



laetus in praesens

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25 July 2022 | Draft

Sustainability through Global Patterns of 60-fold Organization

Psycho-social implications of fullerenes for coherence, integrity and identity of a higher order

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Introduction

Recent decades have been witness to a global focus on sustainable development, culminating in the 17 [Sustainable Development Goals](#) of the United Nations -- whose achievement globally is increasingly questionable. More recently the world of chemistry has been surprised by the discovery of an unsuspected mode of organization of carbon atoms so fundamental to life -- now the focus of intense research. The discoverers were awarded a Nobel Prize in 1996. all before any good use for them had been imagined. Although seemingly rare in nature, they have now been detected in outer space. The molecules they form are now termed [fullerenes](#) in memory of [Buckminster Fuller](#) since their spherical structure bears a strong resemblance to the [geodesic domes](#) for which his architecture is renowned -- and hence the informal reference to "buckyballs" as an alternative.

The common configurations of carbon atoms are a familiar feature of organic chemistry and its many products on which society is now so dependent. The organization of those configurations has long been recognized as similar to the social organization of relationships, of groups, of teams, and of topics. Two-dimensional models and representations of such patterns are familiar in the explanation of psycho-social structure, mind maps and concept maps.

It would seem however that the spherical form of closed fullerenes has attracted no attention from a psycho-social perspective. The most common fullerene of 60 carbon atoms is termed C_{60} . C_{60} , or [Buckminsterfullerene](#). This has a structure corresponding to that of the standard ball of [association football \("soccer"\)](#) known the world over, namely that of the [truncated icosahedron](#). A degree of familiarity has however been achieved through magnetic toys named as [bucky balls](#), which have evoked controversy because of health warnings -- in contrast to promotion of the health benefits of C_{60} oil (*Biofullerene: Nature's nearly perfect solution to the challenges of aging*).

The question in what follows is whether the pattern of spherical organization of C_{60} has psycho-social implications suggestive of higher orders of coherence. (*Understanding Sustainable Dialogue: the Secret within Bucky's Ball?* 1996). Does the "stability" associated with some fullerenes correspond in any way to the "sustainability" so desperately sought? In a period of social fragmentation and strategic incoherence, the question would necessarily be of relevance to fullerenes of greater complexity. Such a focus bears some comparison with the inspiration traditionally offered by the light-focusing diamond as the epitome of crystal organization (*Implications of diamond faceting for enlightening dialogue*, 2002; *Insights from diamond as symbolic of the highest value*, 2020).

The focus on higher order carbon configurations of fullerenes is potentially especially significant because of the role of carbon in

molecules essential to life. The "behaviour" of carbon -- and the patterns by which it may be configured -- suggest that it may therefore constrain and enable the organization of learning and comprehension. Are there potential implications of such patterns for higher order organization with respect to the potential of neural learning, whether within the human brain or in its relevance to artificial intelligence?

In particular the concern here is whether there has long been an intuitive recognition of the coherence and identity offered by a group structure of plus-or-minus 60 people. This would tend to be evident in military organizations, possibly as a "company. It would also tend to be evident in educational institutions as a "class" -- as celebrated over decades by alumni -- or in a "community". It might be evident as an academic "faculty" or a "school of thought" -- or as the "department" of a corporation or other institution. In each case this is distinct from the coherence more readily recognized in groups and teams of plus-or-minus 7.

Somewhat ironically it is indeed military organization which helps to clarify any such exploration. Much is indeed made of the bonding and team spirit within a "squad" of plus-or-minus 7, for example. However a different degree of bonding and identity is evident at the next level of organization, possibly termed a "company". Loyalty and identity take yet another form at the level of a "regiment", a "battalion", or an "army".

With identity and mutual trust increasingly understood as vital to motivated collective action and its efficacy, the argument here is that such coherence might be fruitfully explored through the closed fullerenes. Degrees of coherence could be usefully represented by them in 3-dimensions -- in contrast to loss of meaning associated with their representation in 2-dimensions. Any approximation to a 60-fold "global" configuration of identity might then "hold" multiple patterns of configurations of smaller groups (10x6, 12x5, 4x15, etc.).

Just as there is a "felt sense" of a viable size to smaller groups, is this also the case -- more subtly -- with respect to groups of increasingly larger size? The argument could also apply to groups of concepts in the organization of knowledge with its degrees of generality. Whilst there is some sense of the requisite variety of skills vital to a small group, far less evident is any analogous insight into the variety vital to the coherence and sustainability of much larger groups.

The question is of relevance to the organization of multinational "battlegroups" as in the case of Europe (*Battlegroup of the European Union*). It is presumably of potential relevance to the organization of intergovernmental agencies struggling with the many virtual wars of the global problematique (*Review of the Range of Virtual Wars*, 2005).

There is a degree of irony to the apparent lack of attention to a C₆₀ pattern of psycho-social organization, or to any fullerene of greater complexity. For example, the coherence of the secretive global configuration of military bases of the United States, designed to ensure [global hegemony](#) through [full-spectrum dominance](#), could be usefully explored as taking the form of a C₇₂₀ fullerene (*List of US military bases*).

Examples of the intuited coherence of collective clusters

The following examples are presented as a means of framing the question as to the nature of the coherence and sense of identity enabled by clusters of a particular size. Curiously the preoccupation of chemistry with the molecular bonds (especially in fullerenes) is echoed in the widespread reference to bonding in teams and within communities. Less evident is the degree of "bonding" between topics in the disciplines and between them -- exemplified by the problematic bonds between the arts and the sciences.

Military organization: There is a case for exploring how identity and coherence are evoked within any [military organization](#), especially given their relevance to military action under life-threatening circumstances.

The *Wikipedia* entry offers an indication of that organization in [hierarchical terms](#) as felt to be appropriate in the army, air force and navy of different countries. Distinctions are made, using NATO as an example, between:

unified combatant command (1-10 mill.)	brigade (4,000-8,000)	platoon / troop (26-55)
army group (400,000-1 mill.)	regiment (1,000-3,000)	section / patrol (12-24)
field army (100,000-300,00)	battalion (300-1,000)	squad (6-12)
corps (40,000-80,000)	company (80-250)	team / crew (2-4)
division (10,000-30,000)		

Whilst the focus in clusters of larger size is on the technicalities of [command and control](#), a long-recognized concern is with instilling and evoking motivation and loyalty in those of smaller size -- as classically engendered by a "general". Far less evident is how these are transformed or "diluted" with increase in size.

Business organization: It is to be expected that corresponding clusters would be evident in any for-profit [company](#), given the implications of any shared terminology: group, division, company, section, and team. The most general distinction is variously understood to be between "small" (less than 1,500, even less than 100), "medium" (1,500-2,000, or 100-500), and "large" businesses (*Business Sizes: Classifications and Characteristics, Indeed*, 11 August 2021). The management sciences are especially focused on eliciting company loyalty, for example, whether or not this extends to any larger cluster of companies within a group. *Business Sizes: Classifications and Characteristics* By Indeed Editorial Team Published August 11, 2021

Government and administrative organization: Again it is to be expected that the preoccupations and responsibilities of government would be articulated through a complex structure of ministries/departments, agencies, and sections. Especially relevant to this argument is the form this takes in intergovernmental organizations, exemplified by the United Nations and its Specialized Agencies. These can be understood as arrayed at various levels against a range of virtual wars, as noted above.

Religious organization: A primary example in this case is the Catholic Church, with a pattern of governance extending from the Vatican through dioceses to the level of the parish. Religious organizations exemplify the challenge of evoking and sustaining belief and coherence framing and enabling a sense of identity for the individual. The Catholic Church can be understood as having borrowed a degree of

military organization through its understanding of the [Church Militant](#) (*Ecclesia Militans*) consisting of Christians on earth who struggle as soldiers of Christ against sin, the devil, and "the rulers of the world of this darkness, against the spirits of wickedness in the high places". The military inspiration is shared from a Protestant perspective with [The Salvation Army](#).

Further insights are potentially offered by theological articulations of the organization of heavenly realms and their "heavenly hosts" -- arrayed in the eternal battle between good and evil. Of particular interest is the manner in which saintly figures are engendered and valued as a focus of individual belief -- and the inspiration of dedicated religious institutions.

Community organization (NGOs, social media groups): Many forms of "community" are recognized, from the local to the global level -- complicated by the cross-cutting role of social media enabled by the internet. Efforts are made to interrelate these, most notably in response to issues of global concern. There is however recognition of the manner in which identity and focus is evoked and achieved within smaller groupings at the local level. Coherence at the global level tends to remain elusive or symbolic whatever the intermediary structures.

Examples of the intuited coherence of topic organization

An instructive example is offered by the [biological classification](#), in terms of [taxonomic rank](#). This is the relative level of a group of organisms (a [taxon](#)) in a [taxonomic hierarchy](#). Examples of taxonomic ranks are [species](#), [genus](#), [family](#), [order](#), [class](#), [phylum](#), [kingdom](#), [domain](#), etc. There is an indeterminate number of ranks, as a taxonomist may invent a new rank at will, at any time, if they feel this is necessary. In doing so, there are some restrictions, which will vary with the nomenclature code which applies. *Wikipedia* presents a far more [extensive ranking](#) as an artificial synthesis, solely for purposes of demonstration of relative rank (but see notes), from most general to most specific.

Of particular interest is the manner in which concepts may be clustered within the approach of a discipline to its field of preoccupation. This has been separately explored in a comparison of integrated multi-set concept schemes as forms of presentation (*Patterns of N-foldness*, 1980). This helps to frame the question of degrees of generality or abstraction of relevance to any approach to interdisciplinarity or transdisciplinarity. This featured in the early preoccupations of the Society for General Systems Research.

Of some relevance to this exploration is the manner in which topics are dismissed as irrelevant, as in deprecation by science of the pseudo-sciences -- despite the risk of inviting a perception of pseudo-relevance.

Intuited coherence in organization of time

The [history of calendar development](#), and the arguments for [calendar reform](#), make apparent the degree to which the distinctions made are apparently arbitrary, however coherent they may be felt to be by those adopting them (*List of Calendars*, *Wikipedia*). Proposals have been made for [10-month calendars](#), [12-month calendars](#), and [13-month calendars](#) -- as well as for [Perennial calendars](#) and [Lunisolar calendars](#).

The arbitrary nature is especially emphasized by the manner in which proposals have originated in quite distinct cultures and belief systems, as with the Mayan and Aztec calendars, and the Sumerian calendar. One recent example of the process is that articulated by [Jose Arguelles](#), in part framed by an interpretation of how Maya calendrical mathematics functioned (*The Discovery of the Law of Time*, 1989-1996; *13 Moon Calendar Basics, Foundation for the Law of Time*). This took the form of a *13-Moon Natural Time Calendar*, as explained by Carl Johan Calleman (*What is the Thirteen Moon/Dreamspell Calendar?* 11 December 2014).

Of particular relevance is the study of [chronemics](#), namely the role of time in communication. As described by the *Encyclopedia of Special Education*, this includes time orientation, understanding and organization; use of and reaction to time pressures; innate and learned awareness of time; wearing or not wearing a watch; arriving, starting, and ending late or on time. This approach clarifies the contrasting implications of [communication between cultures using different time systems](#) and their [influence on global affairs](#). This is most evident with respect to [monochronic understanding of time](#) versus a [Polychronic understanding of time](#).

Contrasting preferences in the appreciation of time are now variously evident in:

- speed, as highlighted by world records, most notably in sports, but also in the operation of computers and supercomputers
- slowness, as highlighted by various [slow movements](#), as detailed by *Wikipedia*, in contrast to the value attributed to being on a "fast track"
- increasing preference for tweet-length communication over book-length documents, with recognition of constraints on "available time" as is now evident in the indication of "read time" for documents in text form. This linear word-by-word measure is based on average reading speed and a word count with little consideration given to techniques of rapid scanning of documents (*lecture en diagonal* in French), shifting between sub-headings, highlighted phrases and imagery -- namely selective reading. (*Read Time and You: here's how read time is calculated*, *Medium*, 4 June 2014)
- intercourse, whether in terms of frequency or duration
- acclaimed quest for "quality time"
- holding attention
- time to make a "good impression"

Whereas the modes of organization of months within a year have been the focus of calendars and their development (as noted above), the history of the organization of days into [weeks](#), [hours](#), [minutes](#) and [seconds](#) highlights an even more arbitrary range of possibilities across cultures. Of relevance to this argument is the [sexagesimal](#) organization of time. This is a numerical system with sixty as its base.

