance	
Fig. 11a: Traditional method of transportation (of worldly goods by by women)	Fig. 11b: Dynamic suspension within the "cognitive magnetosphere" ***
	

Fig. 11a usefully highlights the reality for millions obliged, like Atlas, to bear the weight of their own world -- in this case with characteristic elegance. A torus then provides the cushioning junction whilst walking over a flat surface.

Fig. 11b offers a provocative image of the potential challenge of sustainability represented as a dynamic balance -- well-recognized in electromagnetic suspension. The lower torus might indeed represent achieving successful design and operation of a toroidal nuclear fusion reactor as the acclaimed key to the planet's future energy needs (such as ITER). The upper torus might then represent a corresponding form of "cognitive fusion" containing the challenging dynamics of the Knight's move and overcoming the disruptive instabilities of "monkeying" and "blame-games". The relation between the physical and cognitive tori is discussed separately (*Enactivating a Cognitive Fusion Reactor: Imaginal Transformation of Energy Resourcing (ITER-8)*, 2006). Presented in this way, the image echoes the arguments for the elusive psychosocial paradoxical recognition of living "in between" contrasting understandings -- as many are increasingly obliged to do (*Living as an Imaginal Bridge between Worlds: global implications of "betwixt and between" and liminality*, 2011).

Symbols: vital psychoactive focus

The emerging future is one in which increasing social chaos is widely expected, with the possibility of societal collapse (Jared M. Diamond, *Collapse: how societies choose to fail or succeed*, 2005). It is therefore fruitful to reflect, like the Long Now Foundation, on how current essential insights may be communicated through such chaos to the future (*Minding the Future: a thought experiment on*

presenting new information, 1980).

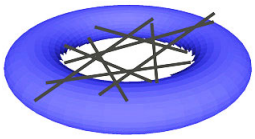
One such possibility is through embodying insight into some form of (metaphorical) Rosetta stone (*Systemic Crises as Keys to Systemic Remedies: a metaphorical Rosetta Stone for future strategy?* 2008; *Potential Psychosocial Significance of Monstrous Moonshine: an exceptional form of symmetry as a Rosetta stone for cognitive frameworks*, 2007). Various systems of beads may offer inspiration to that end (*Designing Cultural Rosaries and Meaning Malas to Sustain Associations within the Pattern that Connects*, 2000)

These considerations all reflect a concern with integrative symbols in which patterns of meaning can be anchored to facilitate their future comprehension. Various symbols central to different cultures, may be considered as comparable in this light.

Crown of Thorns: Central to Christian mythology is the mock crown made of thorn branches that Roman soldiers placed on the head of Jesus before the Crucifixion -- purportedly in response to his claim to be "King of the Jews" (*When Jesus was on trial for his life, the soldiers twisted thorns into a makeshift crown and jammed it on his head*, John 19:2). The Crown of Thorns is held to symbolize the set of human afflictions that cause great suffering -- to be borne like a "crown of thorns".

The cognitive significance of a "crown" has been discussed separately (*Engaging with Globality through Cognitive Crowns*, 2009). In the light of the above argument, that dysfunctional crown is here presented as an indication of the extremely painful consequences of the currently fragmented "crowning" configuration of global strategies. This could be said to derive from linear ("thorn-like") mindsets, ill-adapted to the non-linear nature of the dynamics characteristic of the global system. Here the possibility of replacing this dysfunctional linearity by forms of curvature is more consistent with those dynamics. Ironically the interplay between the imperial mindset, the Christians and the Jews continues to be played out with little sign of resolution.

The torus offers a form by which this cognitive crown can be reframed, notably with respect to collective pain and the implications of loss of a larger integrative harmony. The linear characteristic of the multiplicity of ill-coordinated projects of limited duration is then reconfigured into a sustaining pattern of circularity. The mapping of the Knight's moves onto the torus, curves passing through 12 points, offers a relationship to the various symbolic uses of that number within the Christian tradition (Stations of the Cross, Twelve Apostles, etc).

Fig. 12: Representations of the "Crown of Thorns"	
Fig. 12a: Traditional form (as conventionally imagined)	Fig. 12b: Indication of unjoined-up understanding (failure to map Knight's moves to curvature)
	

N.B. The representation of Fig. 12b calls for greater graphical skills requiring the straight lines to go through the hole of the torus and to stick out above and below -- "thorn-like" -- as a cause of pain when the torus is worn as a crown. This graphical inadequacy also applies to Fig. 14.

Star of David: Within the Jewish tradition fundamental significance is of course attached to the 6-pointed Star of David. The Crown of Thorns might well have been intended as a mocking reference to that. Is there a case for exploring the geometrical commonalities of that star with those emerging from the pattern of Knight's moves projected into a space of higher dimensionality -- in this case onto a torus?

