



# laetus in praesens

Alternative view of segmented documents via Kairos

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## Encoding Coherent Topic Transformation in Global Dialogue

### Memorability of cognitive implication in symmetry-preserving operations on polyhedra

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#### Introduction

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## Introduction

Few would question the importance of dialogue in this period of global crises and poisonous divisiveness. There are of course many approaches to dialogue although it is less evident how their insights translate into the challenges of inter-national, inter-faith, inter-cultural, inter-sectoral, and inter-disciplinary dialogue in practice. There are many calls for dialogue as a substitute for conflict. There is however a sense in which the appreciation of dialogue is somewhat superficial -- reinforced by exercises in which it only takes token and symbolic form as a feature of public relations.

Arguably the investment in dialogue has not reached the "critical mass" necessary for it to effect the difference for which it is held to be vital. It could also be argued that no particular approach to dialogue has emerged as especially relevant to the dimensions of the conflicts so evident at this time -- nor that any has achieved the appreciative support which may be held to be warranted by its promoters.

There is little sense that dialogue of a superior quality is readily evident, of how it might be recognized -- and by whom. Potentially to be upheld as a vital attractor, who would be attracted to that experience? (*Transforming the Art of Conversation: Conversing as the transformative science of development*, 2012).

To provide a context for the following exploration, it is appropriate to indicate some distinctive approaches to dialogue, recognizing that there is relatively little relationship between them -- especially when they are reflective of proprietary interests. Each merits recognition in terms of its strengths and weaknesses, and the contexts in which it is valued. The role of sophisticated technology in enabling it also calls for consideration.

The following exercise is however a **speculative confrontation between approaches to ordering insight which may be indicative of more fundamental approaches to dialogue**. The focus is especially on recognition of what gets transformed in dialogue and how. Most obviously this could be a topic or theme -- typically an issue of some kind. More generally it could be any kind of whole -- even understood as a holon. This could indeed extend to an integrative world view and to any understanding of unity or globality. Clearly any

understanding of an integrated whole could include a sense of personal identity and selfhood -- and how it might be transformed through dialogue.

As a speculative confrontation between potential ways of ordering dialogue, **the particular bias of this exercise is to "follow the numbers" by which articulated insights are ordered** -- as a key to what is tantamount to the confrontation of different "languages". This may be relevant to some approaches to dialogue, but it may be even more evident in methodologies which are considered quite unrelated to dialogue. The assumption here is that in cognitive terms, and irrespective of content, preferred patterns of order bear some relation to the manner in which the human brain articulates the patterns that are recognized, following the arguments of [George Lakoff](#) and [Rafael Núñez](#) (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2000).

It is from such a cognitive perspective that the challenge of dialogue explored here is considered more fundamental than is otherwise assumed -- involving degrees of self-reference and embodiment which have not received due consideration ([George Lakoff and Mark Johnson](#), *Philosophy In The Flesh: the embodied mind and its challenge to western thought*, 1999; [Hilary Lawson](#), *Reflexivity: the post-modern predicament*, 1986). The challenge may be framed in terms of radical recursion, as argued separately (*Radical recursivity of cognitive implications of topology and geometry*, 2021). The reference to geometry and topology points to a related challenge of dialogue, namely how any comprehension of coherence of what is discussed relates to the challenges of memorability -- especially as the complexity of the topic increases

With respect to memorability as it relates to dialogue, this suggests the particular value of interpreting distinctive approaches to order as metaphors of a more fundamental understanding which may well be elusive in terms of conventional modes of descriptive articulation (*Engaging with Elusive Connectivity and Coherence*, 2018). There is even the possibility that engagement in a disparate variety of domains may constitute a form of cognitive displacement through which fundamental forms of dialogue are denatured and embodied in symbolic surrogates.