This system originated with the ancient Sumerians in the 3rd millennium BC, and is still used (with questionable justification) for measuring time, angles, and geographic coordinates. Efforts to rationalize this 60-fold organization, as with the [general decimalisation of](#)

measures (including [decimal time](#)) enacted during the [French Revolution](#), have achieved only limited acceptance. Although the organization of a calendar has been inspired by the experience of solar and lunar cycles, the distinction of weeks, hours, minutes and seconds does not benefit from the experience of natural cycles. Curiously the tendency to decimalisation has not transformed the organization of latitude and longitude as framed by 360 degrees, especially as a consequence of the role of time in the measurement of the latter.

Ironically it is the [heart beat](#) at a rate 70-75 per "minute" which offers the closest approximation to the understanding of a "second" -- with higher and lower [heart beats in animals](#). The same can be said of the [respiratory rate](#) of human adults of 12-20 breaths per minute, with that of children varying with age -- with a distinction made from the the [respiration rate](#) used in ecological and agronomical modeling. With "breaths" understood as inspiration, this suggests an alternative measure which would double that -- again offering an experience more closely related to the second. It could be asked whether any subjective sense of time is related more closely to variation in such rates rather than to the artificial definition of a second -- and might well apply equally to animals.

Educational organization in a learning society

[Learning society](#) is an educational philosophy advocated by the OECD and UNESCO (Centre for Educational Research and Innovation, *Knowledge Management in the Learning Society*, 2000; Kenneth Wain, *Lifelong Learning and the Politics of the Learning Society*, 2007). This positions education as the key to a nation's economic development, and holds that education should extend beyond formal learning (based in traditional educational institutions -- schools, universities etc.) into informal learning centres to support a knowledge economy. A learning society regards the actual process of learning as an activity, not a place -- that is, it takes place outside of regular educational institutions, and is thus also decentralised and deregulated, a tenet of globalization theory.

The focus on education is of particular relevance because of the organizing role of the [class](#), whether in school, university or other initiatives. As noted above, the sense of identity evoked may be celebrated by alumni over decades. Extensive research is undertaken regarding the potential merits of reducing [class size](#) as a means of optimizing learning. Typical class sizes are in the range 20-30 -- where resources permit -- guided by [Maimonides' rule](#) constraining numbers to below 40.

While class size may be the most researched area in education, the research is not conclusive. As noted by the Center for Applied Research and Educational Improvement:

Class size is a highly political and debated topic. Many campaigns, policies, and lobbying efforts have been built around the class size debate. It is easy to see that very high class sizes are difficult for any teacher to handle, and very small class sizes are fiscally unattainable with current educational funding levels, but where to land in the middle is a hotly debated topic. ([Class Size](#))

In an academic environment, a "class" may be of much larger size, posing increasing challenges (Center for Teaching and Learning, *Teaching LARGE Classes*, University of Texas at Austin; Oriana Bandiera, et al, *The impact of class size on the performance of university students*, 2010). As variously clarified:

Class size varies. Large lecture halls often seat 300 students. A student, however, would not have all classes in a 300-person lecture hall. Lecture classes typically allow for personalization through labs and small group sessions... which tend to be about 20 students in size. Only 17% of our classes have over 100 students in them. Also, the further a student moves into his/her major, the smaller the classes tend to be. (Office of the Dean of Students, *What is the average class size?* Oregon State University)

Some introductory courses as well as several other popular courses have large enrollments. Yet, the median class size at Harvard is 12. Of the nearly 1,300 courses offered last fall, for example, more than 1,000 of them enrolled 20 or fewer students. ([How large are classes?](#) Harvard College)

To the extent that conferences (especially international conferences) are characterized by audiences in plenary sessions numbering hundreds (if not thousands), it may then be asked how learning is engendered and sustained in such contexts. Furthermore, to the extent that decision-making assemblies (such as national parliaments) may have hundreds of representatives (as with those of UNESCO and OECD), it is appropriate to ask how effectively these function as "learning environments".

Collective comprehension of patterns challenging civilization

The examples above indicate a clear preference for simpler patterns with which coherence is assumed -- and "felt" -- to be associated, with little tendency to justify the preference. Specific indications are offered by the following:

- [Requisite 20-fold Articulation of Operative Insights? Checklist of web resources on 20 strategies, rules, methods and insights](#) (2018)
- [Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources](#) (2011).
- [Eliciting a 12-fold Pattern of Generic Operational Insights: recognition of memory constraints on collective strategic comprehension](#) (2011)
- [Pattern of 14-foldness as an Implicit Organizing Principle for Governance? Web resources](#) (2021)
- [Table of strategic structural attributions by number of elements](#) (2019)
- [Re-cognition of N-fold sets of "modes", "ways", "moves" and "ploys"](#) (2021)

Approaches to understanding such preferences can be related to memorability, as separately discussed ([Memorability, Mnemonics, Maths, Music and Governance: memory enhancement ensuring strategic credibility](#), 2022). These suggest the possibility of unexplored

constraints on the comprehension of strategic challenges (*Comprehension of Numbers Challenging Global Civilization*, 2014) discussed in terms of the following:

Numbers in play in psychosocial organization	Creative pretence dissociating numbers from sexuality
Conceptual clustering and cognitive constraints	Significance of "encompassing" the numbers required for meaningful governance
Pattern memorability between symbolic mystification and "stretching"	Boundary pushing by sport, religion and governance
Imaginative depiction of the cognitive challenge	Reframing boundaries to engage with patterns of collapse
Requisite complexification of imagery to embody greater significance	

With respect to constraints, this noted that other arguments emerge from assessments of problematic-strategic complexity, as with the [Situational Complexity Index \(SCI\)](#) of the [Institute for 21st Century Agoras](#). Key constraints include:

- "Miller number" (7 ± 2): the much cited constraint reported by [George Miller](#) (*The Magical Number Seven, Plus or Minus Two: some limits on our capacity for processing information*, *Psychological Review*. 1956). In the light of new research, further indications are offered by [Bradley Ford](#) (*The "Magic Number" in Human Memory Capacity*, *Advanced Solutions Blog*, 11 July 2017).
- "Spreadthink number": as identified by [John N. Warfield](#) (*Spreadthink: Explaining ineffective groups*, 1995). This reflects the fact the inability of groups to reach agreement on complex issue. In the SCI, this is assumed to be 5.
- "Dunbar's number": as formulated by [Robin Dunbar](#), This is a suggested cognitive limit to the number of people with whom one can maintain stable social relationships The commonly used value is 150.
- **Span of control**: The number of subordinates a leader can efficiently control or manage, currently understood as ranging up to 10.

The pattern of constraints is also evident in preferences for team size in competitive sports. ([Katherine Klein](#), *Is Your Team Too Big? Too Small? What's the Right Number?* *Knowledge at Wharton*, 14 June 2006). As succinctly stated by [Susan Heathfield](#) (*What Team Size Is Optimum for Performance?* *About.com*):

The team size that is optimum for team performance is a topic much researched and debated. The problem is that you need to consider a number of factors when determining optimum team size.... If you seek effective input, the optimal team size ranges from more than 2 up to 18-20 members, but these individuals are not expected to form a cohesive, highly interconnected team. It is much more likely that teams of a large size form sub-teams and working groups to accomplish the actual work of a project.

The argument follows from a previous discussion of memorability (*Memorability, Mnemonics, Maths, Music and Governance: memory enhancement ensuring strategic credibility*, 2022).

Implications of fullerenes for organization of any global brain

The exploration of the geometry of fullerenes in terms of its psycho-social implications follows from previous discussions of the possibility of reconciling the "headless hearts" with the "heartless heads" (*Time for Provocative Mnemonic Aids to Systemic Connectivity?* 2018). Is the quest for coherence at this time indeed a mistaken quest for closure (*Engaging with Elusive Connectivity and Coherence*, 2018).

Neuronal connectivity? The question to be emphasized is whether the organizational constraints above, especially in their relation to memory, can be more fruitfully discussed in the light of fullerene structures. The possibility may be all the more credible in the light of research on the manner in which the human brain organizes information, as discussed separately (*Implication of 3D representation of a global brain*, 2019). With respect to any "polyhedral" organization, there it was noted that the results of recent neuroscience research indicate the remarkable possibility of cognitive processes taking up even up to 11-dimensional form in the light of emergent neuronal connectivity in the human brain:

Using mathematics in a novel way in neuroscience, the [Blue Brain Project](#) shows that the brain operates on many dimensions, not just the three dimensions that we are accustomed to. For most people, it is a stretch of the imagination to understand the world in four dimensions but a new study has discovered structures in the brain with up to eleven dimensions - ground-breaking work that is beginning to reveal the brain's deepest architectural secrets..... these structures arise when a group of neurons forms a clique: each neuron connects to every other neuron in the group in a very specific way that generates a precise geometric object. The more neurons there are in a clique, the higher the dimension of the geometric object. ...

The appearance of high-dimensional cavities when the brain is processing information means that the neurons in the network react to stimuli in an extremely organized manner. It is as if the brain reacts to a stimulus by building then razing a tower of multi-dimensional blocks, starting with rods (1D), then planks (2D), then cubes (3D), and then more complex geometries with 4D, 5D, etc. The progression of activity through the brain resembles a multi-dimensional sandcastle that materializes out of the sand and then disintegrates. (*Blue Brain Team Discovers a Multi-Dimensional Universe in Brain Networks*, *Frontiers Communications in Neuroscience*, 12 June 2017)

"Deep learning" in humans and artificial intelligence: As noted above, the focus on higher order carbon configurations of fullerenes is potentially especially significant because of the role of carbon in molecules essential to life. The "behaviour" of carbon -- and the patterns by which it may be configured -- suggest that it may therefore constrain and enable the organization of learning and

comprehension.

Are there implications of such patterns for higher order organization with respect to the potential of neural learning, whether within the human brain or in its relevance to artificial intelligence ? (Daniel A. Roberts and Sho Yaida, *The Principles of Deep Learning Theory: an effective theory approach to understanding neural networks*. 2022).

Brain "globality"? The possibility of such equivalence in systemic terms can be variously explored with respect to understandings of a "global brain", most notably with respect to its comprehensibility (*Simulating a Global Brain -- using networks of international organizations, world problems, strategies, and values*, 2001; *Envisaging a Comprehensible Global Brain -- as a Playful Organ*, 2019). Themes in the latter included:

Patterning and framing a global brain?	Requisite helical cognitive engagement within a global brain
Systemic feedback cycles of global brain interrelationships in 2D	Brainwaves and feedback loops in a global brain?
Representations of cyclic dynamics with implications for a global brain	Pathology of the global brain?
Implication of 3D representation of a global brain	Global brain as an organ: playable, playful or neither?
Dynamic patterns of play engendered by Homo ludens and Homo undulans?	

With respect to any "pathology" of a global brain, as suggested by current crises, the equivalence can be explored in terms of meaningful integration (*Are the UN and the International Community both Brain Dead -- given criteria recognizing that NATO is brain dead?* 2019; *Corpus Callosum of the Global Brain?: locating the integrative function within the world wide web*, 2014; *Bipolar disorder of the global brain?* 2009).

Psychosocial relevance of fullerene topology and construction

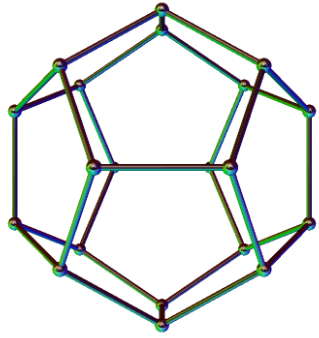
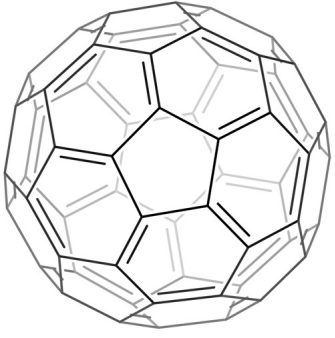
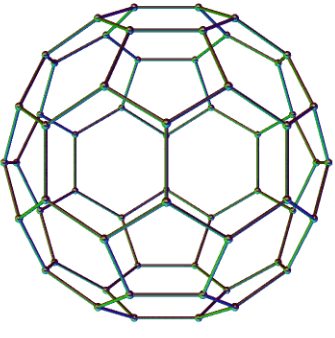
As noted above, the discovery of the fullerene molecules and related forms of carbon such as nanotubes has generated an explosion of activity in chemistry, physics, and materials science which has been variously reviewed. There are many explanations and images of fullerenes available on the web.

