4 August 2015 | Draft

Eliciting Memorable Spheres and Polyhedra from Hyperspace

Integrative connectivity of problems, strategies, themes, groups or people

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Introduction

Much is made of the recognition that everything is now connected to everything, especially as reflected in patterns of links in cyberspace (Albert-Laszlo Barabasi, *Linked: how everything is connected to everything else and what it means for business, science, and everyday life*, 2014; David Easley and Jon Kleinberg, *Networks, Crowds, and Markets: reasoning about a highly connected world*, 2010).

This insight has notably been explored through small-world network graph theory and through understandings of "six degrees of separation", popularized by the parlour game *Six Degrees of Kevin Bacon* (Duncan J. Watts, *Six Degrees: the science of a connected age*, 2004). Various efforts at offering an idea of the complexity of this connectivity have been made based on internet traffic, web links, and the like.

Systematic efforts to document the connectivity of problems, strategies and international organizations as networks have been made in developing the online databases associated with the *Encyclopedia of World Problems and Human Potential* and the *Yearbook of International Organizations*. Extensive efforts have been made to analyze these networks and to find fruitful ways of visualizing them to render them more meaningful as a basis for new modes of action appropriate to the challenges of the times (*Visualizing Relationship Networks: international, interdisciplinary, inter-sectorial*, 1992; *Visualizing Latent Significance in Patterns of Relationships: a case study in relative incompetence*, 2012).

Considerable progress in this respect has been recently made in this respect by Tomá Fülöpp and Jacques de Mévius (*Loop mining in the Encyclopedia of world problems*, 2015), as reported at the conference on Futures Studies Tackling Wicked Problems (Turku, 2015). In the light of these efforts, the concern here is with the issue of how to render memorable and communicable the connectivity which emerges from such analysis. The concern is partially highlighted by possible responses to information overload (*Optimizing Web Surfing Pathways for the Overloaded: polyhedral insights from the travelling salesman problem of operations research*, 2015).

Web users have an increasing concern with how best to manage their web surfing experience given the constraints of time and information overload. The question explored here is a possible means of moving beyond a browser checklist of links ("favourites") and bookmarks, whether or not these are carefully nested within menus and organized by theme. This followed from earlier concern with the challenge of organizing relationships between websites in terms of the conversation threading of internet exchanges (*Interweaving Thematic Threads and Learning Pathways: noonautics, magic carpets and wizdomes*, 2010).

The complexity of some efforts at such network visualization has been deprecated through resulting in so-called "hairballs" (*Graphs Beyond the Hairball*; Lynn Cherny, *Visualizing Graphs: beyond the "hairball"*, 2012; Martin Krzywinski, *Hive Plots: Rational Network Visualization -- Farewell to hairballs*; Arlind Nocaj, et al, *Untangling Hairballs*, 2014; Stephen Few, *From Giant Hairballs to Clear Patterns in Networks*, *Visual Business Intelligence*, 2013; *Hive plots and hairballs, Seeing Complexity: visualizing complex data*, 2011; Hans-Jörg Schulz and Christophe Hurter, *Grooming the Hairball: how to tidy up network visualizations?* 2013; Jeff Johnston, *Embracing the Hairball*, *Exapative*, 2015). Special software applications, such as sigma.js, have been developed to address the current challenge of graph drawing (*Sigma.js Cleans up Hairball Network Visualizations*). Hairballs are inherently unmemorable.

The particular focus here is on using analytical data on triangles of relationships in such a way as to construct memorable networks of

these triangles based on any shared edges. This effectively weaves together those triangles detected into a "flat" 2-dimensional "carpet". The further possibility then derives from detecting "folding" of that carpet into the form of a 3-dimensional "bowl", as a consequence of connectivity at its outer edges forcing a degree of closure. As a result there is a "curling up" of the edges of the carpet with the possibility of detecting some such patterns which close completely to form a sphere. Approximations to such spherical closure, based on triangles, could take the form of polyhedra, whether simple or more complex -- based on loops of more than three relationships (squares, pentagons, hexagons, etc).

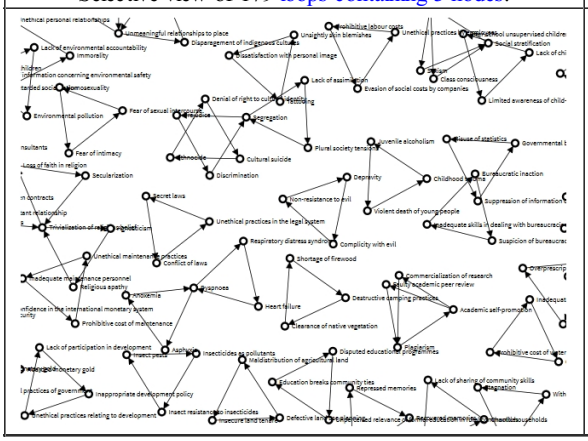
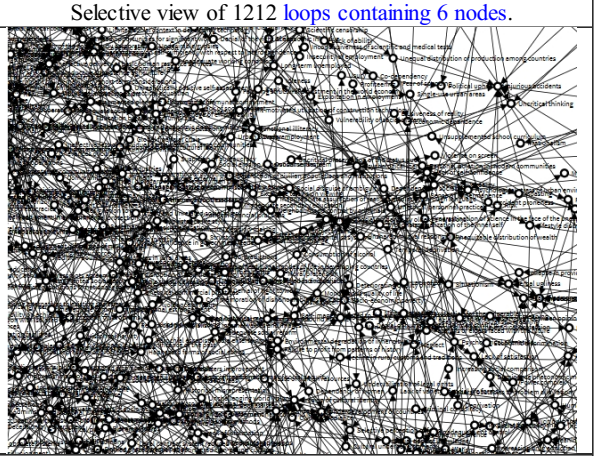
Through their characteristic symmetry, this approach offers a means of detecting and communicating visually memorable forms of order within very complex networks -- however they need to be labelled or colour coded. A particular feature of the approach is the use of force-directed graph drawing, charactetistic of Data-Driven Documents (d3.js), to elicit self-organizing convex polyhedra -- without the conventional prerequisite for vertex coordinates.