Islamic star: A 5-pointed star is extensively used by Islam, typically accompanied by a crescent. Less frequent use is made of an 8-pointed star (especially in Morocco), although Islam also recognizes value in the 6-pointed star. With respect to the latter, Muslim legend recounts Solomon using the star to capture *djinns*, genies, the immaterial counterparts to humans. The 8-point star, called *khatim* or *khatim sulayman*, is widely used as a symbol in Islamic art (*Origins and Meanings of the Eight-Point Star*). Moroccan *zillij* artisans also refer to this as *sibniyyah*, *sabniyyah*, which is a derivative of the number seven *sab'ah*. According to Sufi tradition, with respect to the 8-pointed star, as noted by Ian Alexander *(Sufi*):

> Form is symbolised by the square. Expansion is symbolised by the square with triangles pointing outwards (an 8-pointed star). Contraction is symbolised by the square with triangles pointing inwards (a 4-pointed star). The two star-shapes together symbolise the cycle of creation, 'the breath of the compassionate

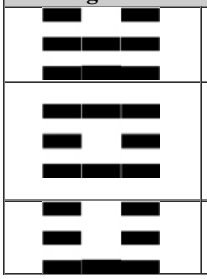
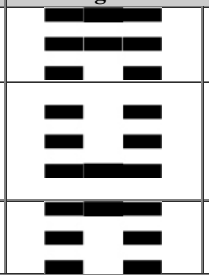
Again there is a case for exploring the geometrical commonalities of these stars with those emerging from the pattern of Knight's moves projected into a space of higher dimensionality.

The pattern of Fig. 7a is indeed an 8-pointed star.

Mandala: There is an extensive literature on the role of the mandala in the Buddhist and Hindu traditions as an integrative mindmap, especially in support of meditation and notably the quest for emptiness or mindlessness (Giuseppe Tucci, *The Theory and Practice of the Mandala*, 1973). Its role is of course echoed in the significance attached in the Christian tradition to the rose window (Painton Cowen, *The Rose Window*, 2005). In relation to the argument above the circular mapping of the Knight's moves (in Fig. 7a) may be understood as such a mandala.

Ba Gua: The possible relationship between the pattern of Knight's moves of Fig. 7a and the Chinese system of 8-fold *Ba Gua* was previously discussed (*Reframing "monkeying" in terms of Knight's move patterns*, 2011). In that previous discussion, the eight cells

within which the Knight moves were used to position the 8 trigrams of the Chinese Ba Gua system. Fig. 13a and Fig. 13b show the traditional alternative arrangements. A reduced version of Fig. 1 is placed in the central position as a reminder of the possibility of further exploration.

Fig. 13: Ba Gua Arrangements	
Fig. 13a: Ba Gua: Earlier Heaven Arrangement	Fig. 13b: Ba Gua: Later Heaven Arrangement
	

Freemasonry: Of interest in relation to the Knight's move is the fundamental symbolic importance attached to the relation between the square and the circle in Freemasonry (Jean-Michel David, *The Square -- the Masonic Square*):

- **square**: The Masonic square is typically depicted in two ways: one with equal length arms - or sides, and the other by arms or sides of unequal lengths. This latter is sometimes referred to as the ***gallows square*** (similar in shape to the character Yod, the tenth letter of many Semitic alphabets, including Phoenician, Aramaic, and Hebrew). The gallows square, with arms in the 3:4 or Pythagorean ratio, is the traditional emblem of the Master. In medieval Europe that shape was used in ecclesiastical script to represent the capital letter G, because it was exactly the same shape as the Greek letter Gamma and equivalent to G in the Roman alphabet, standing alike for God and the attribute "Justice". (W. M. Don Falconer, *The Symbolism of the Square*, 1998)
- **circle**: The traditional Masonic emblem of the square and compasses can be taken as a symbolic representation of the solution to the Jungian problem of squaring the circle. A Mason is one who learns to square the circle. Within this framework, the square can be seen to be a guide to full self-development.

Given the relation of the Knight's move to the golden ratio (mentioned above), the process of learning how to "square the circle" might be understood as a reference to understanding the circular pattern of Knight's moves.

Mathematical theology: It is most curious that the Abrahamic religions (Christianity, Islam, Judaism) are notable in the significance they attach to distinct symbolic patterns and the numbers associated with them. This is evident in their distinct preferences for "stars", as mentioned above. It is astounding that no detectable effort is made to reconcile their differences through the geometry and number theory to which the greatest mathematical insight has contributed -- despite the role of mathematicians of those faiths in elaborating those insights (*And When the Bombing Stops? Territorial conflict as a challenge to mathematicians*, 2000).

With respect to those religions, it might be appropriately said that the Crown of Thorns helps to characterize their current abysmal failure to offer any integrative perspective. The inherent linearity of their various approaches is indeed "thorn-like" and at variance with any integration offered by the curvature characteristic of the torus. As the principal force behind the faith-based governance variously driving and constraining global strategy, together their interpretation epitomizes the currently fragmented "crowning" configuration of global strategies. This is most clearly represented in Jerusalem (*Reframing Relationships as a Mathematical Challenge: Jerusalem as a parody of current interfaith dialogue*, 1997). More specifically it is evident in the "thorny" nature of the relationship between Christian denominations with conflicting claims to "space-time" within the Church of the Holy Sepulchre -- where Jesus was crucified, where he is alleged to have been buried (possibly with what he originally represented).

Cross-cultural considerations: It is for example interesting to explore the correspondences between the hexagram representation favoured by the *I Ching* / *Ba Gua* system and the Star of David (*Double triangular representation of hexagrams: Star of David*, 2008; *Mapping of I Ching hexagram coding onto Star of David*, 2008).