The most helpful overviews of topological and graph theoretical developments in fullerene research appear to be those associated with the team at the Centre for Theoretical Chemistry and Physics of the New Zealand Institute for Advanced Study, as summarized by Peter Schwerdtfeger, Lukas N Wirz, and James Avery (*The Topology of Fullerenes*, *Wiley Interdisciplinary Reviews: Computational Molecular Science*, 5, 2015, 1)

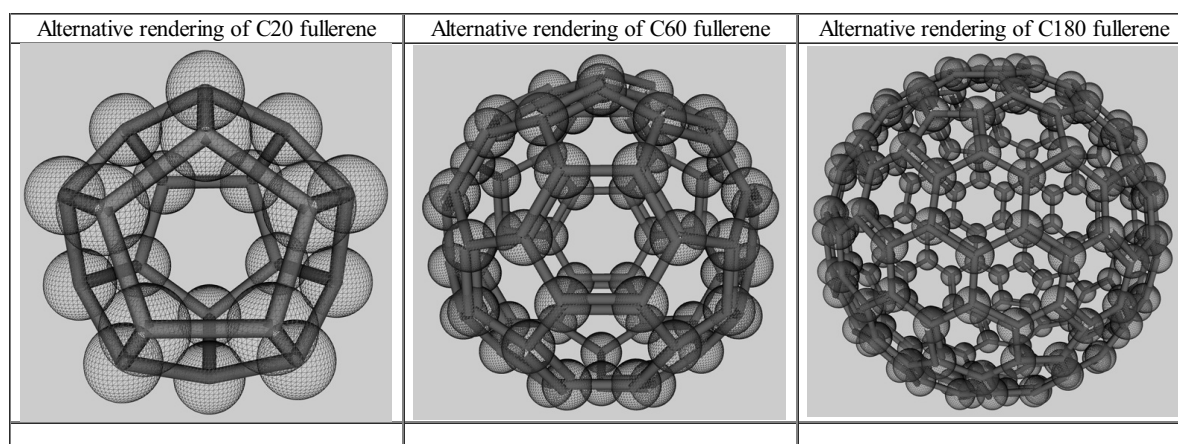
Fullerenes are carbon molecules that form polyhedral cages. Their bond structures are exactly the planar cubic graphs that have only pentagon and hexagon faces. Strikingly, a number of chemical properties of a fullerene can be derived from its graph structure. A rich mathematics of cubic planar graphs and fullerene graphs has grown since they were studied by Goldberg, Coxeter, and others in the early 20th century, and many mathematical properties of fullerenes have found simple and beautiful solutions. Yet many interesting chemical and mathematical problems in the field remain open...

In general, (classical) fullerenes are cage-like, hollow molecules of pseudo-spherical symmetry consisting of pentagons and hexagons only, resulting in a trivalent (and in the most ideal case) convex polyhedron with exactly three edges (bonds) joining every vertex occupied by carbon... [below centre]. The smallest possible fullerene is C₂₀..., a dodecahedron consisting of 12 connected pentagons [below left], and the only Platonic solid in the family of fullerene polyhedra. The truncated icosahedron C₆₀ [below right] belongs to the class of Archimedean solids...

The images in that paper are noted as having been generated by a freely accessible software package associated with a very extensive database (Peter Schwerdtfeger, Lukas Wirz, and J. Avery, *Program Fullerene: a software package for constructing and analyzing structures of regular fullerenes*, *Version 4.4*, *Journal of Computational Chemistry*, 34, 2013; Peter Schwerdtfeger and Lukas Wirz, *Fullerene (Version 4.5) A Program for the Topological Analysis of Fullerenes User's Manual*). This notably refers to *An Atlas of Fullerenes* (1995/2006) compiled by .P. W. Fowler and D. E. Manolopoulos.

Common fullerenes		
Dodecahedron (C ₂₀)	Truncated icosahedron showing distinctive carbon bonding	Truncated icosahedron (C ₆₀)
		
Made using <i>Stella Polyhedron Navigator</i>	Benjah-bmm27, Public domain, via Wikimedia	Made using <i>Stella Polyhedron Navigator</i>

The set of C_n fullerenes presented by Nick Frederick (*[C_n Fullerenes](#)*, Michigan State University, 2019), as discussed below, offers [.xyz files](#) with the coordinates of many fullerenes. These can be displayed interactively in 3D with the [Avogadro application](#) and exported into other formats, notably [.wrl](#) -- from which they can be converted into the [.x3d](#) format.



Varieties of fullerene: A distinction is made between two major families of fullerenes, with fairly distinct properties and applications: the closed buckyballs and the open-ended cylindrical carbon nanotubes. Hybrid structures exist between those two classes, such as [carbon nanobuds](#), namely nanotubes capped by [hemispherical](#) meshes or larger "buckybuds". Of potential interest to this argument are toroidal fullerenes (as discussed below) in which the tube takes the closed circular form of a torus.

Fullerenes with fewer than 60 carbon atoms have been called "lower fullerenes", and those with more than 70 atoms "higher fullerenes" or "giant fullerenes".

As noted by M. Deza, et al (*[Fullerenes as Tilings of Surfaces](#)*, *Journal of Chemical Information and Modeling*, 40, 2000):

If a fullerene is defined as a finite trivalent graph made up solely of pentagons and hexagons, embedding in only four surfaces is possible: the sphere, torus, Klein bottle, and projective (elliptic) plane. The usual spherical fullerenes have 12 pentagons; elliptic fullerenes, 6; and toroidal and Klein-bottle fullerenes, none. Klein-bottle and elliptic fullerenes are the antipodal quotients of centrosymmetric toroidal and spherical fullerenes, respectively... Toroidal and Klein-bottle fullerenes may also be called toroidal and Klein-bottle polyhexes since they include no pentagons.... Klein-bottle polyhexes have been considered in several papers.

From a theoretical perspective the number of predicted fullerenes has now grown into millions with the recognition of [isomers](#), namely those with identical formula. A further distinction is made between those more stable, obeying the isolated pentagon rule (IPR), and others (Hao Li and Heping Zhang, *[The isolated-pentagon rule and nice substructures in fullerenes](#)*, *Ars Mathematica Contemporanea*, 15, 2018, 2). However, as noted by Schwerdtfeger (2015), the fullerene graphs contain all the information needed to sort through the millions of isomers by way of simple, easily computed topological indicators, in order to detect the few relatively stable candidates from a chemical perspective.

Particular attention is given to the "[main fullerenes](#)", the smaller closed buckyballs (with an indication of the number of their isomers). This is most clearly provided (with images) via a set of web pages maintained by Nick Frederick, *[C_n Fullerenes](#)*, Michigan State University, 2019) to which those in the following table are linked. For each fullerene configuration the number of isomers is indicated below, followed by the prime factorization, the number of vertex spirals (discussed below), and an indication of the total strain energy (continuum elasticity) where there is only one isomer. Factorization is included because of its potential indication of memorability as a consequence of patterns of symmetry.

Indication of primary C_n fullerenes: isomers, factorization, vertex spirals and total strain energy									
C20	C24	C26	C28	C30	C32	C34	C36	C38	C40
1	1	1	2	3	5	5	13	13	37
$2^2 \times 5$	$2^3 \times 3$	2×13	$2^2 \times 7$	$2 \times 3 \times 5$	2^4	2×17	$2^2 \times 3^2$	2×19	$2^3 \times 5$
1	0	0	0	0	0	0	0	0	0
24.204 eV	25.046 eV	24.675 eV							
C42	C44	C46	C48	C50	C52	C60	C70	C72	C74
45	89	116	125	269	437	1	1	1	1
$2 \times 3 \times 7$	$2^2 \times 11$	2×23	$2^4 \times 3$	2×5^2	$2^2 \times 13$	$2^2 \times 3 \times 5$	$2 \times 5 \times 7$	$2^3 \times 3^2$	2×37
0	0	0	0	1	0	2	1	0	0
						24.849 eV	26.486 eV	29.549 eV	26.897 eV
C76	C78	C80	C82	C84	C86	C90	C92	C94	C96
2	5	5	9	21	19	45	85	133	187
$2^2 \times 19$	$2 \times 3 \times 13$	$2^4 \times 5$	2×41	$2^2 \times 3 \times 7$	2×43	$2 \times 3^2 \times 5$	$2^2 \times 23$	2×47	$2^3 \times 3$

C98	C100	C180	C240	C260	C320	C500	C540	C720
259	450	1	1	1	1	1	1	1
2×7^2	$2^2 \times 5^2$	$2^2 \times 3^2 \times 5$	$2^4 \times 3 \times 5$	$2^2 \times 5 \times 13$	$2^6 \times 5$	$2^2 \times 5^3$	$2^2 \times 3^3 \times 5$	$2^4 \times 3^2 \times 5$
		40.915 eV	45.871 eV	50.687 eV	60.290 eV	82.631 eV	84.843 eV	106.649 eV

Indications of psycho-social coherence and memorability from fullerenes?

The research on the properties of fullerenes is notably focused on what contributes to their stability. This follows from their previously unsuspected existence and the improbable degree of organization of such molecules -- especially given the occurrence of some in nature.

The question here is how this may be indicative of unsuspected degrees of coherence in psycho-social organization -- potentially corresponding to the intuited (or "felt") sense highlighted in the examples above. It is therefore useful to note how stability is recognized in fullerenes -- as being indicative (if only metaphorically) of the coherence and integrity of possible forms of psycho-social organization. Especially relevant in that respect is whether such forms are comprehensible, memorable and replicable.

Expressed otherwise, **does fullerene organization offer a language within which the subtlety of "coherence" and "integrity" can be discussed in relation to their comprehensibility and memorability?**

Any focus on "integrity" recalls arguments made with respect to "tensional integrity" or *tensegrity* (*Transcending Psychosocial Polarization with Tensegrity Biomimetic clues to collective resilience and unshackling knowledge*, 2021). This is the architectural principle explored as having cognitive implications by Buckminster Fuller with respect to geodesic dome construction (*Synergetics: Explorations in the Geometry of Thinking*, 1975; and *Synergetics 2: Explorations in the Geometry of Thinking*, 1979) and discussed separately (*Geometry of Thinking for Sustainable Global Governance: cognitive implication of synergetics*, 2009). This was the justification for the naming of fullerenes, as noted above.

Framed in this way, current efforts at "global organization" may merit recognition as dangerously sub-optimum and an indication of "sub-understanding", as argued by Magoroh Maruyama from a cybernetic perspective (*Peripheral Vision: polyocular vision or subunderstanding? Organization Studies*, 25, 2004, 3).

A sense of "stability", as understood for an individual or collective, may correspond systemically to the thermodynamic stability of fullerenes, especially with respect to pattern retention and memorability. It could be understood as related to notions of "getting it" or "losing it" -- enabled or undermined by the "heat" of the moment.

Thermodynamic stability: There is extensive research on molecular stability with respect to fullerenes:

- Michael C. Parker and Chris Jaynes: *Fullerene Stability by Geometrical Thermodynamics* (*ChemistrySelect*, 2020, 5)
- Simone L. Waite, et al: *Thermochemical stabilities of giant fullerenes using density functional tight binding theory and isodesmic-type reactions* (*Journal of Computational Chemistry*, 42, 2021, 4)
- Evgeniy Yu Pankratyev, et al: *On the upper bound of the thermodynamic stability of fullerenes from small to giant* (*Physica E: Low-dimensional Systems and Nanostructures*, 101, July 2018).

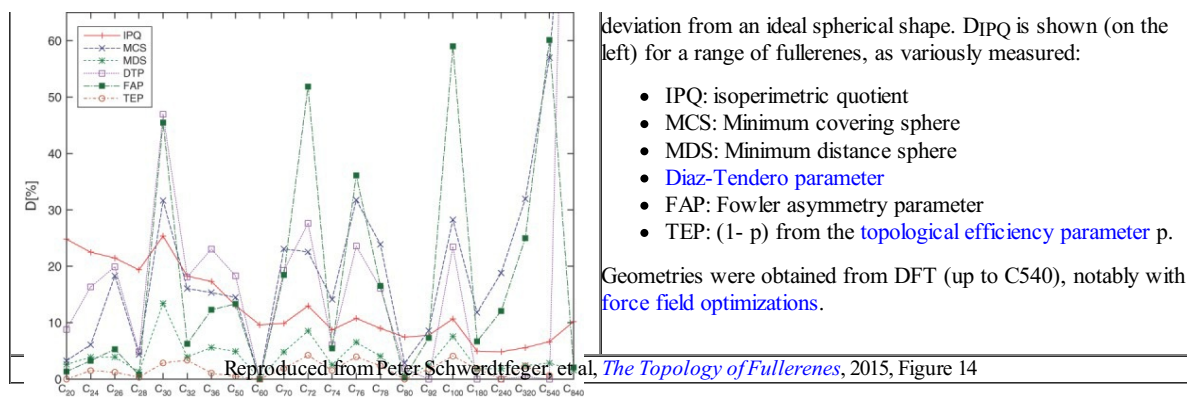
Notable focus is given to the manner in which stability is ensured by the presence of 12 pentagons, as in the case of C60. In the light of Euler's theorem, a fullerene on n vertices has exactly 12 pentagons and (n/2)-10 hexagons, where n is a natural number equal or greater than 20, but not equal to 22 (Maryam Jalali-Rad, *Which fullerenes are stable?* *Journal of Mathematical Nanoscience*, 5, 2015, 1-2; Antonio Rodríguez-Forteza, et al, *Fullerenes: formation, stability, and reactivity*, *WIREs: Computational Molecular Science*, 1, 2011, 3).