Weaving carpets of triangles into spheres and simpler polyhedra

Use is made of "hyperspace" in the title, rather than cyberspace, in order to stress the connectivity associated with hyperlinks within cyberspace. This follows from an early argument (*From Information Highways to Songlines of the Noosphere: global configuration of hypertext pathways as a prerequisite for meaningful collective transformation*, 1996). As such it offers an emphasis on conceptual entities and integrative cognitive engagement with them -- hence its appriateness for consideration of problems, stategies, and the like, as noted above (*Spherical Configuration of Interlocking Roundtables: Internet enhancement of global self-organization through patterns of dialogue*, 1998).

Loop detection: The approach here is based on the analysis by Tomá Fülöpp using the strongly connected components algorithm of graph theory developed by Robert Tarjan (*Depth-first search and linear graph algorithms*, *SIAM Journal on Computing*, 1972). This exercise was applied to networks of relationships between world problems perceived by international constituencies. It notably builds sets of loops linking three, four, five, and more, such entities -- extending to hundreds. Such entities, potentially labelled (as shown below) are associated with the nodes.

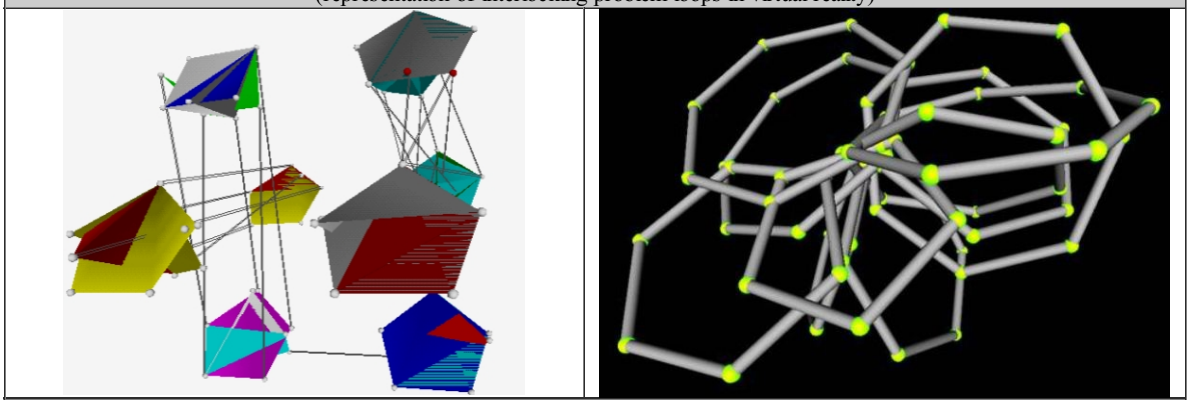
Screen shots of vicious problem loops (or cycles)
(screen shots from dynamic representations by Tomá Fülöpp, EWP Editing Platform)

Selective view of 179 loops containing 3 nodes.	Selective view of 1212 loops containing 6 nodes.
	

Clearly three-fold loops correspond to a triangular pattern, four-fold to a square pattern, and five-fold to a pentagonal pattern. These can of course be recognized as the face components of the simpler polyhedra. Earlier work extended to exploration of links between problems and strategies (*Feedback Loops Interlinking World Problems*, 2000).

Loop interlocking: Earlier experiments focused on simple interlocking between polygons, as illustrated below (reproduced from *World Problem Loop Interlocks*, 2000)..

Screen shots of earlier experiments
(representation of interlocking problem loops in virtual reality)



In the recent analysis by Tomá Fülöpp (*EWP Editing Platform*, 2015), more systemic detection of loop interlocking has been enabled.

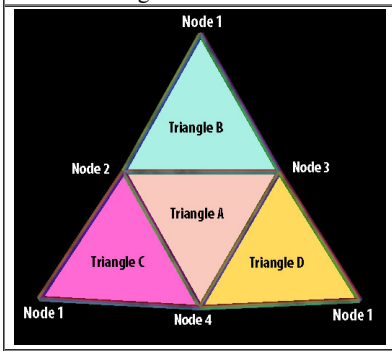
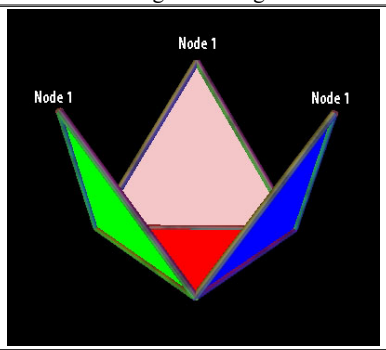
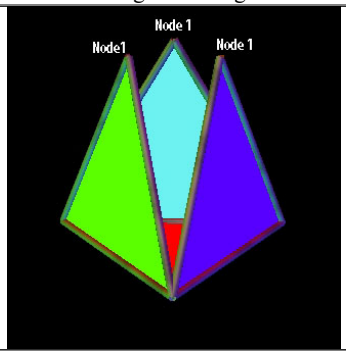
Polyhedral patterns: The question is how to employ these sets in a further step to build patterns of a higher order. In particular, is it possible to detect approximations to a sphere composed of a selected subset of the triangular patterns -- and how is that subset to be detected? This implies ordering the triangles, as in a jigsaw puzzle, joining two of them on the basis of a shared edge (side) -- meaning with distinct nodes at each end of the edge, common to the two triangles. Detection of subsets having such commonality is relatively trivial.