There would appear to many features of symbolic geometry which remain to be explored as a means of giving a memorable sense of coherence (*Sustainability through Magically Dancing Patterns: 8x8, 9x9, 19x19 -- I Ching, Tao Te Ching / T'ai Hsüan Ching, Wéiqí (Go)*, 2008) This exploration follows from a series of earlier papers on the *I Ching* and the *Tao Te Ching* (*9-fold Higher Order Patterning of Tao Te Ching Insights: possibilities in the mathematics of magic squares, cubes and hypercubes*. 2003; *9-fold Magic Square Pattern of Tao Te Ching Insights experimentally associated with the 81 insights of the T'ai Hsüan Ching*, 2006).

Toroidal symbols: With respect to the toroidal form discussed above, there is a curious symbolic relationship between:

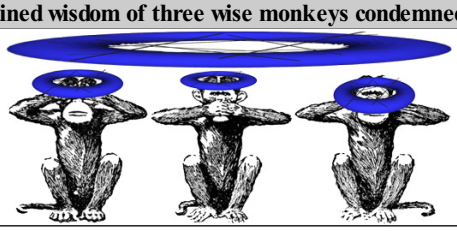
- the halo as an indication of the spiritual insight to which it might be expected that the religions would aspire. It is notably used Hellenistic Greek, Roman, Hindu, Buddhist and Christian sacred art, where individuals may be depicted with the form of a circular glow
- the lifebuoy designed to be thrown to a person in the water, to provide buoyancy in order to prevent drowning. At a time when a variety efforts are made to provide "flotation" to financial schemes threatened with disaster, there (E. S. Browning, *DeLong vs. Grant: Is QE3 a Lifebuoy or Torpedo? The Wall Street Journal*, 16 June 2011). The "torpedo" reference is reminiscent of the "thorn-like" linearity mentioned above.
- the torc torc was a sign of nobility and high social status and possibly a divine attribute -- in significant contrast to the neck shackle. As with the original slave collar used to identify and discipline slaves, in a BDSM context, a collar is a device of any material placed around the neck of the submissive partner.

Mythological indicators: As a complement to purely geometrical symbols, consideration can also be given to the role of myth, as previously argued (*Relevance of Mythopoeic Insights to Global Challenges: cognitive integration implied by the Lord of the Rings*, 2009; *Cognitive Fusion through Myth and Symbol Making: archetypal dimensions*, 2006). Myth may notably be used to explore the dysfunctionalities of blame-games and "monkeying" (*The "Dark Riders" of Social Change: a challenge for any Fellowship of the Ring*, 2002), but especially of end times scenarios of eschatological writings (*Spontaneous Initiation of Armageddon: a heartfelt response to systemic negligence*, 2004).

Understood as global psychosocial functions, the death of the "gods" as explored in the Norse myth of Ragnarök, is especially appropriate to a much-challenged global civilization. As author of *Ragnarök: the end of the gods* (2011), A. S. Byatt remarks:

> But if you write a version of *Ragnarök* in the 21st century, it is haunted by the imagining of a different end of things. We are a species of animal which is bringing about the end of the world we were born into. Not out of evil or malice, or not mainly, but because of a lopsided mixture of extraordinary cleverness, extraordinary greed, extraordinary proliferation of our own kind, and a biologically built-in short-sightedness.... I read of human projects that destroy the world they are in, ingeniously, ambitiously engineered oil wells in deep water, a road across the migration paths of the beasts in the Serengeti park, farming of asparagus in Peru, helium balloons to transport the crops more cheaply, emitting less carbon while the farms themselves are dangerously depleting the water that the vegetables, and the humans and other creatures, depend on. (*Ragnarök: the doom of the gods*, *The Observer*, 5 August 2011)

Fig. 14: Representation of constrained wisdom of three wise monkeys condemned to linear, disintegrative thinking



The functional identity of the 3 monkeys has been discussed separately (*Monkeying with Global Governance: emergent dynamics of three wise monkeys in a knowledge-based society*, 2011). They might refer to the 3 Abrahamic religions, or to the traditional estates of the realm, or to other institutionalised system functions. With each might be associated a set of asystemic "thorns". Thus the "crown of thorns" worn by the monkey of the "first estate" -- religion -- might be acknowledged as the unfortunately disparate linearity of the principal religions, as described by Stephen Prothero (*God Is Not One: the eight rival religions that run the world -- and why their differences matter*, 2010). Ironically, "one" is "beyond" each religion -- perhaps fortunately for God. Any problematic interfaith discourse might then be usefully explored through a pattern of Knights moves -- with the prospect of a fruitful pattern of a subtler order.

There is an irony to the current condition of what is alleged to remain of the historic Crown of Thorns (as described in the *Catholic Encyclopedia*). It has been treated as the holiest of relics, following removal of the "holy thorns" -- of which some 700 are variously claimed to have been widely distributed. To the extent that various institutionalised functions and systemic problems, are now represented by such a "crown", their status as "holy relics" of dubious authenticity is appropriate.

Comprehension

The implicit concern here is with how collective comprehension of greater complexity and subtlety is to be enabled.