In that sense, engagement with those domains may offer suggestive metaphors which might include:

- ball-games: in which dialogue can be understood as the movement of a ball between participants, with the manner of throwing or pitching being such as to challenge the capacity of the receiver; the ball being the point of the dialogue
- board-games: involving strategic ploys, as exemplified by chess as a form of dialogue
- polyhedral transformation: given the well-defined transformations to which any given polyhedron may be subject; the polyhedral form being the topic complex of the dialogue -- the memplex
- dance: as a form of interaction readily interpreted as dialogue
- crafts: in which a whole is engendered or transformed, as with pottery, cooking, diamond cutting -- a dialogue with matter

As a speculative exercise it is appropriate to avoid any presumption that it is conclusive. Rather, **the purpose here is to juxtapose potential indications with the suggestion that they may constitute complementary insights into a more fundamental approach to dialogue**. In particular, if each approach is considered primarily as a metaphor, the question is whether it is in metaphorical terms that the relation between them may be clarified in indicating the nature of a more elusive fundamental approach (*Transcendent Integrity via Dynamic Configuration of Sub-understandings?* 2014).

The exploration follows from previous exercises (*Forthcoming Major Revolution in Global Dialogue: challenging new world order of interactive communication*, 2013; *Multi-option Technical Facilitation of Public Debate: eliciting consensus nationally and internationally*, 2019).

In their discussion of the implications of new technologies, these noted the future of AI-mediated dialogue: *AI Dialogue: beyond the Turing Test to the Buber Test?*. Those earlier exercises were followed by others in which the role of AI was especially emphasized (*AI-enhanced Future of the Conferencing Process: meeting design through interactive incorporation of participants and content*, 2020; *From Zoom Organization to Zome Configuration and Dynamics*, 2020). The latter focused especially on polyhedral configurations of relevance to the dialogue process.

## Overview of a variety of approaches to dialogue and conversation

A valuable overview of a number of approaches to dialogue is provided by [Pioneers of Change Associates](#) (*Mapping Dialogue: a research project profiling dialogue tools and processes for social change*. 2006) -- with "mapping" primarily understood as providing an overview of the range of initiatives -- rather than their use of mapping techniques. The intention with that project was to map a number of dialogue approaches relevant for social change practitioners, and to create an initial toolbox for Nelson Mandela Foundation and others intending to facilitate or convene dialogue processes. The project compared: Appreciative Inquiry, Change Lab, Circle, Deep Democracy, Future Search, Open Space, Scenario Planning, School for Peace, Sustain Dialogue, and World Cafe.

**Approaches:** Distinctive approaches and understandings of dialogue are implicit in the following (in no particular order and potentially overlapping):

- **Dialogue analysis** (*International Association for Dialogue Analysis*):
  - **Conversation theory**: a cybernetic and dialectic framework that offers a scientific theory to explain how interactions lead to "construction of knowledge", or "knowing": wishing to preserve both the dynamic/kinetic quality, and the necessity for there to be a "knower". This work was proposed by [Gordon Pask](#) in the 1970s ([Agoston Nagy](#), *Learning through Conversation*, *Medium*, 24 July 2017; [Andrew Pickering](#), *Cybernetics and The Mangle: Ashby, Beer Aand Pask*, *Social Studies of Science*, 32, 2002, 3)
  - **Conversation analysis**: an approach to the study of social interaction, embracing both verbal and non-verbal conduct, in

situations of everyday life.