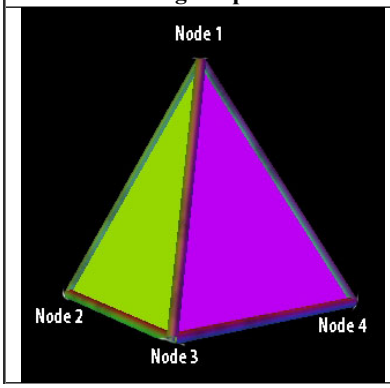
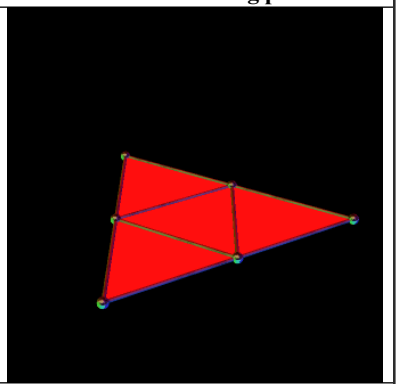
The challenge may be how to combine 2 triangles found in this way, with any larger pattern -- excluding those which do not "fit" at any one stage in the process. The procedure may involve building up in "pools" of memory sets of 2 triangles, 3 triangles, and more, as pieces which may be useful at some stage in the elaboration of the larger pattern. Clearly many such patterns could be "under construction" in parallel in the course of the process -- since different spheres might eventually emerge, at different stages after many iterations. Tomá Fülöpp already offers access to "interlocking loops".

As noted above, the process could detect simpler triangulated polyhedra where closure of the pattern is detected earlier in the selection -- prior to those triangulated polyhedra which could take more spherical form of geodesic spheres -- even more commonly recognized as geodesic domes (as noted below).

The process of fitting the triangles together is usefully illustrated by the following, prepared with Stella Polyhedron Navigator. Pairs of triangles sharing an edge can be identified (like Triangles A and B, A and C, and A and D). It is then a question detecting patterns of 4 triangles with one triangle in common (as in the case of Triangle A) in the image on the left below. Amongst that set, it is then a question of identifying those in which the "external" node is identical (as would be the case with Node 1 in the image on the left). Those meeting that condition then effectively form a tetrahedron of 4 triangles -- and can be considered as having been folded into 3 dimensions as variously illustrated below.

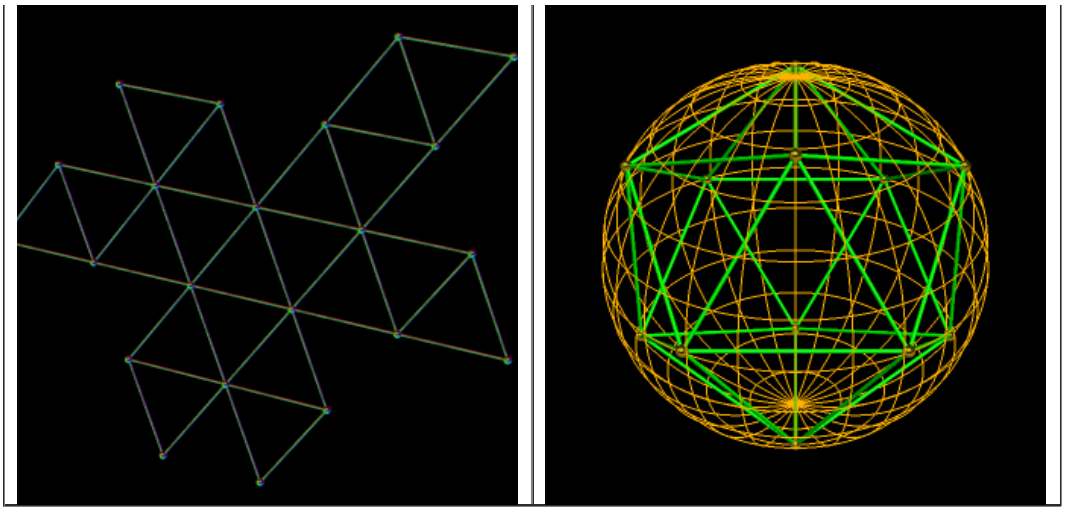
Screen shots of stages in folding of simplest polyhedron variously sharing edges and nodes

4 triangles in 2 dimensions	Stage in folding	Stage in folding
		

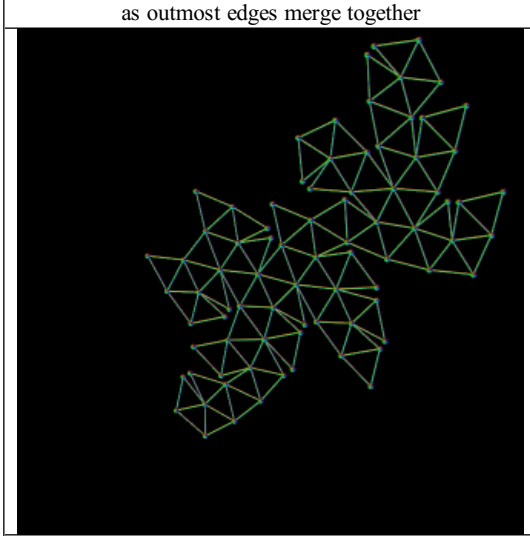
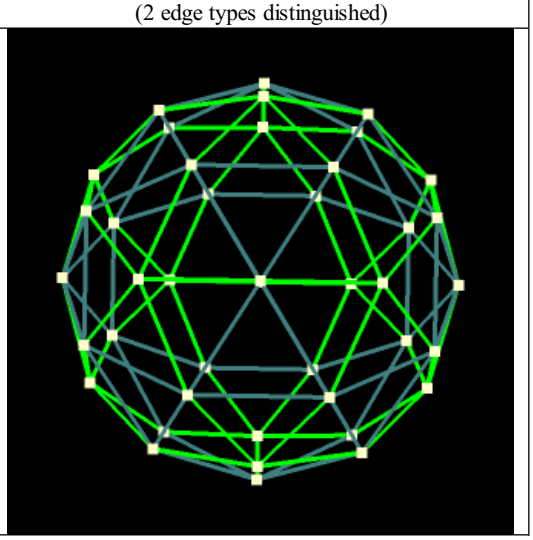
Folding completed	Animation of folding process
	

The possible relation to a sphere can be understood using a more complex polyhedron in the group of 5 Platonic polyhedra of which the tetrahedron above is the simplest. Again the argument is usefully illustrated by the following images (prepared with Stella Polyhedron Navigator). The degree of approximation to a sphere is evident in the animation on the right by addition of a circumsphere..