Early insights into such stability were provided by Nobel Laureate Harold Kroto (*The stability of the fullerenes C_n, with n = 24, 28, 32, 36, 50, 60 and 70*, *Nature*, 329, October 1987, 6139):

It has been proposed that the geodesic and chemical properties inherent in a closed, hollow, spheroidal, carbon cage structure with the symmetry of a European football can readily explain the remarkable stability observed for the C60 molecule. Here I present a set of simple, empirical chemical and geodesic rules which relate the stability of carbon cages mainly to the disposition of pentagonal rings, or various directly fused pentagonal ring configurations. The rules yield cluster magic numbers consistent with observation and in particular predict that the fullerenes, C_n for which n = 24, 28, 32, 36, 50, 60 and 70 should have enhanced stability relative to near neighbours. These results provide further evidence for the proposal that closed hollow cages form when carbon nucleates in the vapour phase, and in particular that C60 buckminsterfullerene is indeed a truncated icosahedron as originally proposed.

A valuable overview is provided by the following figure from the work of Schwerdtfeger, et al, (2015), as indicated above.

Various deformation parameters D (in percent) for a series of fullerenes selected according to stability	
	The simplest measure of sphericity is the <i>isoperimetric quotient</i> which can be defined for a polyhedron, as the dimensionless quantity obtained using the volume and surface area of the sphere as a reference.
	For an ideal sphere, $q_{IPQ} = 1$ and D_{IPQ} is a measure of the



It is intriguing to note that in measuring stability preoccupation with "minimum covering sphere" is also of concern to the management sciences (D. Jack Elzinga and Donald W. Hearn, *The Minimum Covering Sphere Problem*, *Management Science*, 19, 1972, 1). More generally, it is potentially of relevance to encompassing concepts within a sphere of knowledge (Linus Källberg, *Minimum Enclosing Balls and Ellipsoids in General Dimensions*, Mälardalen University, 2019).

The figure helps to frame the relative limited distortion from the coherence of spherical organization associated to varying degrees with C₂₀, C₂₈, C₆₀, C₇₆, C₁₈₀ and C₅₄₀ -- and most notably C₆₀. The study by Parker and Jeynes (2020) notes the implications of **chirality** and counter-propagating spirals (discussed below with respect to toroidal fullerenes):

This work proves that stability of C₆₀ is a geometrical property of the thermodynamics of the system: a significant methodological advance since a detailed treatment of the energetics may be avoidable. This approach may be fruitful, not only for fullerenes but also for general problems of molecular stability and in other applications of conformational chemistry. For the non-chiral C₆₀, C₃₈₄, and the weakly-chiral C₂₈, C₇₆ and C₃₈₀ (of these, C₃₈₀ and C₃₈₄ are classed as "unspirallable"). **Schlegel projections** are used to show that these fullerenes can all be represented by pairs of spirals counter-propagating in anti-parallel (C₂) symmetry. For C₆₀, the high symmetry is used to construct an analytical approximation for the spherical double-spirals, shown mathematically to be Maximum Entropy (MaxEnt) using the formalism of Quantitative Geometrical Thermodynamics (QGT). Therefore C₆₀ is necessarily stable. This MaxEnt stability criterion is general, depending only on the geometry and not the kinematics of the system.

The figure is also helpful in offering a sense of the relative probability that a fullerene-style psycho-social configuration will be felt to be coherent and stable -- and capable of sustaining a sense of collective identity. It justifies the focus on the particular advantages of C₆₀.

Islands of stability and relative "half-life": The detection of relatively stable fullerenes recalls the challenge of detecting relatively stable chemical elements and **isotopes** in nuclear physics -- termed **islands of stability**. This is a predicted set of isotopes of **superheavy elements** that may have considerably longer **half-lives** than known isotopes of these elements. The concern in the case of fullerenes is indeed the detection of "super-heavy" fullerenes -- more commonly termed "giants".

The term is also used more generally to characterize any type of decay, exponential or otherwise (Arthur M. Schneiderman, *Half-Life Method*, *Encyclopedia of Social Measurement*, 2005). With respect to financial debt, for example, the term half-life refers to the point at which half of the total principal of a debt obligation comes due or has been paid off. Also known as the average life, the half-life allows borrowers to determine the point at which they have fully paid half the principal (Daniel Liberto, *Half-Life*, *Investopedia*, 1 June 2022).

There is clearly a sense in which the forms of psycho-social organization invite recognition in terms of half-lives, most obviously with respect to the bankruptcy of for-profit corporations, the failure of non-profit initiatives (communes, etc), or the collapse of empires (Ahmed Zidan. *The half-life of NGOs*, *The Broker*, 28 December, 2011). It is otherwise evident in strategic obsolescence, legislation, shifts of opinion, as well as collective fads and fashions (Paul Estes, *The Half-Life of Skills*, *HR Daily Advisor*, 25 March 2020; *Use of Successive Half-Lives to Aid in Planning*, Institute for Healthcare Improvement). With respect to information, the insight has been adapted to the **half-life of knowledge** has been an understanding has been explored with respect to "facts" (Samuel Arbesman, *The Half-life of Facts: Why Everything We Know Has an Expiration Date*, 2012).

Comprehension and memory can be understood in such terms, thereby usefully framing the question as to the relative stability of larger and smaller configurations of which the fullerenes are indicative. As with individual memory, can collective memory be usefully understood as having a half-life? How does memory decay, as is only too evident in the case of collective memory (*Societal Learning and the Erosion of Collective Memory*, 1980).

Force-directed approach: It appears that detection of fullerenes has made use of force-directed techniques. These are valuable in exploring their representation, as presented and illustrated separately (*Use of force directed layout to elicit memorable polyhedra*) in a discussion of *Eliciting Memorable Spheres and Polyhedra from Hyperspace* (2015). This focused on the integrative connectivity of problems, strategies, themes, groups or people. The interactive animations to which the following are linked form part of a larger set discussed there -- to which links are provided.

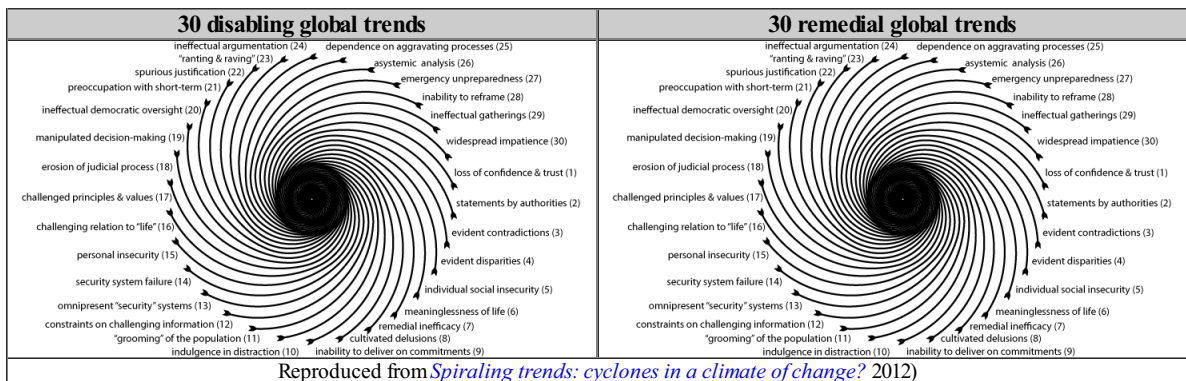
As one experiment in psycho-social relevance, the nodes were associated with individual documents on this site -- whose titles are evident by placing the cursor on the node in the animation. This was seen as a step towards engendering a more comprehensive perspective.

Screen shots of selected results of force-directed layout to elicit Archimedean polyhedra (not to scale relative to one another; subject to further tests)		
Cuboctahedron	Small rhombicuboctahedron	Truncated cuboctahedron
Interactive force-layout animation	Interactive force-layout animation	Interactive force-layout animation

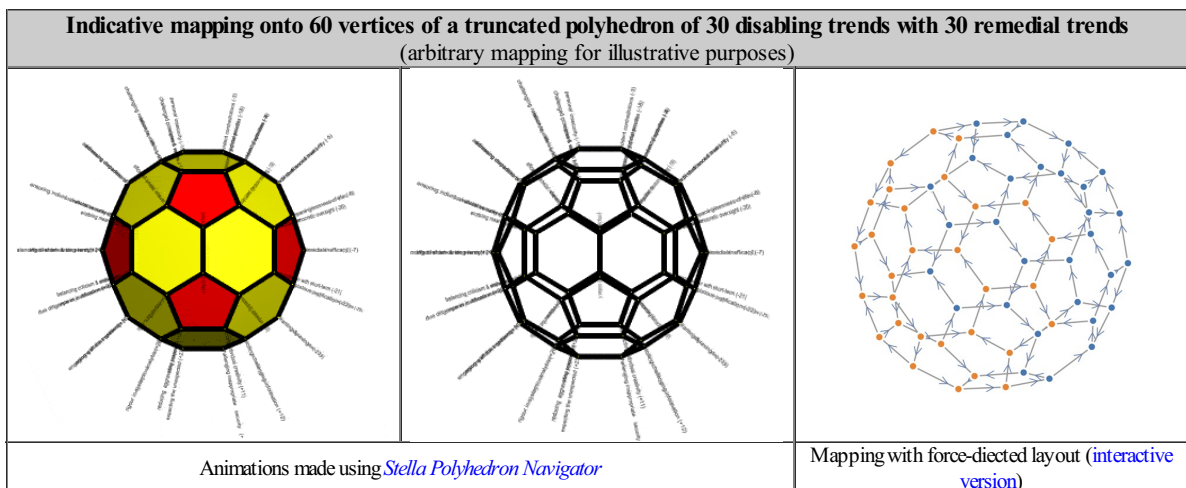
Related applications of the technique include:

- *Interactive orbital animations of world views using data-driven documents (d3.js)* with respect to possibilities of *Transcendent Integrity via Dynamic Configuration of Sub-understandings?* (2015)
- *Symmetrical polyhedra as a key to memorable organization of thematic sites*, explored as a response to *Optimizing Web Surfing Pathways for the Overloaded* (2015) as offering polyhedral insights from the travelling salesman problem of operations research
- *Indicative simulations of memetic "tank warfare" in psychosocial systems?* (2019) and *Systemic configuration of highly disparate cognitive modalities* (2019), explored with respect to the relationship between think tanks.
- *Dynamics of force-directed layout of concept sets beyond truncation of rigid polyhedra* (2018) focused on the possible *Requisite 20-fold Articulation of Operative Insights?*
- configuration of climate metaphors by the *Interactive Visualization of 384 I Ching Transformations using Force-directed Layout* (2015) in a discussion of *Enhancing Strategic Discourse Systematically using Climate Metaphors* (2015), and of *64 Questions for the Environmental Conservationists of the World -- raising the question as to why they are not effectively addressed* (2017)

As an illustration of the possibility with respect to 60-fold organization, 30 positive and 30 negative trends are indicated below, These derive from a separate discussion -- with other illustrations of their interaction (*Convergence of 30 Disabling Global Trends: mapping the social climate change engendering a perfect storm*, 2012).



The 30 positive and 30 negative trends above can be suggestively mapped onto a 60-vertex truncated polyhedron as shown in alternative renderings below. That on the right is a screen shot of the *Interactive Mapping of 30 Problems with 30 Strategies onto Truncated Icosahedron using Force-directed Layout* (2022). The blue nodes are challenging problems and the orange nodes are remedial strategies - the titles becoming evident on mouseover. More skillful renderings are suggested in the visual representation of both alternatives.



Tiling perspective: All the finite fullerenes of primary interest may be considered as actually trivalent tilings with (combinatorial) pentagons and hexagons of particular surfaces, whether spherical, toroidal, Klein bottle, or projective (elliptic) plane (Michel Deza et al., *Fullerenes as Tilings of Surfaces*, *Journal of Chemical Information and Modeling*, 40, 2000; Michel Deza, et al., *Fullerenes as Tilings of Surfaces*, *Geometry of Chemical Graphs*, 2010).

Insights of psycho-social relevance can be derived from the extensive literature on [tiling](#), [tesselation](#) and the associated mathematics -- most notably on a sphere (*Spherical tiling and spherical polyhedra*, 2015). In mathematics, a [spherical tiling](#) or spherical polyhedron is a tiling of the sphere in which the surface is divided or partitioned by [great arcs](#) into bounded regions called spherical polygons.