This is a theme that has been variously explored in relation to the templates offered by geometry and to the work of others:

- Embodiment of Change: Comprehension, Traction and Impact? Discovering enabling questions for the future (2011)
- Sustaining a Community of Strange Loops: comprehension and engagement through aesthetic ring transformation (2010)
- Harmony-Comprehension and Wholeness-Extending (2010)
- Comprehension of Requisite Variety for Sustainable Psychosocial Dynamics: Transforming a matrix classification onto intertwined tori (2006)
- Hyperaction through Hypercomprehension and Hyperdrive: necessary complement to hypertext proliferation in hypersociety (2006)
- Imagination, Resolution, Emergence, Realization and Embodiment: iterative comprehension ordered via the dynamics of the Mandelbrot set (2005)
- Applying Mathematical Insights to Comprehension of World Problems (1999)
- Discovering richer patterns of comprehension to reframe polarization (1998)
- Comprehension of Appropriateness (1986)

Limitations and constraints on integrative frameworks

In a global knowledge-based society characterized by increasing degrees of information overload, it is a matter of increasing concern as to how who understands what, and why -- and which initiatives they are then enabled to take undertake. The challenge is obvious in an integrative exercise -- as with this one -- where inadequacies are immediately apparent in the light of the insights of others, especially those of far greater competence and expertise.

It is then useful to identify the nature of constraints in such explorations, as a factor to be taken into account in any design. Efforts to do

so include:

- Memory Challenges at the Edge of the World (2008)
- An Inconvenient Truth about any Inconvenient Truth (2008)
- Creative Cognitive Engagement: Beyond the limitations of descriptive patterning (2006)
- Transcending methodological limitations (1995)
- Social organization determined by incommunicability of insights (1995)
- Societal Learning and the Erosion of Collective Memory (1980)
- Pointers to the Pathology of Collective Memory (1980)
- Limits to Human Potential (1976)

Such considerations merit reflection in the light of the challenge of premature closure. Seemingly especially significant is the requisite interplay between closure and openness for the sustainability of that dynamic, as variously argued by Orrin Klapp (*Opening and Closing; strategies of information adaptation in society*, 1978) and by Hilary Lawson (*Reflexivity: the post-modern predicament*, 1985; *Closure: A Story of Everything*, 2002). If only to leave scope for future reflection and development, a degree of incompleteness is required (*Beyond Harassment of Reality and Grasping Future Possibilities* , 1996).

Musical facilitation of integrative comprehension

It is appropriate to note the extent to which music is widely appreciated and constitutes a basis for social integration and coherence. Elsewhere a case has been made for recognizing insights into musical harmony as a key to the elusive coherence required for global governance (*A Singable Earth Charter, EU Constitution or Global Ethic?* 2006; *Polarities as Pluckable Tensed Strings: hypercomprehension through harmonics of value-based choice-making*, 2006; *Structuring Mnemonic Encoding of Development Plans and Ethical Charters using Musical Leitmotivs*, 2001). Insight into polyphony as also been considered (*All Blacks of Davos vs All Greens of Porto Alegre: reframing global strategic discord through polyphony?* 2007).

As mentioned there, rich consideration has been given to the cognitive role of music by Douglas Hofstadter (*Gödel, Escher, Bach: an Eternal Golden Braid*, 1979).

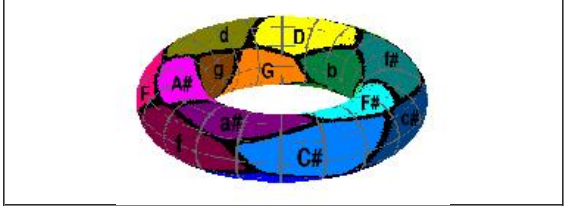
Cognitive role of music: Especially relevant to the above argument are indications from recent research regarding the organization of music, especially in cognitive terms based on the torus. As previously discussed (*Memorability: musical clues to psychosocial system sustainability*, 2006), the relation of music to the functioning of the brain is a theme in the cognitive neurosciences [more].

Research by Petr Janata *et al* (*The Cortical Topography of Tonal Structures Underlying Western Music*, *Science,* 13 December 2002, 298. 5601, pp. 2167-2170) has indicated that knowledge about the harmonic relationships of music is maintained in the rostromedial prefrontal cortex providing a stronger foundation for the link between music, emotion and the brain. The melody used experimentally was crafted to shift in particular ways through all 24 major and minor keys. The relationships between the keys, representative of Western music, create a geometric pattern in the form of a torus (see Petr Janata, *Music Mapped to the Torus*, 2005, and torus dynamics movies).

The piece of music moves around on the surface of the torus offering a means of determining the pure representation of the underlying musical structure in the brain. The work clarified the mapping of melodies in the brain, as it varied from one occasion to another suggesting that the map is maintained as a changing or dynamic topography. This dynamic map may provide the key to understanding why a piece of music may elicit different behaviours at different times [more more] (see also Robert J. Zatorre and Carol L. Krumhansl, *Mental Models and Musical Minds*, *Science*, 13 December 2002: 298. 5601, pp. 2138-2139). Of particular interest was the role of any such mapping in the memorability of favourite tunes.