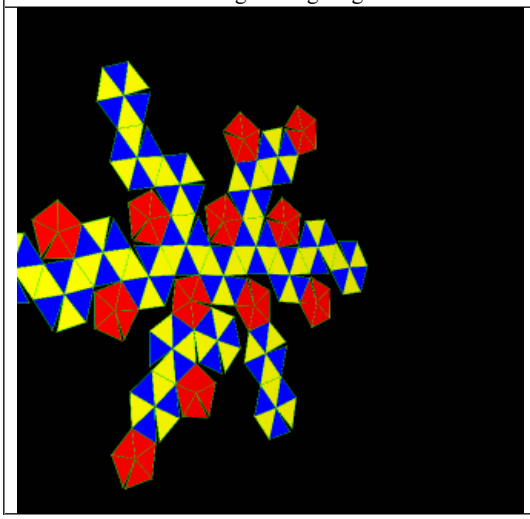
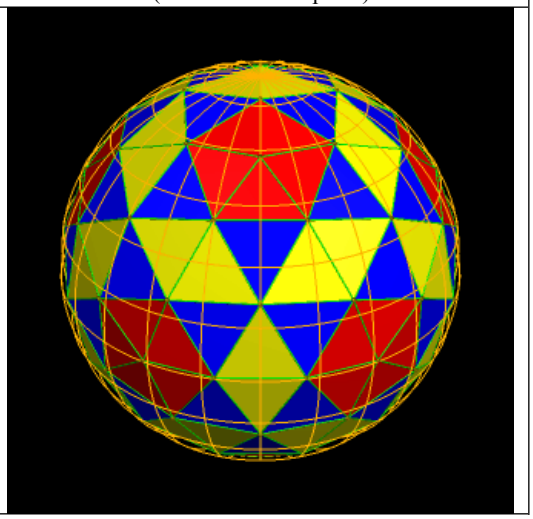
Illustration of process using animations of icosahedron 30 edges; 12 vertex nodes; 20 triangular faces (transparent)	
Carpet of triangles folding into approximation of a sphere as outmost edges merge together	Rotation of folded icosahedron (within a circumsphere)



Geodesic spheres: Using the icosahedron as a base, further insights into the process of gathering triangles into a carpet pattern to be folded can be obtained by considering geodesic spheres. The animation (on the right below) shows the nature of the great circles formed by the edges coloured blue-grey, in contrast to the interrupted great circles by the edges coloured green.

Illustration of process using animations of 2-frequency geodesic sphere based on icosahedron 120 edges (of 2 types); 42 vertex nodes (of 2 types); 80 triangular faces (of 2 types)	
Carpet of triangles folding into approximation of a sphere as outmost edges merge together	Rotation of geodesic sphere (2 edge types distinguished)
	

The illustration may be repeated with the icosahedron as a base. The degree of approximation to a sphere increases by increasing the frequency and therefore the number of interrelated triangles required to constitute that pattern -- as is evident in the image on the right below..

Illustration of process using animations of 3-frequency geodesic sphere based on icosahedron 270 edges (of 5 types); 92 vertex nodes (of 3 types); 180 triangular faces (of 3 distinctly coloured types)	
Carpet of triangles folding into approximation of sphere as outmost edges merge together	Rotation of folded geodesic sphere (within a circumsphere)
	

Eliciting memorable spherically symmetrical semiregular polyhedra

The focus above has been on triangles and the possibility of eliciting spherically symmetrical polyhedra based on triangulation (*Triangulation of Incommensurable Concepts for Global Configuration*, 2011. Also of interest is the possibility of detecting polyhedra that are somewhat irregular because their faces are not necessarily triangles. They may be squares, pentagons, hexagons, or the like. These notably characterize the 13 Archimedean polyhedra -- somewhat less memorable (or well-known) than the Platonic polyhedra. The relevance of spherically symmetrical polyhedral configuration to memorability has been argued separately (*Memorable pattern language for configuring sets of websites?* 2015)

Distinguishing methods for detecting polyhedra: In the light of the above description, various methods may be distinguished:

- ***Method 1***: As described above, namely build up a carpet and see whether the edges curve progressively into closure. This procedure may be used by drawing on the sets of triangle, squares, pentagons or hexagons identified by the analytical process
  - including only faces of one type (eg triangles or squares),
  - including only faces of two types (triangles and squares; or triangles and pentagons; etc),
  - including only three (or more) types

  Thus, if the analytical process detects non-triangular polygons, these can be fitted together, with or without triangles, to form such polyhedra. The question then becomes the procedure for detecting progressive 3D closure of the pattern of polygons through the addition of polygons to the 2D carpet of polygons and enabling connectivity as it proves possible. This could be done without consideration of any guiding "target" patterns -- as with those of the Archimedean polyhedra. A brute force procedure could be used to detect any combination of polygons which gave closure. Whether or not these are memorable through their symmetry characteristics is then a separate consideration. Symmetry constraints could be imposed to filter out those which did not meet such criteria of memorability.