- **Relational dialectics**: a communication theory which could be interpreted as “a knot of contradictions in personal relationships or an unceasing interplay between contrary or opposing tendencies.
- **Discourse analysis**: an approach to the analysis of written, vocal, or sign language use, or any significant semiotic event.
- **Dialogical analysis**: or more precisely dialogical interaction analysis, refers to a way of analyzing human communication which is based on the theory of **dialogism**. This makes several assumptions. It assumes that human communication entails the interaction of diverse perspectives and is embedded in a socio-historical context, that the meaning of a communication can be different to the various participants, that it is important to examine the consequences of a communication, and that each participant in a communication is, to some degree, orienting to the orientation of the other. Dialogical analysis is an interpretative methodology which closely analyzes spoken or written utterances or actions for their embedded communicative significance. It is distinct from discourse analysis and conversation analysis because its focus goes beyond the question of how people speak and what they achieve by speaking. Dialogical analysis uses dialogue as a metaphor for understanding phenomena beyond communication itself, such as the self
- **Transactional analysis**: a psychoanalytic theory and method of therapy wherein social transactions are analyzed to determine the ego state of the communicator (whether parent-like, childlike, or adult-like) as a basis for understanding behavior. An approach popularized by Erice Berne (*Games People Play*, 1964) which clarifies its relevance to dialogue
- **Dialogue for learning**:
  - **Dialogic pedagogy**: a theory and practice of teaching in which dialogue is central. Teachers and students are in an equitable relationship and listen to multiple points of view.
  - **Dialogic learning**: is learning that takes place through dialogue. It is typically the result of **egalitarian dialogue**; in other words, the consequence of a dialogue in which different people provide arguments based on validity claims and not on power claims
  - Instructional conversation design: notably as articulated by Rocci Luppichini (*Handbook of Conversation Design for Instructional Applications*. IGI Global, 2008). This recognizes the role conversation in instruction, particularly in the design and development of technologically advanced educational environments.
  - Task-oriented conversations: Ananlada Chotimongkol *Dialog Structure for Task-Oriented Conversations* by Language Technologies Institute School of Computer Science Carnegie Mellon University, 2003
- **Decision-making dialogue**:
  - **Structured dialogue**: as originally developed by Alexander Christakis and John Warfield (*A Brief History of Interactive Management and Structured Dialogue*; Alexander Christakis, *An Epic Learning Journey: from the Club of Rome to Dialogic Design Science and Demosophia* (*Social Systems and Design*, 2014)
    - *Structured Democratic Dialogue Process (SDDP)* (*Culture and Cultural Heritage*)
    - *Structured Dialogic Design (SDD)* (NCDD, 24 December 2008)
  - Deliberative dialogue:
    - Martha McCoy and Patrick Scully: *Deliberative Dialogue to Expand Civic Engagement: What Kind of Talk Does Democracy Need* (*National Civic Review*, 92, 2002. 2)
    - Peter McBurney, David Hitchcock and Simon Parsons (*The Eightfold Way of Deliberation Dialogue* (2007). Deliberation dialogues occur when two or more participants seek to jointly agree an action or a course of action in some situation. The resulting protocol is suitable for dialogues between computational entities, such as autonomous software agents.
  - Strategic conversation: to which there are many references, notably including Alex Lascarides (*Strategic Conversation, Proceedings of the SIGDIAL Conference: 12th Annual Meeting of the Special Interest Group on Discourse and Dialogue*, 2011)
  - Conflict resolution: as notably promoted by UNDP (*Why dialogue matters for conflict prevention and peacebuilding*, 2009)
    - Johan Galtung: *Conflict Transformation by Peaceful Means: the Transcend Method* (United Nations Disaster Manual Training Program, 2000)
    - **Conflict Dialogue Project**: Supports a culture of resolving conflicts non-violently in all sections of society.
    - *Dialogue and Facilitation* (Mediators Beyond Borders)
    - Susan Stigant: *National Dialogues: A Tool for Conflict Transformation?* (United States Institute of Peace, 2015)
    - Peter M. Kellett (*Conflict Dialogue: working with layers of meaning for productive relationships*, 2006)
- **Conversation as an art** -- on which there are numerous references, including:

- [Michel de Montaigne](#): *On the Art of Conversation, Essays*, 3 (1588); *Of the Art of Conference*, translated by Charles Cotton (circa 1686)
- Patricia Romney: *The Art of Dialogue (Animating Democracy)*
- Noelle Aarts: *The Art of Dialogue* (Wageningen University, 2015)
- Margaret Shepherd: *The Art of Civilized Conversation: A Guide to Expressing Yourself with Style and Grace* (2005)
- Melanie Chan: *The Dying Art of Conversation – has technology killed our ability to talk face-to-face?* (*The Conversation*, 29 March 2019)