Toroidal harmonic space: The torus may be used as a representation of harmonic space. A piece of music moves around in this space [more]. The results of psychoacoustic experiments by C L Krumhansl and E J Kessler (*Tracing the dynamic changes in perceived tonal organization in a spatial representation of musical keys*, Psychological Review 89(4), 1982, pp. 334-368) of the inter-key relations of all major and minor keys can be represented geometrically on a torus -- as shown by Benjamin Blankertz, Hendrik Purwins and Klaus Obermayer (*Constant Q Profiles and Toroidal Models of Inter-Key Relations -- ToMIR*, 1999) in the following image

Fig. 15: Geometric representation of the inter-key relations
of all major and minor keys
(derived from psychoacoustic experiments by Krumhansl and Kessler)



Thomas Fiore (*Music and Mathematics*; *Beethoven and the Torus*) addresses one of the central concerns of music theory, namely to find a good way to hear a piece of music and to communicate that way of hearing. He notes that music theorists make much use of mathematics in creating conceptual categories enabling them to develop taxonomies and classifications of the various sets that arise in music. In particular he shows how group theory offers a way of describing the ways that sets and pitches relate and how they can be

transformed from one to another. Fiore focuses in particular on the musical relevance of the PLR group:

> This is a set of functions whose inputs are major and minor chords and whose outputs are major and minor chords. These musical functions go back to the music theorist Hugo Riemann (1849-1919). As a result, the PLR group is sometimes called the neo-Riemannian group [see also Edward Gollin, *Neo-Riemannian Theory*, 2005].

Using the PLR group Fiore shows that the harmonic progression in the second movement of Beethoven's *Ninth Symphony* traces out a path on a torus. Beethoven's *Ode to Joy* (in the fourth and ninth movement of that symphony) has been adopted as the official anthem of Europe (Council of Europe, 1972; European Union 1985).

Given the passing recognition by the European Commission of the role of the torus in multi-agent modelling relating to sustainable development (see above), it might be asked whether a key to sustainability for Europe lies not "under the noses" of policy makers but rather "behind their ears". Of course there is indeed the possibility that odour may play an unsuspected role as suggested by Chris C. King (*Fractal and Chaotic Dynamics in Nervous Systems*, 1991) and may also be usefully mapped onto a torus. There may be unsuspected higher orders of significance to references in political discourse as to whether strategies "stink" or "smell right"!

Torus and tonnetz: In a helpful overview Justin London (*Some Non-Isomorphisms Between Pitch and Time*, *Music Theory Midwest*, April 2001) points to the recognition of the role of the torus by Brian Hyer (*Re-Imagining Riemann*, Journal of Music Theory 39(1), 1995, pp. 101-38) and Richard Cohn (*Neo-Riemannian Operators, Parsimonious Trichords, and their Tonnetz Representations*. Journal of Music Theory, 41(1), 1997, pp. 1-66). The tonnetz on which he focuses is a tonal lattice invented by Hugo Riemann as a model for just intonation. He notes that:

> Music theorists are not alone in recognizing the torroidal shape of tonal space. Researchers in music perception and cognition have empirically measured the goodness-of-fit for notes and chords in a tonally-primed context, and they too have mapped tonal space onto the surface of a torus.

London concludes:

> ... metric space is planar, tonal space is non-planar; therefore the two spaces are non-isomorphic. And if the two spaces are non-isomorphic, then there are fundamental problems in trying to map elements or relationships (i.e., functions which employ those elements) from one space to another....What is gained in this exercise is that by trying to follow the same "rules" in constructing graphic representations of tonal and metric relationships, we are forced to confront the differences between them. We also are reminded of how the topologies of ... the metric tree and the tonal tonnetz -- arise from the combination of formal relationships among their component elements as well as the way human beings hear and understand those relationships. As in all of our musical representations, what we can hear and what we can imagine are intertwined and interdependent.

Justin Hoffman (*Listening with Two Ears: conflicting perceptions of space in tonal music*, 2011): ***

> The Tonnetz is a spatial model of tonal pitch, constructed by placing fifths along the horizontal axis of a coordinate plane and thirds along the vertical axis.... Though the Tonnetz can be constructed in several different ways, all of these representations have in common that they appear in some type of space (whether a coordinate plane or a non-Euclidian torus) with fifth-related pitches along the horizontal axis and third-related pitches along the vertical axis.... The second space, the pitch-class Tonnetz, is a torus, a finite space representing relationships among the twelve pitch classes that result from equal temperament

Music and sustainability: Given the remarkable sustainability of religious orders, monasteries and ashrams, it is worth exploring the role that regular use of bells -- audible throughout the community -- have played in ensuring coherence. Such possibilities are linked to understandings of the cognitive organization associated with any understanding of "attunement" in relation to individual or collective meditation (as well as to the torus-like halo associated with holy people, and even a particular form of clerical tonsure).

Is it possible that particular patterns -- mappable onto a torus -- are more appropriate to ensuring community sustainability? What is to be learnt from bell tuning in ancient China in this respect -- in sustaining scattered communities within the imperial domain? Especially noteworthy was the function of the Yellow Bell in ancient China -- a carefully tuned instrument upon which all other pitches and measurements were based, which was carefully reworked by every new emperor [more | more]. Might this be a key to the challenges of development in Africa (cf *Knowledge Gardening through Music: patterns of coherence for future African management as an alternative to Project Logic*, 2000)

Orbifolds: Especially relevant to this argument is the work of Dmitri Tymoczko (*The Geometry of Musical Chords*, *Science*, 313. 5783, 7 July 2007, pp. 72 - 74):

> A musical chord can be represented as a point in a geometrical space called an orbifold. Line segments represent mappings from the notes of one chord to those of another. Composers in a wide range of styles have exploited the non-Euclidean geometry of these spaces, typically by utilizing short line segments between structurally similar chords. Such line segments exist only when chords are nearly symmetrical under translation, reflection, or permutation. Paradigmatically consonant and dissonant chords possess different near-symmetries, and suggest different musical uses.