- ***Method 2***: Use triangles to achieve closure, as with the geodesic approach, and then determine whether the squares, pentagons, etc can be elicited by suppressing selected links engendered by the triangulation. Thus convex polyhedra, composed of multiple triangles (as above), can be tested to determine whether some nodes are effectively "redundant" -- in that subsets of triangles can be considered as having outer links defining the squares, pentagons, etc in the pools of those collected by analysis

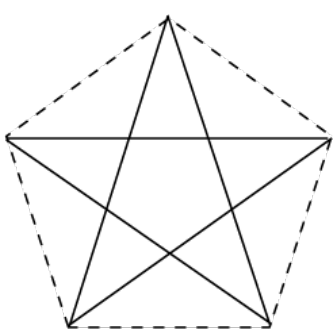
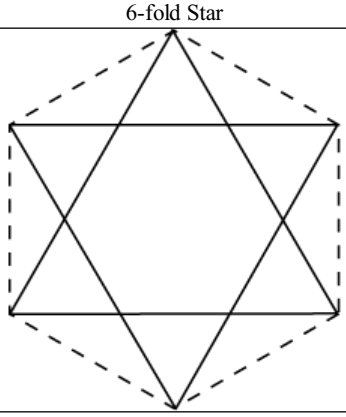
- ***Method 3***: Use patterns of connectivity derived from the range of known simpler and more complex polyhedra as templates within which to test whether the detected triangles, squares, or pentagons, etc fit -- namely whether the pool of loops enables those patterns to be built. Thus a library of known patterns of links is used to test and filter combinations of triangles, squares, and the like. This approach could be applied using the patterns of the 13 semi-regular Archimedean polyhedra, for example.

More complex procedures: The various procedures explored above take no account whatsoever of existing sophisticated mathematical algorithms by which patterns and cycles can be detected "through" numerous links, as discussed in the extensive literature in graph theory on Hamiltonian cycles. (Brent M. Dingle, *Finding Hamiltonian Cycles in the Inner n-Cube*, 1999; Carlo H. Séquin, *Patterns on the Genus: 3 Klein Quartic*; A. Ya. Belov-Kanel, et al *Interlocking of Convex Polyhedra : towards a geometric theory of fragmented solids*, *Moscow Mathematical Journal*, January 2009). Of related interest are insights from unfolding polyhedra (Brendan Lucier, *Unfolding and Reconstructing Polyhedra*, University of Waterloo, 2006). Through such procedures the detection of polyhedra could no doubt be rendered much more efficient.

Use of insights from spherical tiling are considered below.

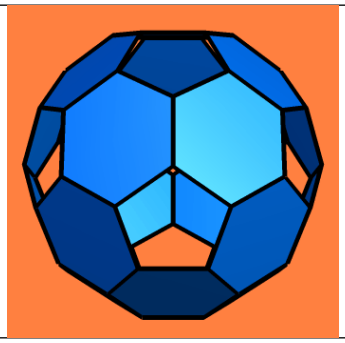
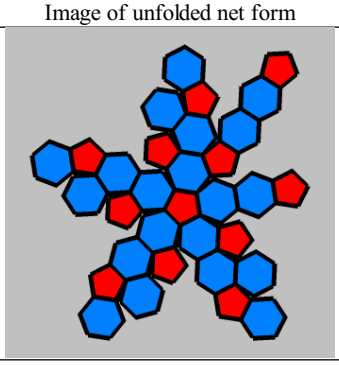
Illustrative application to symbolic reconciliation

Aspects of the methods above, and of possible applications, are usefully illustrated by a previous exercise (*Middle East Peace Potential through Dynamics in Spherical Geometry: engendering connectivity from incommensurable 5-fold and 6-fold conceptual frameworks*, 2012). Although discussed separately in detail, the approach can be briefly summarized by the following images. The argument there is focused on the symbolic significance associated with the 5-fold and 6-fold stars of Islam and Judaism in 2D -- and how these might be reconciled in 3D.

Symbolic stars embedded in corresponding polygons	
5-fold Star	6-fold Star
	

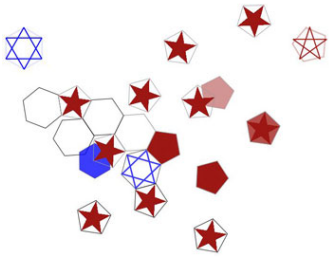
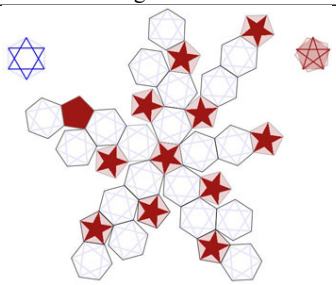
Truncated icosahedron "reconciling" pentagons and hexagons in 3D

(images produced using Stella Polyhedron Navigator)

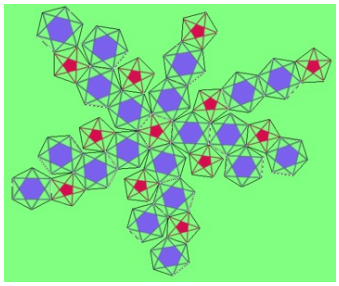
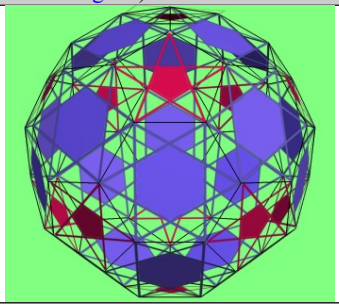
Animation of folded form	Image of unfolded net form
	