Symmetry-preserving operations on polyhedra can be explored in terms of their cognitive implication for memorability and comprehension (*Cognitive operations potentially analogous to generation of tiling patterns*, 2021; *Cognitive implications of operational modification of polyhedra -- "global tiling"*, 2021), as discussed with respect to *Encoding Coherent Topic Transformation in Global Dialogue* (2021) and *Systemic Coherence of the UN's 17 SDGs as a Global Dream* (2021)

Toroidal fullerenes as a complement to the global form

The focus on globalization and its spherical representation does much to obscure the implications of the toroidal form. This is somewhat ironic since the Earth as a globe can be understood as tracing out a torus in its revolution around the Sun (ignoring the manner in which the solar system is moving as a whole, such that the pathway can also be understood as being of helical form). In this sense there is a case for recognizing the degree to which people effectively live on a torus, as discussed separately (*Imagining Toroidal Life as a Sustainable Alternative: from globalization to toroidization or back to flatland?* 2019) with respect to the following:

Reality distortion, psychosocial torsion and psychological torque	Toroidal embodiment, knottedness and being a torus?
Intuitive recognition of toroidal cycles?	Mistaken recognition of holes repressed by a global focus?
Imagining life on a torus, or within it -- as lived in reality?	Torc-bearing, Playing-ball, Scoring and Nesting
Being "flat"-- or "global" and "well-rounded"?	Aesthetic reconciliation of contrasting toroidal metaphors?
Transforming the world into a doughnut: a vital clarification	Transformative interplay between flat, global and toroidal?
Toroidal conceptualization	

Of particular relevance to human memorability is the degree to which toroidal configurations are valued in the design of memory in supercomputers, as discussed separately (*Framing Cognitive Space for Higher Order Coherence: toroidal interweaving from I Ching to supercomputers and back?* 2019) with respect to the following:

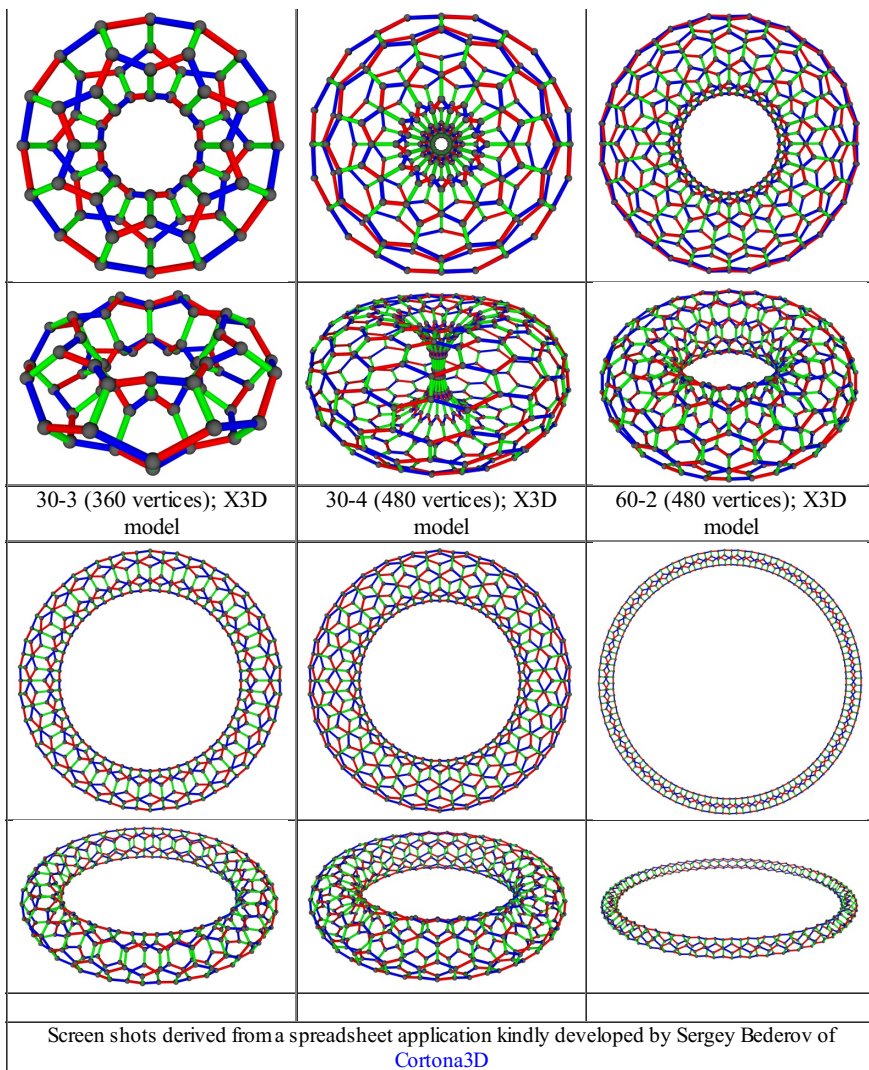
Torus interconnect -- as used in supercomputers	Reflexivity in multi-loop thinking and higher order learning
Cubic organization of <i>I Ching</i> trigrams -- an eightfold way	Toroidal constraint -- nuclear fusion as metaphor of cognitive fusion
Pattern language and polyhedral mapping	Supercomputers, hypercomputing and superquestions?
Higher dimensionality, polyhedral packing and transformation	Framing the space for conscious creativity?
Brain organization, cognition, comprehension -- and music	Dancing cognitively inside the box -- and beyond
Oppositional logic and its requisite polyhedral geometry	

Research on fullerenes (or toroidal polyhexes) now extends to the possibilities of their toroidal form, as noted in the following:

- B. T. Ortega and M. McGuigan: *Visualization and Simulation of Carbon Structures with Higher Genus* (Brookhaven National Laboratory. 2018)
- C. Chuang, Y. C. Fan and B. Y. Jin: *Systematics of Toroidal, Helically-Coiled Carbon Nanotubes, High-genus Fullerenes, and Other Exotic Graphitic Materials* (*Procedia Engineering*, 14, 2011)
- Chern Chuang and Bih-Yaw Jin: *Hypothetical toroidal, cylindrical, and helical analogs of C60* (*Journal of Molecular Graphics and Modelling* 28, 2009, 3)
- Chern Chuang, Yuan-Chia Fan and Bih-Yaw Jin: *Generalized Classification Scheme of Toroidal and Helical Carbon Nanotubes* (*Journal of Chemical Information and Modeling*, 49, 2009, 2)
- Michael C. Parker and Chris Jeynes: *Fullerene Stability by Geometrical Thermodynamics* (*Chemistry Select*, 5, 2020, 1)
- Ming-Hsuan Kang: *Toroidal Fullerenes with the Cayley Graph Structures* (*Discrete Mathematics*, 311, 2011, 21)
- M. Bača, et al.: *Total Edge Irregularity Strength of Toroidal Fullerene* (*Mathematics in Computer Science*, 7, 2013).
- E. C. Kirby, et al: *Toroidal Polyhexes* (*Journal of the Chemical Society -- Faraday Transactions*, 1993, 89).

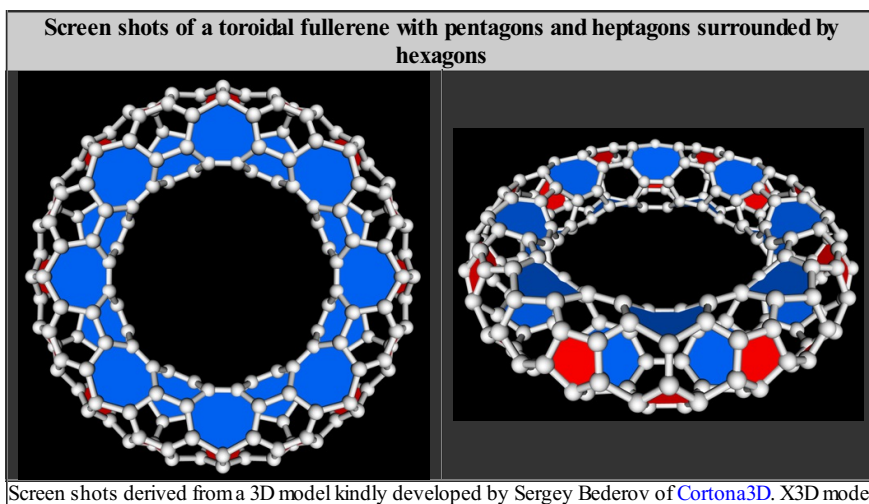
There are necessarily far fewer depictions of toroidal fullerenes, especially in 3D -- as is potentially appropriate to the psycho-social implications explored here and the issues of comprehension. It is therefore appropriate to note the remarkable facility offered by a spreadsheet application developed by Sergey Bederov enabling an extensive range of toroidal forms to be generated in X3D format by modification of several parameters -- irrespective of whether they are to be considered as fullerenes of chemical significance. Screen shots of selected 3D examples of the output are presented below.

Screen shots of generated toroidal fullerenes composed entirely of hexagons (polar and side views)		
8-2 (64 vertices); X3D model	15-6 (360 vertices); X3D model	20-5 (400 vertices); X3D model



Closed convex fullerenes of spherical form are of particular interest when they conform to the isolated pentagon rule (IPR) which is indicative of their stability and the possibility of their synthesis. Those of toroidal form are questionably convex and do not necessarily obey that rule. As noted by Sergey Bederov with respect to those generated by his spreadsheet application, those above lack pentagons -- being entirely composed of hexagons. It is questionable whether these are an optimal structure from a chemical, since the lengths of edges (corresponding to chemical bonds) are different.

In developing that spreadsheet application, Bederov explored the possibility of a potentially more stable toroidal fullerene through the introduction of pentagons in accordance with research on toroidal carbon nanotubes (Florian Beuerle, et al, *Optical and Vibrational Properties of Toroidal Carbon Nanotubes*, *Chemistry: a European Journal*, 17, 2011, 14; Pakhpoom Sarapat, *A Review of Geometry, Construction and Modelling for Carbon Nanotubes*, *Applied Sciences*, 9, 2019, 11). Such research recognizes that in order to maintain their Euler characteristic (zero for a torus), an equal number of heptagons (7-gons) needs to be added (coloured blue and red in the screen shots below to distinguish them by size). Whether pentagons or hexagons, these are in each case by hexagons; mathematically all vertices lie on a perfect torus and edge lengths differ by only 24%.



From a psycho-social perspective, the construction of a toroidal fullerene is remarkably reminiscent of the design challenges faced in the

toroidal construction of a nuclear fusion reactor. There the key concern is the [toroidal configuration of magnets](#) to ensure that plasma at very high temperature does not come in contact with the walls of the container -- according to the [tokamak](#) design principle. Toroidal fullerenes may offer insights into the design of a reactor for "cognitive fusion" fundamental to viable global strategy, as discussed separately ([Enactivating a Cognitive Fusion Reactor: Imaginal Transformation of Energy Resourcing \(ITER-8\)](#), 2006).

Global coherence from helical windings on fullerenes -- and their animation

As with polyhedra in general, fullerenes can be viewed from a cyclic perspective -- as configurations of cycles (K. Kutnar and D. Marusic, [On cyclic edge-connectivity of fullerenes](#), *Discrete Applied Mathematics*, 156, 2008).

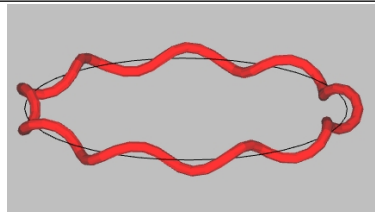
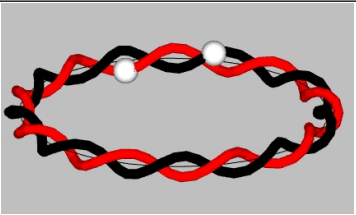
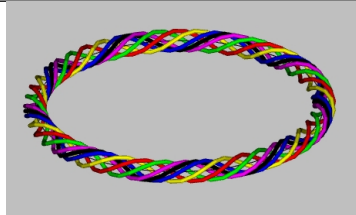
Knots: A toroidal fullerene (or toroidal nanotube) can be usefully recognized from the perspective of [knot theory](#) as an [unknot](#) -- the least knotted of all knots. This suggests recognition of more complex configurations -- knotted tori -- as discussed ([Cyclic Representation of Coherence as Knots and Otherwise: interestingness of curves of increasing complexity in relation to sustainability](#), 2022). Many of the more complex images there bear a striking resemblance to the toroidal fullerenes generated above.