Of interest to a potential ambiguity in the significance attached to the Knight's move as mapped onto a torus, in a subsequent work Dmitri Tymoczko (*The Generalized Tonnetz*, 2011) relates two categories of music-theoretical graphs, those in which points represent chords and those in which points represent notes.

> Music theorists typically represent voice-leading possibilities using two different types of graph. In note-based graphs, notes correspond to points and chords are represented by extended shapes of some kind; the prototypical example is the Tonnetz, where major and minor triads are triangles, and efficient voice leadings are reflections ("flips") preserving a triangle's edge. In chord-based graphs, by contrast, each point represents an entire chord and efficient voice leading corresponds to minimal motion in the space, typically along an edge in a discrete graph.

Harmonic relationship between Knight's moves: Such considerations highlight the possibility of insights into ways of ordering the Knight's moves in terms of their harmonic significance -- potentially through their appropriate sonification. Is appropriate integration of relevance to governance then to be understood through patterns of resonance as previously argued? (*Liberation of Integration through pattern, oscillation, harmony and embodiment*, 1980 with a *Fugitive Integration: a musical addendum*, 1980). Is it through such insights that a sense of how a "crown" may "work" -- as with a "Round Table"? More provocatively, with what sense of shared harmony do "keynote speakers" now address an "audience".

Self-reflexivity essential to appropriate design

The argument above focuses however on the dynamics associated with the geometry, of which the geometry may itself be an expression -- as with a standing wave. This then raises self-reflexive questions such as:

- how one gives expression to the geometry
- how one's identity is implied and shaped by the geometry
- how one's comprehension of dynamic engagement with the environment is constrained by understanding of the geometry
- how symmetry equality ****

Whilst seemingly abstract, this offers a means of discussing:

- the nature of any sense of focus and identity -- the experience and meaning of being centered rather than fragmented
- how one makes a "point", develops "lines" of argument, and considers the resulting construct -- in relation to those "made" by others

Especially intriguing is the sense of a "sphere" of influence, possibly based on the intersection of several "circles" of associates, friends, allies, etc. More challenging is any sense of "pointlessness" and "emptiness" to life -- and how this relates to any sense of plenitude. These may be variously associated with some sense of "globality" in the integrative, psychosocial and planetary senses (*Future Generation through Global Conversation: in quest of collective well-being through conversation in the present moment*, 1997; *Implication of Personal Despair in Planetary Despair*, 2010). The relationship between the simpler geometries thus gives expression to the curious interplay between the "flatland" of board games -- echoing the experience of life constrained to an urban gridwork -- as compared with any all-encompassing existential sense of emptiness.

It is in this sense that "flatland" offers no unique position. But curiously a "point" may then be made by "scoring" in some way -- possibly in relation to the geometry of goal posts, nets, boundaries, etc. The argument above, transforming point-making over a flat surface onto a torus, engenders a central focal point for the pattern -- within a space of emptiness. It achieves this by designing in curvature, giving expression to challenges to comprehension -- where one point is invisible to another because of horizon effects.

It is in this sense that the torus offers a form of "missing link" between the necessities of a flat earth mentality and those of a more challenging engagement with globality -- dependent geometrically on a fundamentally inaccessible central point. Such an argument can also be related to the challenge of integrative thinking as variously understood in which a distinction can be made between:

- organization of knowledge "on the flat", as in curricula, spreadsheets and tabular presentations of topics
- circular configurations of knowledge, most notably as in mandalas and yantras -- of which the Fig. *** circular pattern of Knight's moves "on the flat" offers an example. Here a degree of concentration is implied, with knowledge being given a focus.
- the toroidal representation of Knight's moves
- embodiment of knowledge in spherical form. as implied by the possibilities of meditation (mindlessness, emptiness ***). The implications for global governance are suggested by spherical organizations of knowledge (*Spherical Configuration of Categories -- to reflect systemic patterns of environmental checks and balances*, 1994; *Spherical Accounting: using geometry to embody developmental integrity*, 2004)

There is therefore a case for comprehending a "design exercise" like this as an embodiment of essential cognitive principles. Does requisite self-reflexivity imply that each specific geometric attribute of cognitive significance should be uniquely associated with one of the 12 Knight's of any cognitive Round Table -- centre, place, orientation, etc? Curiously the argument for managing and focusing "distraction" can be clarified by reference to the necessary and well-studied configuration of arrays of aerials or solar panels.

Again, however, use of "geometry" obscures the dynamics from which particular forms and patterns emerge through a degree of self-organization. The cognitive implications have been well-explored by George Lakoff and Rafael Nuñez (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2001).