Screenshots indicating phases in SVG portion of animation sequence

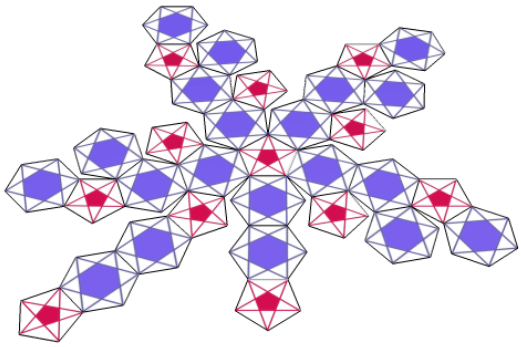
[click for separate SVG animation -- right-click on active animation for options -- improvements welcome
NB: Animation works in Firefox and Opera; effects do **NOT** work properly in Google Chrome, Internet Explorer or Safari]

Early stage in animation	Late stage in animation
	

Animation sequence subsequent to the SVG phase
screenshots showing extremes of the folding (and unfolding) process on a green background
(animation generated using Stella Polyhedron Navigator)

Animation sequence subsequent to the SVG phase
screenshots showing extremes of the folding (and unfolding) process on a green background
(animation generated using Stella Polyhedron Navigator)



The paper from which these animations were reproduced includes related commentaries on:

Fundamental cognitive patterns assumed to be characteristic of belief systems

Dialogue implications of design and geometry

Systems perspective on three-dimensional cognitive configuration

Animation of interaction and interlocking between cognitive patterns
Interactive design of cognitive pattern animation
Visualizing alternative stories through manipulation of animation design options
Polyhedral catalysts of global imagination
Design, pattern language and geometry
and communication
Triangulation, connectivity and "stitching": enabling coherent global system dynamics
Resonance, fullerenes and the Middle East?
Incommensurable cognitive patterns and their symbolism
Peace as a meta-pattern of resonance: psychosocial, implicit and emergent

Spherical tiling and spherical polyhedra

In discussion of the above illustration references was made to insights to be derived from the extensive literature on tiling, tesselation and the associated mathematics -- most notably on a sphere. In mathematics, a spherical tiling or spherical polyhedron is a tiling of the sphere in which the surface is divided or partitioned by great arcs into bounded regions called spherical polygons.

As noted and illustrated by *Wikipedia*, the most familiar spherical polyhedron is the soccer ball (outside the USA and Australia, a football), thought of as a spherical truncated icosahedron. The *Wikipedia* entry offers a systematic array of such polyhedra.

All the regular, semiregular polyhedra and their duals can be projected onto the sphere as tilings. Given by their Schläfli symbol {p, q} or vertex figure a.b.c.

Examples of spherical tilings reproduced from Wikipedia								
Schläfli symbol	{p,q}	t{p,q}	r{p,q}	t{q,p}	{q,p}	rr{p,q}	tr{p,q}	sr{p,q}
Vertex figure	pq	q.2p.2p	p.q.p.q	p.2q.2q	qp	q.4.p.4	4.2q.2p	3.3.q.3.p
Tetrahedral (3 3 2)	33	3.6.6	3.3.3.3	3.6.6	33	3.4.3.4	4.6.6	3.3.3.3.3
		V3.6.6	V3.3.3.3	V3.6.6		V3.4.4.4	V4.6.6	V3.3.3.3.3
Octahedral (4 3 2)	43	3.8.8	3.4.3.4	4.6.6	34	3.4.4.4	4.6.8	3.3.3.3.4
		V3.8.8	V3.4.3.4	V4.6.6		V3.4.4.4	V4.6.8	V3.3.3.3.4
Icosahedral (5 3 2)	53	3.10.10	3.5.3.5	5.6.6	35	3.4.5.4	4.6.10	3.3.3.3.5
		V3.10.10	V3.5.3.5	V5.6.6		V3.4.5.4	V4.6.10	V3.3.3.3.5
Dihedral example p=6 (2 2 6)	62	2.12.12	2.6.2.6	6.4.4	26	4.6.4	4.4.12	3.3.3.6

These images point to possible understandings of relationship links portrayed as curving arcs -- rather than as straight lines, as is the tendency in network representations.

Use of force directed layout to elicit memorable polyhedra

A particular feature of the approach is the use of force-directed graph drawing, charactetistic of Data-Driven Documents (d3.js), to elicit self-organizing convex polyhedra -- without the conventional prerequisite for vertex coordinates (Elijah Meeks, *D3.js in Action*, 2015). A preliminary illustration of the approach was made in an earlier document (*Optimizing Web Surfing Pathways for the Overloaded: polyhedral insights from the travelling salesman problem of operations research*, 2015). This demonstrated the feasibility of eliciting a dodecahedron in this way -- a classic example of a spherically symmetrical polyhedron.

As noted by *Wikipedia*, force-directed methods in graph drawing date back to the work of W. T. Tutte (*How to Draw a Graph*, 1963), who showed that polyhedral graphs may be drawn in the plane with all faces convex by fixing the vertices of the outer face of a planar

embedding of the graph into convex position, placing a spring-like attractive force on each edge, and letting the system settle into an equilibrium. Because of the simple nature of the forces in this case, the system cannot get stuck in local minima, but rather converges to a unique global optimum configuration. Because of this work, embeddings of planar graphs with convex faces are sometimes called Tutte embeddings.

The concern here is to extend the dodecahedron in 3D to explore the possibility with the 13 semi-regular Archimedean polyhedra -- given newly demonstrated capacity of Tomá Fülöpp to detect triangles, squares, and other polygons in a large set of data using the algorithm of Robert E. Tarjan (*Depth-first search and linear graph algorithms. SIAM Journal on Computing*, 1972).