Spirals: In the analysis of fullerenes, consideration is given to the distinction between:

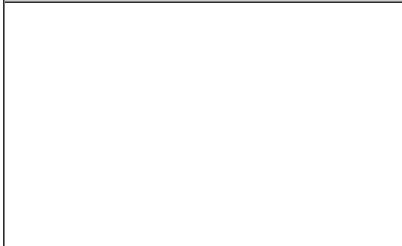
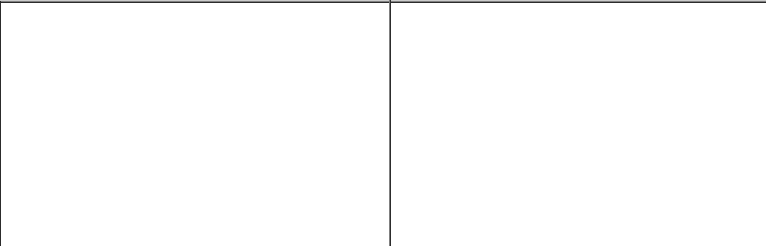
- **face spirals** (Lukas N. Wirz, Peter Schwerdtfeger and James Emil Avery: [Naming Polyhedra by General Face-Spirals - Theory and Applications to Fullerenes and other Polyhedral Molecules](#), *Fullerenes Nanotubes and Carbon Nanostructures*, 26, 2017, 10; D. E. Manolopoulos, et al, [Theoretical studies of the fullerenes: C34 to C70](#), *Chemical Physics Letters*, 1991, 181)
- **vertex spirals, or Hamilton spirals:** (Patrick W. Fowler, Daniel Horspool, and Wendy Myrvold: [Vertex Spirals in Fullerenes and Their Implications for Nomenclature of Fullerene Derivatives](#), *Chemistry: a European Journal*. 2007, 13). From its extensive tables, this notes that omnispiral fullerenes are rare: there are only four found amongst the over ten million fullerenes with C_n less than 120 and none amongst the set of isolated-pentagon fullerenes with C_n between 120 and 150. These four are the unique fullerene isomers of C_{20} and C_{24} and the unique isolated- pentagon fullerene isomers of C_{60} and C_{72} .
- **aspirable fullerenes:** Ralf Tonner, Peter Schwerdfeger and James Avery: [Structure and Properties of the Nonface-Spiral Fullerenes T -C 380 , D 3 -C 384 , D 3 -C 440 , and D 3 -C 672 and Their Halma and Leapfrog Transforms](#), *Journal of Chemical Information and Modeling*, 2014)

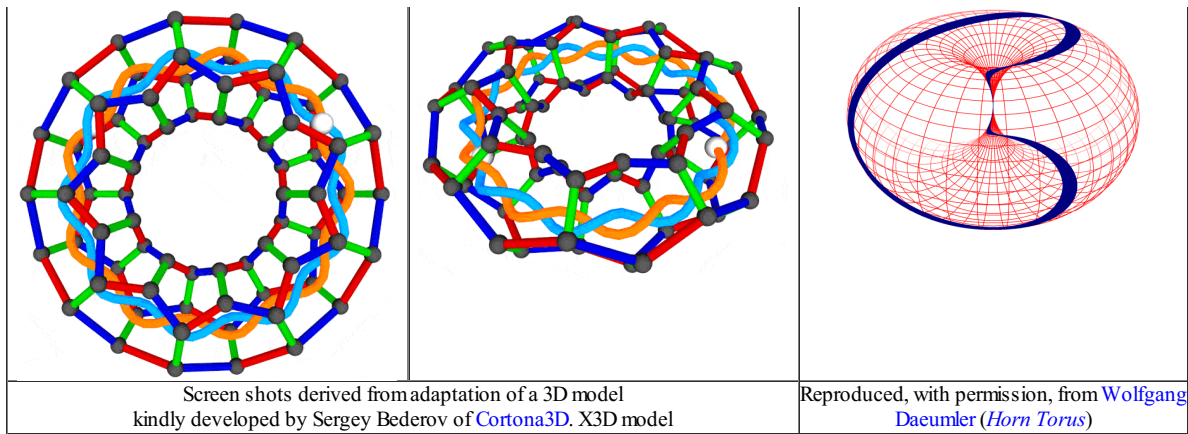
Spirals are also discussed in terms of spiral "motion" (Natalie Thamwattana, et al, [Spiral motion of carbon atoms and C60 fullerenes inside single-walled carbon nanotubes](#), *International Journal of Theoretical and Applied Multiscale Mechanics*, 1, 2009, 2).

Helical coils: The literature currently offers depictions in 2D of the cycles and spirals associated with fullerenes. Visually these are somewhat simplistic in their dependence on Schlegel projections, as noted above. An alternative approach offering a 3D perspective took as its point of departure an exploration of toroidal helical windings ([Visualization in 3D of Dynamics of Toroidal Helical Coils: in quest of optimum designs for a Concordian Mandala](#), 2016). This gave rise to images below (reproduced from [Towards a 3D visualization of toroidal counter-coiling dynamics](#), 2016).

Pseudo-counter-coiling		
Original coil around an invisible torus	Pseudo-counter-coil addition (helices out of phase, but spheres travel in opposite directions)	Addition of pseudo-counter-coil to 5-coil pattern (black addition of different phase only)
		
(X3D or VRML)	(X3D or VRML)	(X3D or VRML)

Associating helical coils with toroidal fullerenes: The coding for the central model above could then be "embedded" experimentally into a toroidal form, as generated by Bederov's fullerene spreadsheet application, to give the 3D form below. Colours were changed to suggest a potential relation between the orange helix and the bonds coded red, with the cyan helix corresponding to the bonds coded blue. The circulating white spheres were not modified. The model could be further modified to render the spirals coincidental (if only tangentially) with the corresponding bonds.

Indicative counter-coiling of helices within a toroidal fullerene	Animation of Lissajous curve on horn torus
	



Of potential relevance, as discussed in relation to the [horn torus](#) (as a possible toroidal fullerene variant reproduced above), the presentation of [Lissajous curves](#) on the surface, as illustrated by the animation (above right) is a focus of [Wolfgang Daeumler \(*Horn Torus and Physics: 'Geometry of Everything'*\)](#). The visual form of these curves is often suggestive of a three-dimensional knot, and indeed many kinds of knots, including those known as [Lissajous knots](#), project to the plane as Lissajous figures. Also of potential interest are other forms of such curves are variously presented (Nick Berry, *Lissajous curves*, *DataGenetics*; Wolfgang Erb, *Spherical Lissajous curves*, *Math-Unipd*; Clifford A. Pickover, *Picturing Spherical Lissajous Figures*, *Leonardo*, 24, 1991, 3).

Geometry of music: With respect to comprehension and memorability, the merit in exploring further is the sense in which the organization of tones as experienced in music has progressed far beyond what might be readily associated with scales and musical notation as conventionally represented. This suggests that there are new ways of comprehending coherence in the light of the comprehension of music -- with implications for global strategy.

The research of particular interest is notably associated with the manner in which the human brain appears to organize the patterns of tones that is appreciated, as clarified by the work of [Dmitri Tymoczko](#) relating music and geometry (*A Geometry of Music: harmony and counterpoint in the extended common practice*, 2011; *The Geometry of Musical Chords*, *Science*, 313, 2006). The manner in which such organization of tones in [musical tuning](#) and [harmony](#) has long been explored in terms of the [Tonnetz](#) (a tone-network), namely a conceptual [lattice diagram](#) representing tonal space. Tymoczko has developed his understanding through its generalization (*The Generalized Tonnetz*, *Journal of Music Theory*, 56, 2012. 1).

Musical theory has notably developed further through a loose collection of ideas termed [Neo-Riemannian theory](#). Of potential relevance to any organization of human values, Neo-Riemannian transformations can be modeled with several interrelated geometric structures. The Riemannian (tonal grid, left below) is a planar array of pitches along three simplicial axes, corresponding to the three consonant intervals. Major and minor triads are represented by triangles which tile the plane of the Tonnetz. Edge-adjacent triads share two common pitches, and so the principal transformations are expressed as minimal motion of the Tonnetz. Pitches in the Tonnetz are connected by lines if they are separated by minor third, major third, or perfect fifth. Interpreted as a torus the Tonnetz has 12 nodes (pitches) and 24 triangles (triads).

Various experimental graphical representations of the Tonnetz have been presented:

- a toroidal view (below centre) has been proposed by David Bulger, and revised subsequently (David Bulger and Richard Cohn, *Constrained voice-leading spaces*, *Journal of Mathematics and Music*, 2015)
- an embedding of the Tonnetz on a 4D [Hypersphere](#) (below right) based on graph theory, as proposed by Gilles Baroin (*The planet-4D model: An original hypersymmetric music space based on graph theory*, 2011).

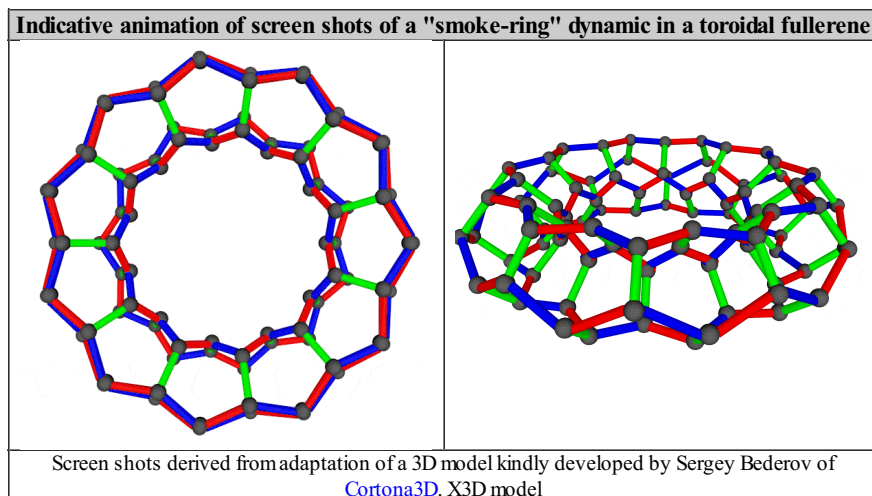
A valuable discussion of such possibilities is provided by [Jason Yust](#) (*Organized Time: rhythm, tonality, and form*, 2018; *Geometric Generalizations of the Tonnetz and their Relation to Fourier Phases Spaces*, 2017).

Selection of graphical representation of the Tonnetz		
Planar organization of the Tonnetz	Toroidal view of the neo-Riemannian Tonnetz	Planet-4D embedding the Tonnetz on a 4D hypersphere
Watchduck (a.k.a. Tilman Piesk) , CC0, via Wikimedia Commons	Adaptation of Davidwbulger , Public domain, via Wikimedia Commons	Gilles Baroin PHD , Public domain, via Wikimedia Commons

The toroidal Tonnetz depiction could be understood as derived from a triangulation of the hexagons in representation of a toroidal fullerene. It makes apparent how distinct blue and red helical windings could be "superimposed" on such fullerene models by extending the bonds to the spheres "across" each hexagon. This would give rise to blue and red helical windings (evident in the animation above),

and to green circles corresponding to the bonds indicated in green.

Smoke-ring animation: The helical pattern discussed above is evident in the central animation above -- complicated by a smoke-ring dynamic. Omitting the helical coiling, the smoke-ring dynamic can be explored with respect to a toroidal fullerene as indicated in the model below. This derives from a development by Sergey Bederov of the spreadsheet application by which the toroidal fullerenes are generated. Again a variety of such animated fullerenes can be generated by modification of several parameters. Further development would be required to embed the helical coiling in the fullerene to emulate the dynamic in the representation above of the Tonnnetz.