Conclusion

The argument has highlighted the extremely painful "thorn-like" consequences of unjoined-up linear thinking. In effect the "thorns" are the various pieces of a global system which lack appropriate connectivity, resulting in unsustainability and instability -- and potentially chaos and collapse. This is evident with the (re)current collapse of the financial system. More challenging is the demonstrated incapacity -- of those empowered to do so -- to dialogue effectively about the condition, or even to focus on why that dialogue is so ineffective.

In the midst of another financial crisis, the nature of the underlying "thinking crisis" -- a "thundering absence of bright ideas" -- is underlined by Julian Glover (*The cry for leadership goes out - and no one answers*, *The Guardian*, 7 August 2011):

> Despite their differences, capitalists, socialists, liberals and conservatives are united by a common idea. It is the assumption of linear progress for human civilisation: the belief, seldom stated because rarely challenged, that things can only get better or - if they seem not to be - can get better if we choose the right policies....What, once summoned back to their offices, do we expect ministers to do?... No one can agree, of course, but it's worse than that: no proposed strategy carries complete conviction even with those who propose it. There is a thundering absence of bright ideas, of the "hey, here's how" variety. Hence the frightening silence.... The crises we face in the summer of 2011 are no less sharp or scary [than those of the past], but what's missing isâ? leadership, not so much by people asâ?by ruling ideas. The best, as Yeats said, lack all conviction.

The last phrase is derived from the first verse of a much-cited poem (*The Second Coming*, 1919):

Turning and turning in the widening gyre
â? â? The falcon cannot hear the falconer;
â? â? Things fall apart; the centre cannot hold;
â? â? Mere anarchy is loosed upon the world,
â? â? The blood-dimmed tide is loosed, and everywhere
â? â? The ceremony of innocence is drowned;
â? â? The best lack all conviction, while the worst
â? â? Are full of passionate intensity.

This is suggestive of a toroidal psychosocial dynamic of global proportions -- the ultimate Crown of Thorns for a knowledge-based civilization -- in which inadequate efforts at "linearity" will be forced into forms of curvature characteristic of a "black hole" with its fundamental "event horizons".

The focus here on "design considerations" has endeavoured to "design in" factors of inherently problematic connectivity -- what mathematics has recognized as "moonshine" connectivity -- as offering the requistie subtlety to encompass the dynamics in question. The resultant coherence might be usefully understood as a poorly tuned instrument -- a challenge to further reflection. However there is also the recognition of a degree of necessary incompleteness and inadequacy -- of a work in progress and an invitation to future learning. Premature closure -- inflexible claims to having the "right" answer -- are itself a characteristic inadequacy of many models on offer.

Events at the time of writing suggest that the methods of the global governance process are essentially inadequate -- not only the dependence on "casino capitalism" but what might be recognized as a "casino conceptual system" that enables and sustains it. Especially striking, beyond the capacity to engender a sustainable "solution", is the sense that any such candidate would be packaged in hope-mongering "spin" in exactly the same way as the processes of empty promise-making which have engendered the current pattern of crises. There is seemingly little capacity to deliver global solutions, despite claims to the contrary -- and even those of a technical nature.

This is well articulated with respect to the UK's own effort to develop a comprehensive information system for its National Health Service. Launched in 2002, with a budget of £11.4bn, with the aim of replacing paper medical records with a centralised national electronic database:

> It never happened. The scheme quickly degraded into a mass of regional and incompatible systems, provided by two companies... Neither has been able to deliver even the reduced capability specified in their contracts.... There are lessons in the report for all policymakers. As the failure of the NHS private finance scheme has also shown, the government is an inept purchaser of private services: indecisive, ponderous, overambitious and wasteful. Mass centralisation does not reduce costs, but it kills flexibility. Under the national scheme, NHS professionals were given expensive systems with little discussion of what would actually help them do their jobs. The project was carried along by the momentum of its scale and the sense that having spent so much it would be wrong to pull out (*NHS database: Digital disaster*, *The Guardian*, 3 August 2011

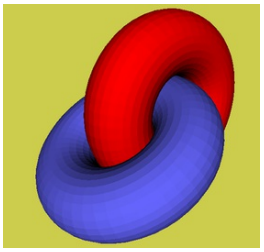
There is an irony to the promissory capacity of governance, at a time of global concern with "emissions", in that that capacity is usefully to recognized as the emission of political "hot air" -- equally threatening to another form of "global warming" (*Sins of Hot Air Emission, Omission, Commission and Promission*, 2009). In that sense, more fundamental than the financial system is the system of confidence with which those active in governance are currently "monkeying" (*Primary Global Reserve Currency: the Con? Cognitive implications of a prefix for sustainable confidelity*, 2011). Increasingly any proposals in the language of the past ring hollow and are completely lacking in credibility (*Abuse of Faith in Governance*, 2009; *Credibility Crunch engendered by Hope-mongering*, 2008).

In these circumstances it is not to be expected that the "global system" will be appropriately enhanced as might have been expected. Nor is it to be expected that appropriately "joined-up thinking" will emerge from any collective global endeavour and acquire "universal" acceptance. This suggests that it is for each to develop the conceptual skills implied by patterns of thinking of a subtler order -- of which the argument above is but one tentative indicator. It is with such "do-it-yourself" models that people (together with their elective affinities) will necessarily have to engage with their world and the other "Knights" therein -- effectively on the basis of a form of "*Declaration of Universal Independence*" (2009).