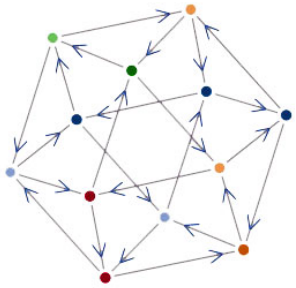
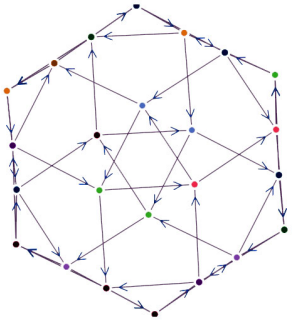
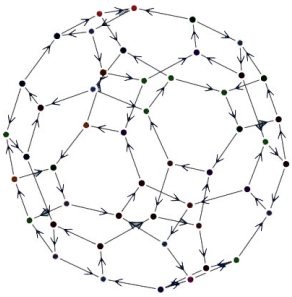
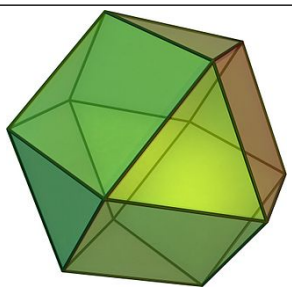
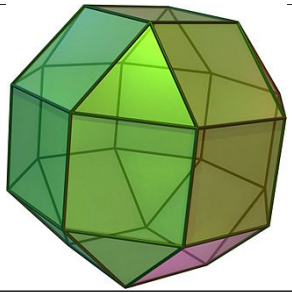
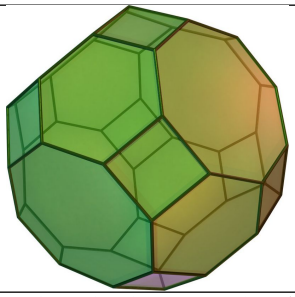
The challenge was how to use the resource pool of disparate polygons to build polyhedra in the absence of the fixed coordinates normally required in define a polyhedron. The point of departure was the inherent connectivity of the polygons which implied the possibility of building larger patterns. The issue was how to use connectivity alone to enable the construction process. One problematic feature of such exploration is that polyhedral coordinates tend to be associated with applications which render the precise set of coordinates subject to copyright. Ironically this may also apply to connectvity data, but to a lesser degree.

The animations above illustrate the principles of the process. The further requirement is then obtaining the patterns of connectivity associated with a wide variety of polyhedra. This data is not readily available. It is typically implicit in formats required to construct such polyhedra for design purposes -- as the manner in which points defined by coordnates are to be connected.

An extremely rich source of polyhedra of every kind is the Stella Polyhedron Navigator. One of the geometry definition export file formats it offers is .OFF. The approach taken here was therefore to export each of the Archimedean polyhedron from Stella. The connectivity data was extracted from each and reformatted as an adjacency list -- namely a document constructed as an edge list of nodes (with names) and links in JSON format. An arbitrary collection of document labels was used in naming the nodes.

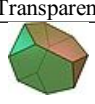
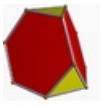
The JSON documents could then be visualized using the force-directed layout facilities of d3.js, with results indicated below -- after the dynamics had converged the connectivity into stable configurations. Preliminary issues with the dynamic displays resulted from a degree of incompetence with d3.js parameters (sizing the on-screen result, adjustment of the charge parameter, etc). The directional arrows and node colouring are again an arbitrary choice for purposes of illustration. Using the cursor in the dynamic displays does indeed bring up the (arbitrary) label of the node, but a hyperlink has not been activated on it. There is some question as to whether all browsers can handle that d3.js facility.

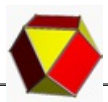
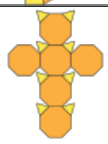
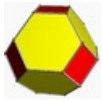
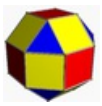
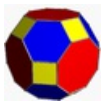
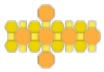
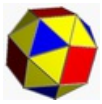
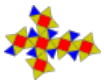
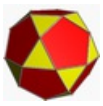
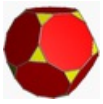
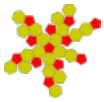
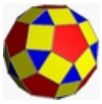
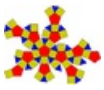
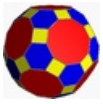
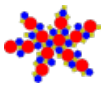
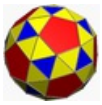
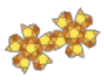
Screen shots of selected results of force-directed layout to elicit Archimedean polyhedra
(not to scale relative to one another; subject to further tests)

Cuboctahedron (force-layout animation)	Small rhombicuboctahedron (force-layout animation)	Truncated cuboctahedron (force-layout animation)
		
		
Rotating animation from *Wikipedia*	Rotating animation from *Wikipedia*	Rotating animation from *Wikipedia*

Access to separate interactive force-directed animations of Archimedean polyhedra (visualization may be dependnt on browser and resizing)

Archimedean polyhedra
(table adapted from original version in *Wikipedia* with added links to force-directed animations)

Polyhedron	Transparent	Solid	Net	Faces		Edges	Vertices
truncated tetrahedron [force-directed version]	 (Animation)			8	4 triangles 4 hexagons	18	12
cuboctahedron					8 triangles		

(rhombitetratetrahedron) [force-directed version]				14	6 squares	24	12
truncated cube [force-directed version]	 (Animation)			14	8 triangles 6 octagons	36	24
truncated octahedron (truncated tetratetrahedron) [force-directed version]	 (Animation)			14	6 squares 8 hexagons	36	24
rhombicuboctahedron (small rhombicuboctahedron) [force-directed version]	 (Animation)			26	8 triangles 18 squares	48	24
truncated cuboctahedron (great rhombicuboctahedron) [force-directed version]	 (Animation)			26	12 squares 8 hexagons 6 octagons	72	48
snub cube (snub cuboctahedron) [force-directed version]	 (Animation)			38	32 triangles 6 squares	60	24
icosidodecahedron [force-directed version]	 (Animation)			32	20 triangles 12 pentagons	60	30
truncated dodecahedron [force-directed version]	 (Animation)			32	20 triangles 12 decagons	90	60
truncated icosahedron [force-directed version]	 (Animation)			32	12 pentagons 20 hexagons	90	60
rhombicosidodecahedron (small rhombicosidodecahedron) [force-directed version]	 (Animation)			62	20 triangles 30 squares 12 pentagons	120	60
truncated icosidodecahedron (great rhombicosidodecahedron) [force-directed version]	 (Animation)			62	30 squares 20 hexagons 12 decagons	180	120
snub dodecahedron (snub icosidodecahedron) [force-directed version]	 (Animation)			92	80 triangles 12 pentagons	150	60

Of some concern was how d3.js would handle connectivity of unequal length -- as is the case with the 2 or more edge types (lengths) in the semi-regular Archimedean polyhedra, Further thought could be given to techniques for weighting the sizes in the converged result.