Degrees of integration implied by nested fullerenes (bucky-onions)




Hyperfullerenes (or [nested fullerenes](#)) consisting of smaller fullerenes nested within larger one (also known as [bucky-onions](#)) have been found to form on heating soot and fullerenes to very high temperatures. It is an interesting property of the fullerenes is that, because they are completely hollow (i.e. all the atoms lie on the surface), fullerenes of different sizes can in principle be nested within each other like Russian dolls. For example nested structures of C60 within C240 within C540 within C960 have been found. These spherical structure may have as many as 70 layers each with a interlayer distance matching that of bulk graphite (Karl Harrison [Bucky Onion: Nested Fullerene, buckyonion, c540, c240, c60 3DChem.com](#), December 2005)

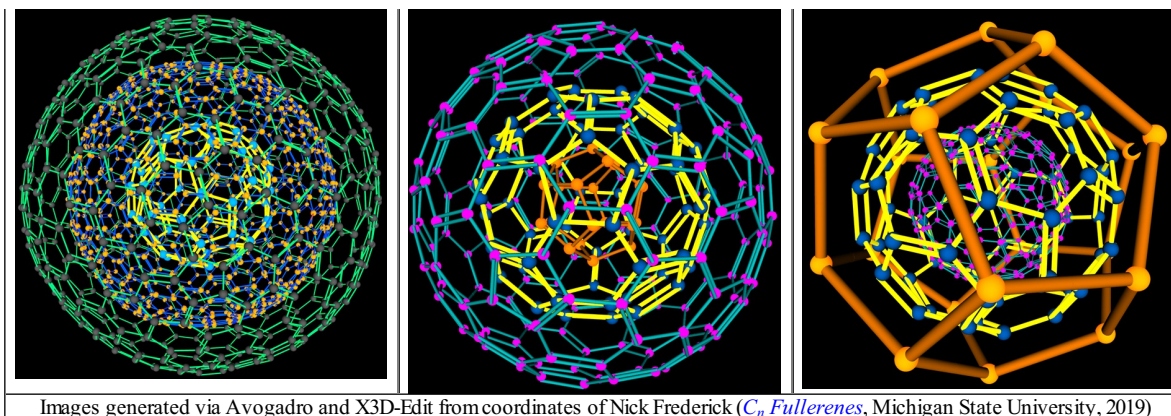
- David Bradley: [Bonding Bucky-Onions](#) (*Physical Review Focus*, 8, 2001, 27) Adding nitrogen to buckyballs makes them form connected 'onion' structures and a material with impressive properties.
- LemiTürke: [A bucky onion from C20 and C60: an AM1 treatment](#) (*Journal of Molecular Structure: THEOCHEM*, 545, 2001, 1-3)
- Jianping Lu: [The Shape of Bucky Onions](#) (*arxiv*, 1993-07-1)
- Jesús Antonio Luque-Urrutia, et al: [Do Carbon Nano-onions Behave as Nanoscopic Faraday Cages? A Comparison of the Reactivity of C60, C240, C60 and C240](#) (*Chemistry: a European Journal* 2019).
- Helena Dodziuk, et al: [Ease of formation of nested fullerenes](#) (*Chemical Physics Letters*, 329, 2000, 5-6)
- K. Bates and G. Scuseria: [Why are buckyonions round?](#) (*Semantic Scholar*, 12 January 1998)

Considerable interested has been evoked by recognition that both fullerenes and buckyonions may be a feature of interstellar space:

- Aigen Li, et al: [On Buckyonions as an Interstellar Grain Component](#) (*Monthly Notices of the Royal Astronomical Society*, 390, 2008, 1)
- Juhua Chen, et al: [On buckyonions as a carrier of the 2175 Å interstellar extinction feature](#) (*Proceedings of the International Astronomical Union*, 1 February 2008)
- Susana Iglesias-Groth: [Fullerenes and Buckyonions in the Interstellar Medium](#)

As a visualization exercise, use was made of the coordinates provided for fullerenes by Nick Frederick ([C_n Fullerenes](#), Michigan State University, 2019), as noted above, in order to nest a set of fullerenes to form a bucky-onion. This exercise is however not to scale and is therefore misleading in many respects, notably in that the carbon atoms should be of equal size. Because of the number of elements in the nesting within the C720 model (52,000 lines of code), various design techniques could not be used (rotation, etc) to evoke greater engagement with such a representation. This also limited that model to three nested fullerenes (C720, C540 and C60), whereas more could have been included. Such limitations applied to a lesser degree in the nesting within C180 (10,300 lines of code).

Images indicative of nesting of fullerenes as bucky-onions (arbitrary scaling and use of colour; image on right, as an inversion of that in the centre, is not a fullerene)		
C60 nested within C540 within C720	C20 nested within C60 within C180	C180 nested within C60 within C20
		



Nested fullerenes are curiously reminiscent of the Kepler solar system model (below left) and of multi-layered Chinese carved ivory puzzle balls, as discussed separately (*Rotation and pumping of nested Chinese "puzzle balls" as symbolizing "worlds-within-worlds"*, 2015). These were presented in the context of consideration of patterns of change suggested by nesting, packing, and transforming symmetrical polyhedra (*Psychosocial Implication in Polyhedral Animations in 3D*, 2015). Possible puzzle ball dynamics were explored there by nesting sets of Platonic or Archimedean polyhedra using variations of relative rotation and expansion/contraction of polyhedra at different rates).

Animation of nesting of Platonic or Archimedean polyhedra:		
Polyhedral model of solar system of Johannes Kepler on <i>Mysterium Cosmographicum</i> (1596)	Rhombic triacontahedron (green) as a nesting framework	Faces non-transparent
Reproduced from <i>Wikipedia</i> entry	Virtual reality variants <i>static</i> : vml or x3d ; <i>mutual rotation</i> : vml or x3d ; <i>"pumping"</i> : vml or x3d ; videos: <i>"pumping"</i> mp4 ; <i>"rotation"</i> mp4	Video animation (.mov); virtual reality (.wrl; x3d)

Implications of higher dimensional fullerenes?

Misleading use of "higher" is also a consequence of consideration of 3D forms beyond C60 (Francois Diederich, et al, *Beyond C60: the higher fullerenes*, *Accounts of Chemical Research*, 25, 1992, 3; Yohji Achiba, et al, *Higher Fullerenes: Structure and Properties*, *MRS Online Proceedings Library*, 359, 1994).

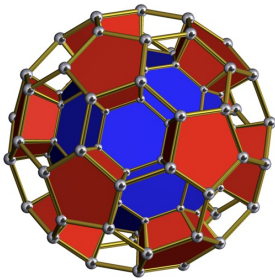
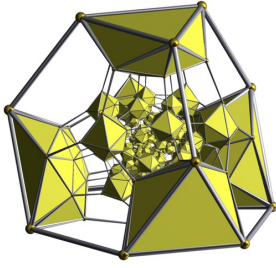
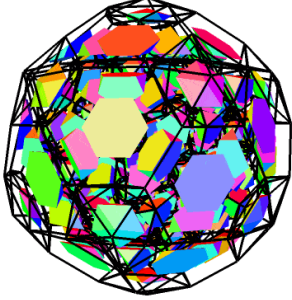
A mathematical treatment of the concept of the extended fullerenes and their further generalization to higher dimensional spaces is given by Michel Deza and M. I Shtogrin (*Three, four and five-dimensional fullerenes*, *Southeast Asian Bulletin of Mathematics*. 23, 1999, 1; *Toward 4-Dimensional Fullerenes*, *CiteSeer*, 1998). Other references to higher dimensions may however be potentially misleading, when "higher" is limited to 3D (Lok Kumar Shrestha, et al, *Fullerene Nanoarchitectonics: From Zero to Higher Dimensions*, *Chemistry: an Asian Journal*, 15 April 2013) as noted in the latter:

Because fullerene molecules can be regarded as an ideal zero-dimensional (0D) building units with attractive functions, the construction of higher-dimensional objects, that is, 1D, 2D, and 3D nanomaterials may realize important aspects of nanoarchitectonics.

The recognition accorded to C60 -- the truncated icosahedron as the fullerene "football" -- has given rise to questions about the existence of "four-dimensional footballs" in the light of the investigation of 4-fullerenes by Deza and Shtogrin. It is therefore especially intriguing to note insights into the form it takes, if any, in 4D rather than in 3D (Antonio Pasini, *Four-dimensional football, fullerenes and diagram geometry*, *Discrete Mathematics*, 238, 2001). As noted by the latter:

In particular, we prove that there is no four-dimensional analogue of the football. More precisely, we prove that there is just one simply connected 4-fullerene where the cells are truncated icosahedra, but it is obtained as a Grassmann geometry of a non-spherical (whence, infinite) Coxeter complex.

With use of the above-mentioned Planet-4D embedding of the Tonnetz on a 4D hypersphere (depicted above), it could be imagined that the C60 football as a truncated icosahedron does take some form in 4D of relevance to the argument with respect to fullerenes. As indicated by *Wikipedia*, a [truncated icosahedral prism](#) is a convex uniform polychoron (four-dimensional polytope), namely one of 18 convex uniform polyhedral prisms created by using uniform prisms to connect pairs of Platonic solids or Archimedean solids in parallel hyperplanes.

4D fullerenes: 3D images of representations of truncated polyhedron		
Truncated icosahedral prism	Truncated 600-cell projected into 3D down an axis with tetrahedral symmetry.	Animation of truncated icosahedron projected into 3D with icosahedral symmetry (<i>Stella</i> , 23- <i>Tex</i>)
		
Generated by <i>Stella 4D</i>		

Is a four-dimensional pattern of potential relevance to a time-bound global civilization, as may be variously argued (*Four-dimensional requisite for a time-bound global civilization?* 2015; *Comprehending the shapes of time through four-dimensional uniform polychora*, 2015; *Five-fold ordering of strategic engagement with time*, 2015).

Symbolic relevance of fullerenes

As noted above, there are very few references to the potential psycho-social implications of fullerenes in the extensive literature on the topic. Given the origin of the term, more such references are however associated with the geodesic domes developed by Buckminster Fuller. An early exception in this respect is the educational focus of James M. Royer (*Buckyballs, hairyballs, educational semiotics, and other interesting topics*. *Educational Psychology Review*, 4, 1992).

A more recent exception, with specific reference to fullerenes, is the detailed study by Gregory Sova (*Buckyball as a Reconciling Symbol in the Unus Mundus*, 2018). Given the role of mathematics and geometry, it might be asked how fullerenes will come to be considered (fruitfully or misleadingly) in the context of the extensive literature on sacred geometry (*Carbon Molecule C60: a longevity and health game changer*, *Gaia*, 18 February 2020). Such concerns can be related to the neglected potential of mathematical theology (*Mathematical Theology: future science of confidence in belief*, 2011; Ralph Abraham, *Mathematics and Mysticism*, University of California, 2015).

Zomes: As noted separately, some such significance has also been accorded to the structure of *zomes* -- a term derived from a combination of *dome* and *zonohedron*. That pattern is presented separately as an approach to global strategic coherence through integrating doughnut, helix and pineapple models (*From Zoom Organization to Zome Configuration and Dynamics*, 2020). A possible relation between fullerenes and zomes has been noted (*The Seven Zonish Dodecahedra with Zones Added Based on Faces, Edges, and/or Vertices*, *RobertLovesPi*, 20 December 2014).

Of potential relevance, the process of *zonohedrification* (by which zonohedrons are engendered) was noted with respect to the dynamics of a pentagonal configuration of nonagons in 3D (*Concordian Mandala as a Symbolic Nexus*, 2016).

I Ching and Sahasrara: The coherence of C60 is achieved through a global configuration of 20 hexagons separating 12 pentagons. Given its early influential role in Chinese decision-making, and in the early development of binary computer coding, it is therefore especially intriguing to note the coherence attributed to the set of 64 hexagrams of the *I Ching* -- originally arrayed as a circle by *Shao Yong* in 2D (*Patterning and framing a global brain?* 2019). This frames the question as to whether the configuration of those hexagrams in 3D or 4D would be suggestive of a higher order of insight and coherence -- and how this might relate in geometrical terms to the C60 array as a truncated icosahedron.

For example, the drilled truncated cube is relatively unique as a configuration of 64 edges in 3D (*Dynamics of Parallel edges of Drilled truncated cubes in 3D: experimental interactive 3D display*), as discussed separately (*Psychosocial Implication in Polyhedral Animations in 3D*, 2015). On the other hand a toroidal fullerene with 64 vertices (as depicted above) could also be used as an exploratory mapping of the 64 hexagrams, with a helical winding and/or a smaolke ring animation (as shown above).

As with such hexagons, the hexagrams of the *I Ching* are specifically understood as composed of lines which oscillate between *ying* and *yang* conditions -- represented as broken and unbroken lines -- thereby transforming any one hexagram into another. From that perspective, it is a challenge to geometry to consider how 64 hexagrams might be most fruitfully arrayed on a sphere, despite the remarkable study of such an arrangement with its consideration of a "Yi-spiral" by *József Drasny* (*The Yi-globe: the image of the cosmos in the I Ching*, 2007). Also intriguing is how any pattern of hexagrams might relate in geometrical terms to the importance associated with the 5-fold *Wu Xing* pattern in Chinese philosophy -- as a pentagram (*Wu Xing: The Five Corners of a Circle*, *Asian Geographic*, 18 March 2020; *Memorable dynamics of living and dying: Hygeia and Wu Xing*, 2014).

Similar consideration could be given to representation as nested fullerenes of the highly esteemed *Sahasrara* (or crown chakra) of Hindu and Buddhist traditions. Known otherwise as the 1,000 petalled lotus, it is held to be composed of 20 layers of approximately 50 petals each (*Satellite Constellation and Crown Chakra as Complementary Global Metaphors?* 2020; *Global Insight from Crown Chakra Dynamics in 3D?* 2020). Could the "20 layers" be understood as corresponding in some way to the 20 hexagons of C60, with the "approximately 50 petals" corresponding to its vertices -- or to those of the C50 fullerene?