It then becomes incumbent on each to embody an asystemic "Crown of Thorns" and to transform it by engendering in it the connectivity appropriate to global systemic cycles. To that end the quest is for "forms" -- geometrical, musical, or otherwise -- that can be used as templates with which thinking of appropriate subtlety can be associated, as previously argued (*¿ Higher Education 8 Meta-education ? Transforming cognitive enabling processes increasingly unfit for purpose*, 2011).

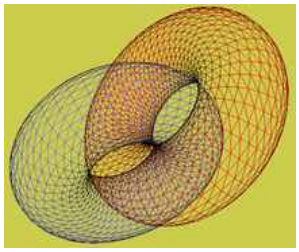
With respect to the emphasis above on the torus, of interest in exploring the possibility of embodying a "crown" of greater integrity, are the following visual developments of the argument. The various images of Fig. 16 endeavour to show the implications of two intertwined tori -- moving with respect to one another -- suggesting ways of considering the relationship between objective and subjective understandings (*¡¿ Defining the objective ∞ Refining the subjective ?!*, 2011)

Fig. 16: Screen shots of a dynamic virtual reality model of intertwined tori
(click on each variant to access and manipulate in 3D;
in the free Cortona VRML viewer, right click for preferences to switch from/to the "wireframe" presentation)



Red torus has a vortex (smoke ring) dynamic in the model

Blue torus has a wheel-like dynamic in the model



VRML animations by Bob Burkhardt.
Reproduced from *Comprehension of Requisite Variety for Sustainable Psychosocial Dynamics* (2006)

The four variants of Fig. 17 offer an understanding of a geometric "compromise" between the linearity of a cube and what is effectively a torus surrounding it, with the whole offering a degree of spherical symmetry. This polyhedron is of particular interest in that it has 64 edges -- allowing the edges to be interpreted as indicative of the dynamics of change according to the extensive system of the Chinese *Ba Gua* / *I Ching* system (see Commentary). As an exercise, without considering their position with respect to one another, the names of those conditions have been associated with individual edges. (The images were generated using *Stella Polyhedron Navigator*)

Fig. 17a: Drilled truncated cube -- a polyhedron approximating a torus, with 64 edges
(Variously rotated with selected faces coloured or not)

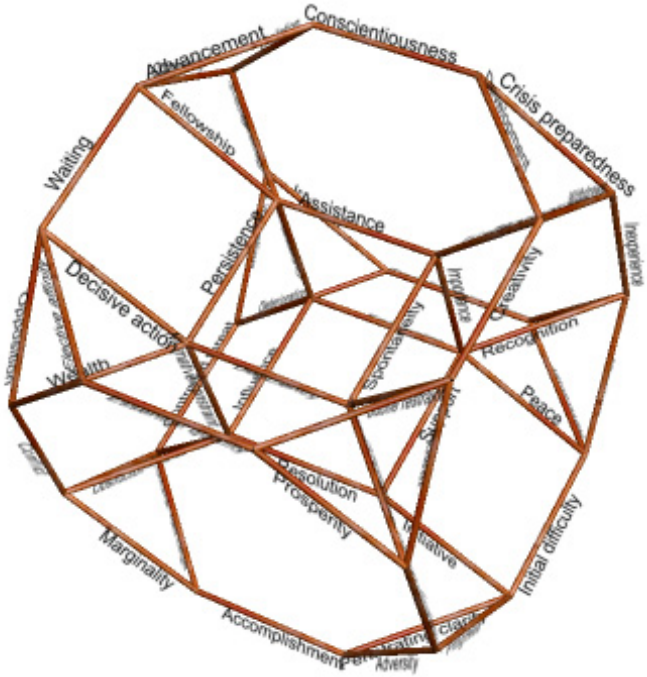


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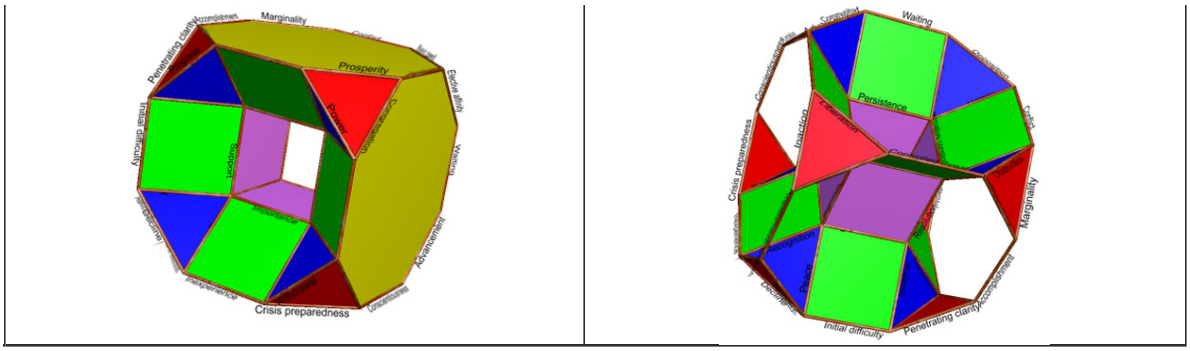
Fig. 17b: Drilled truncated cube -- a polyhedron approximating a torus, with 64 edges
(Variously rotated with selected faces coloured or not)



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