An interesting further possibility is to use the capacity of d3.js to log the coordinates of nodes of the converged result, as variously discussed (Stackoverflow, Airpair). The question would then be how these frozen coordinates might be exported and manipulated, possibly into a ,OFF format. This could then be imported into a mesh or computer-aided design application (such as MeshLab, which uses a JSON format). Within that environment "irregularities" resulting from the force-layout process might then be "normalized" -- with additional graphics effects to enhance memorability

Potential significance of memorable irregularity?

It is appropriate to respond to a potential criticism as to why not avoid the convolutions of the force-directed approach by making direct use of known polyhedral coordinates. Whilst this might be a viable approach under some circumstances, the advantage of eliciting polygonal loops, as currently extracted, is that it does not preclude the identification of polyhedral forms which could be cumbersome to derive from data subject to intellectual copyright and proprietary formats.

There is every possibility that polyhedral forms based on much larger numbers of polygonal loops (and cycles) could be detected. The potential systemic significance of such structures merits distinct reflection where these are composed of:

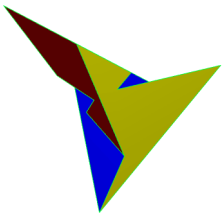
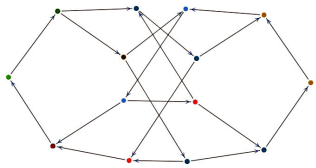
- problems, as in the current analysis based on perception of relationships between world problems -- suggesting relevance to current preoccupation with the wicked problems of policy making
- strategies, as available from a related data set on global strategies
- organizations, as available from related data on relations between international organizations
- concepts, as is typical of thematically interrelated international conferences, or even those cultivated as understandings of human development
- human values, as potentially available from a related data set
- individuals, as currently mined from social media data sets

Rather than the "carpet" metaphor used above, a more relevant metaphor may be associated with current recognition of the need for larger "tents" within which to gather together disparate entities. The metaphor is notably employed with respect to interfaith discourse which is currently proving to be so inadequate to the faith-based conflicts of the world. This argument is developed separately (*Global Brane Comprehension Enabling a Higher Dimensional Big Tent?* 2011). A charming irony noted there is the inspiration that tent-making offered to the polymath Omar Khayyám -- it being the traditional occupation of his family. Hence the relevance of "stitching" to the approach outlined above (*Triangulation, connectivity and "stitching": enabling coherent global system dynamics*, 2012).

Significantly missing from the above discussion is the sense of chirality by which polyhedral forms may be distinguished, namely as directed graphs there may be right- and left-handed variants of some of the forms above. This is increasingly important in molecular chemistry where the two forms may have quite distinct properties. As stressed, the arrows included into the force-directed layouts above are there purely to indicate the need for such clarification.

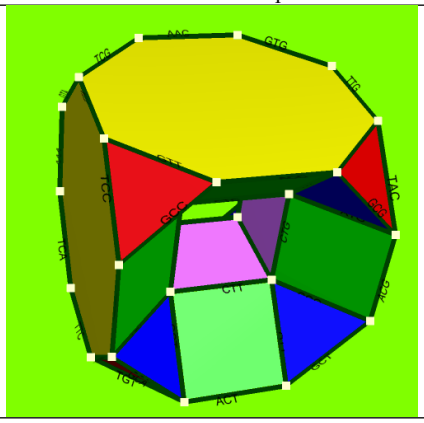
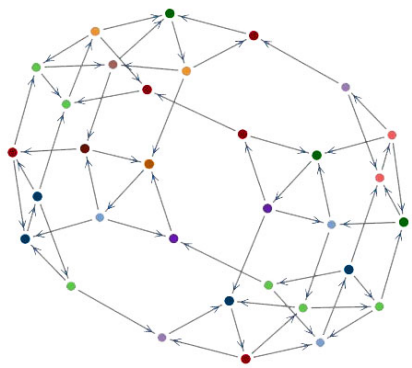
Of particular interest is the relation between memorability and degrees of symmetry, as well as the memorability of some unusual structures with unusual forms of symmetry, such as the Szilassi polyhedron, as discussed separately (*Mapping of WH-questions with question-pairs onto the Szilassi polyhedron*, 2014).

Animations of Szilassi polyhedron
of 7 faces (4 types), 21 edges (12 types), 14 vertices (7 types) [totalling 42=2x3x7; product=$2x3^2 7^3$]
(paired shapes coloured identically; prepared using Stella Polyhedron Navigator)

Folding together of the two complementary nets	Rotation of polyhedron	Screen shot of interactve force-directed version
		

Given the cyclic loops configured by this approach, of particular interest is the potential systemic significance of the polyhedral configurations which might emerge from the process outlined above. In this respect it is appropriate to recall the very extensive range of polyhedra of different degrees of symmetry, somce being convex and others concave. In the case of the drilled truncated cube (presented below), its use as a mapping framework is discussed separately (*Changing Patterns using Transformation Pathways*, 2015).

Drilled truncated cube
of 32 faces (5 types), 64 edges (9 types), 32 vertices (4 types) [totalling 128=2^7]
(animation prepared using Stella Polyhedron Navigator)

Animation with faces non-transparent	Screen shot of interactive force-directed version
	

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