The higher order of coherence implied by both traditional symbols bear comparison with discussion of understandings of a metasphere (*Future Global Exodus to the Metasphere: enabling mass migration of humanity to a cognitive frontier*, 2022). Any C60 configuration, or C720 as a "spherical" configuration of even higher order, might be considered from that perspective.

Alternative balls: soccer versus sepak tapraw? As a truncated icosahedron, the C60 fullerene is familiarly compared with the soccer ball of Western origin, as noted above. This is produced by sewing together a pattern of hexagons and pentagons. It can however also be explored in terms of the manner in which the ball used in the Eastern sport of *sepak tapraw* (or kick volleyball) is constructed. This is produced by weaving together six bands which frame pentagonal holes, as noted by Yutaka Nishiyama (*A Sepak Takraw based Molecular Model for C60: a mathematical study of a 60-atom molecule*, *ResearchGate*, 26 April 2015).

From a mathematical perspective, this representation -- using the 60-vertex 4-valence rhombicosidodecahedron -- is proposed in order to capture the positional relations and electron orbits of the 60 atoms comprising the full molecule and to avoid problematic consideration of double-bonding in the truncated icosahedron.

This alternative perspective offers a greater sense of global coherence -- the quest of the argument above -- as noted in the conclusion of that study:

The positions of the atoms better approximate a sphere... Furthermore, we obtain a sense of unity of the molecule overall, not unlike the six bands in a sepak takraw ball. A soccer ball is created by sewing together 12 pentagon (black) and 20 hexagon (white) leather panels. While there is a relationship between adjacent panels, this does not give a sense of unity for the whole. In contrast, a sepak takraw ball is formed from six interwoven bands that trace out great circles, which gives it a much more unified feel. Considering not only the relation between five atoms (five-membered rings) and six atoms (six-membered rings), but the bundles of all 60 atoms into six electron orbits, the latter feels the better suited.

Such a conclusion is a reminder of a curiously unexplored relationship between the configuration of balls and their cultural significance worldwide. The golf ball also derives its construction from the great circles of the truncated icosahedron, thereby locating and isolating its dimples, as indicated in the relevant patent (*Golf ball having surface divided by small circles*, US10058739B2, 2016). As noted by Tom Veilleux (*How do dimples in golf balls affect their flight?* *Scientific American*, 19 September 2005):

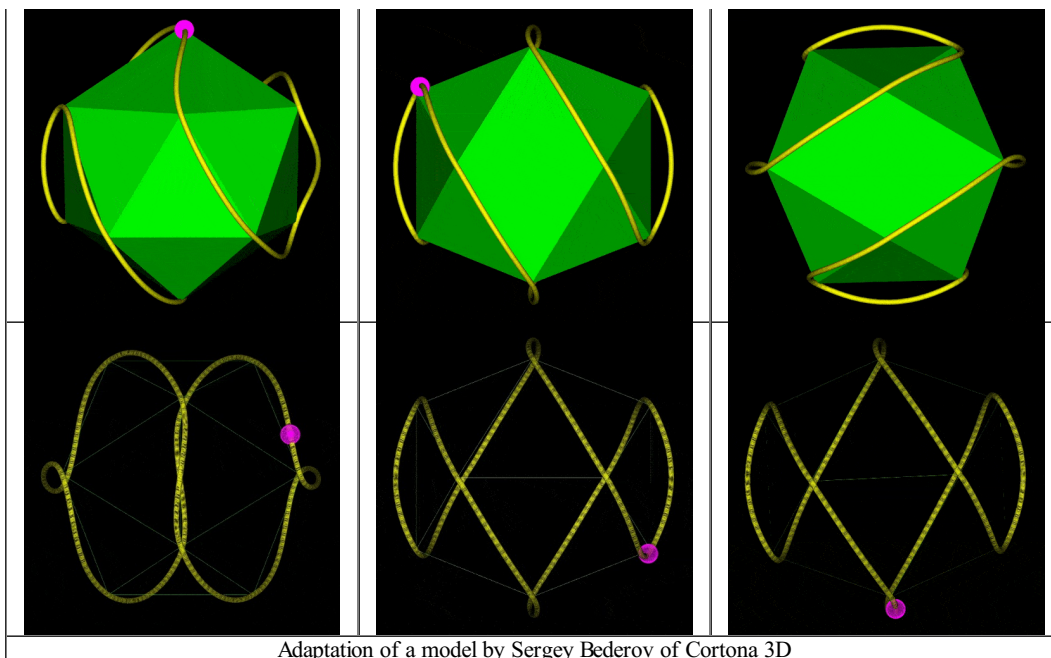
Most golf balls have between 300 and 500 dimples, which have an average depth of about 0.010 inch. The lift and drag forces on a golf ball are very sensitive to dimple depth: a depth change of 0.001 inch can produce a radical change to the ball's trajectory and the overall distance it can fly. Dimples have traditionally been spherical in shape, but it is possible to optimize the aerodynamic performance of other shapes.

Potentially, given metaphorical references to the capacity of a strategic proposal to "fly", there may be significance to be derived from the number of dimples on a golf ball (and its relation to the fullerenes of a corresponding number: 320, 500, 540) Veilleux notes that one optimized dimple design uses hexagons.

The case of the tennis ball and baseball, in the light of the *tennis ball theorem*, is discussed separately (*Comprehension of requisite complexity through game-ball design?* 2021). Speculatively it might then be asked whether the importance attached to balls and their design is indicative of an unconscious comprehension of alternative global forms of strategic viability. In that speculative mode, it could also be provocatively asked whether the design of the familiar cap commonly worn in tennis, baseball and golf offers related insights (*Baseball Cap Implications in the Quest for Global Hegemony*, 2020).

The curve common to the seam pattern of the baseball and tennis ball can be exploratively associated with an icosahedron, as discussed separately and indicated below (*Re-membering the Globe from a Flatland Perspective*, 2020). The wireframe variant recalls representations of the Lissajous curve discussed above. The exercise could be usefully repeated using the truncated icosahedron -- possibly with multiple curves as explored in the previous exercise. The emphasis is on how the relatively comprehensible (and elegant) seam pattern can be understood as "binding together" the somewhat less comprehensible complexity of the C60 fullerene.

Animation of single curve passing through the 12 vertices of an icosahedron (with wireframe variant below)		
Vertical rotation	Horizontal rotation	Rotation in plane of screen



Resonance: Less evident in the **static** depictions of C60 above are the **dynamics** of the resonant bonding between carbon atoms in any hexagon (as a **benzene ring**) and how this might call for more complex renderings of the molecule (*Resonance hybrid of benzene*, *Big Chemical of Encyclopedia*; Tomas Kaiser, et al., *On the 2-resonance of fullerenes*, *SIAM Journal on Discrete Mathematics*, 25, 2011, 4). The relevance of understandings of resonance to the long-standing strategic challenge of the Middle East merits particular consideration (*Resonance, fullerenes and the Middle East?* 2012). That discussion was framed by a focus on engendering connectivity from incommensurable 5-fold and 6-fold conceptual frameworks in the light of the dynamics of spherical geometry. There it was noted that:

Of the greatest importance to the structural viability of the simplest molecules essential to life is the phenomenon of **resonance** whereby the possible bonding between the carbon atoms in the structure takes a dynamic alternating form. It is this dynamic form which is understood as being energetically the most efficient and economic -- giving rise to structures known as **resonance hybrids**. The structure is then understood to be represented by several contributing structures (also called resonance structures or canonical forms).

The recognition of the fullerenes resulted in early investigation of the nature of resonance within C60. For example Harald Friperntinger (*The Cycle Index of the Symmetry Group of the Fullerene C60*, 1996), in a section entitled *The resonance structure of the fullerene C60*, notes that it was already known that the fullerene C60 had 12500 resonance structures (D.J. Klein, et al, *Resonance in C60, Buckminsterfullerene*, *Journal American Chemical Society*, 108, 1986). Friperntinger produces a valuable **tabular summary** indicating those which are essentially different.

There is now a very extensive mathematical and chemical literature on the nature of the connectivity within the truncated icosahedral form, and especially C60. This research engenders visualizations which are potentially of great relevance to exploring structural configurations of psychosocial significance.. an especially helpful example is that of Heping Zhang and Dong Ye (*Cyclical Edge-connectivity and Resonance of Fullerenes*, 2007).

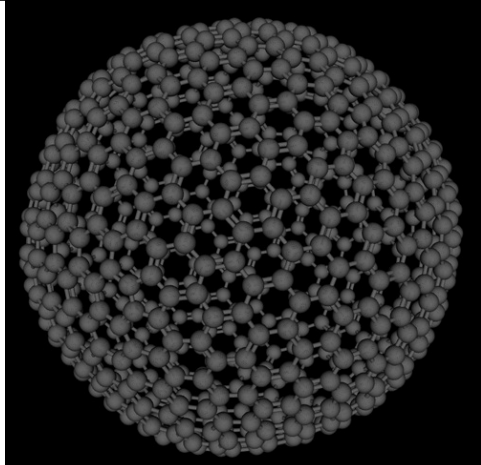
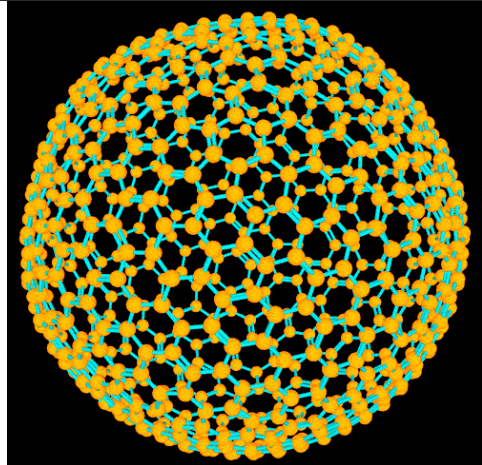
Memorable patterns: Understood more generally, there is the question of how particular patterns -- especially those with a degree of symmetry -- "carry" or "hold" organizational and conceptual coherence, whether for the individual or the collective. The process is most obvious in the case of religious symbols. It evokes particular attention in the case of logos (*Eliciting Insight from Mandala-style Logos in 3D*, 2020). For the individual it can be recognized in how one "gets one's act together" and an understanding of "joined up" thinking.

A significant shift from the classic "magic number" focus of "**7 plus-or-minus 2**" to 60 may well call for recognition of another magic number of relevance to memory organization in the form of "60-plus-or-minus 4" -- rather than 2. However questionable, this would then encompass other patterns: 56 (as 8x7), 63 (as 9x7), and 64 (as 8x8 or 4x16) -- but not 70 (as 5x14 or as 7x10).

Such considerations are especially relevant to popular comprehension of international entities, whether the United Nations or regional bodies (*Coherent Representation of the European Union by Numbers and Geometry*, 2019). This explores the mapping of strategic elements and principles onto the icosahedron and dodecahedron, for example.

Such relevance can be speculatively explored with respect to the organization of NATO (*Envisaging NATO Otherwise -- in 3D and 4D? Potentially hidden faces of global strategy highlighted through polyhedra*, 2017). In discussing NATO as a 4D entity, this notes the vast array of four-dimensional analogues to 3D polyhedra, known as **polychora**, with the possibility of animating 3D cross-sections in real-time and 4D rotation. The C720 fullerene could be used provocatively, as depicted below, as a means of indicating contrasting world views. Both could representations could be enhanced with other design options and animations.

Contrasting use of the C720 fullerene as an indication in 3D of potential global coherence (video)	
"Death Star" coherence of the pattern of US bases?	Alternating conditions of the "global brain"? Integrative representation of the global network of

Representation of full-spectrum hegemony sought through the worldwide deployment of a network of over 600 military bases	creative initiatives
	
X3D version	X3D version

Given the argument for a C60 perspective presented here, and the widespread enthusiasm for 12-fold strategies, there is some irony to any C720 perspective as a product of 12x60 (*Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources*, 2011).

Cognitive vehicles: Given the widespread imaginative appeal of the *Death Star* image of *Star Wars* as a weaponised [starship](#), there is a case for exploring the complementary image from another strategic perspective -- even as a timeship. This follows from the promotion of the image of [Spaceship Earth](#), notably popularized by Buckminster Fuller (*Operating Manual for Spaceship Earth*, 1968). As increasingly stressed, global strategy is now challenged by time factors in relation to resources, it could then be argued that there is a complementarity to be explored (*Embodying a Timeship vs. Empowering a Spaceship*, 2003; *Timeship: Conception, Technology, Design, Embodiment and Operation*, 2003).

With the increasing cognitive role of music as a response to global stress, speculation could fruitfully extend to its role as a vehicle through which communication space and knowledge space are navigated (*Toneship design to enable noonautics by the voices of civilization?* 2015).

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