**Mapping dialogue:** Somewhat distinct from the above, but variously implying a relevance to dialogue, are the variety of efforts to visualize a dialogue through some kind of mapping process, either during the event or thereafter. Their use tends to be constrained by intellectual copyright and licensing arrangements. These include (in no particular order):

- [Discourse Mapping Resources](#) (Digiterp Communications): Discourse mapping is a term used for both the process of analyzing texts in interpreter education and skill development and the features of an interpretation that use linguistic features to create a connected and cohesive product.
- [Dialogue Mapping™](#) (CogNexus Institute)
- [DebateGraph](#): a web-platform for visualizing and sharing networks of thought -- and opening reasoning and action to collaborative learning and iterative improvement.
- [Argument Mapping](#) (Austhink): produces [Rationale™](#), claimed to be the first high-quality software for [argument mapping](#), the diagramming of reasoning and argument.
- Visual Minutes (also known as Graphic Recording or Scribing): A real-time, illustrated record of conferences, events, meetings and debates, involving the art of listening and capturing words in a dynamic, creative and exciting way. (*What are Visual Minutes? Creative Connection*; *Visual Minutes from the Think Tank, Board Intelligence*; *Visual Minutes, Woven Ink*)

**Knowledge management.** This is the process of creating, sharing, using and managing the knowledge and information of an organization. It refers to a multidisciplinary approach to achieve organisational objectives by making the best use of knowledge. It tends to be less directly relevant to the process of dialogue, but it may enable it.

- [Knowledge mapping](#): This is the processes and tools to portray a perspective of the players, sources, flows, constraints and sinks of knowledge within an organization. Such maps are organized using various interconnected nodes to make it easy to find out where to look for information. Various software applications may enable their construction ([Top Knowledge Management Software](#)).
- [Mind mapping](#): Diagram produced to organize information visually. Such maps can be produced collaboratively by teams and in dialogue processes.
- [InfraNodus](#): enables insight to be generated using Text Network Analysis InfraNodus, namely a network thinking tool that reveals the relations and patterns in data. The built-in insight recommender provides a real-time advice on the best ways to develop the knowledge
- Knowledge cartography: A discipline of visually mapping the conceptual structure of ideas, such as the connections between issues, concepts, answers, arguments and evidence (Simon Buckingham and Alexandra Okada, [Knowledge Cartography for Open Sensemaking Communities](#), *Journal of Interactive Media in Education* 2008, 1). The cognitive process of externalising understanding clarifies the grasp of the situation, as well as communicating it to others as a network that invites their contributions.
- Epistemic network analysis: A theory-based approach to digital learning, known as [epistemic games](#), to address the challenge of assessment in new media learning environments (David Williamson Shaffer, [Epistemic Network Analysis: a prototype for 21st Century assessment of learning](#), *International Journal of Learning and Media*, 2010).
- Natural language coding: This may be used for analysis of dialogues (Lars Ahrenberg, Nils Dahlbäck and Arne Jönsson, [Coding Schemes for Studies of Natural Language Dialogue](#), January 1997). Dialogue structure is coded in terms of segments consisting of moves belonging to general illocutionary types, such as initiative and response, and being further specified as to their topical domains. Focus structure is coded in terms of a number of focal parameters, which may differ from one type of dialogue to another.

**What is missing?** As suggested above, there would appear to be a remarkable need for intense research on dialogue as a means of addressing the divisive issues of the times. There are seemingly few reviews on progress in dialogue, other than from within the framework of one of the issues named above. There is little dialogue between those approaches -- each claiming a degree of success -- despite an evident lack of global uptake in response to the challenges of the times. What is missing?

It is curious to note the intense interest worldwide in the movement of a ball in ball games, and those with the skills in that regard. As champions they are exorbitantly rewarded. In the case of dialogue, there is relatively little interest in the capacity to handle the central point of a debate, or to recognize the skills involved -- irrespective of the strategic importance of the issues.

Skill in repartee may be acknowledged, but there would appear to be no context in which dialogue of the highest quality is recognized and appreciated in relation to its relevance to strategic issues. Unlike restaurants, hotels and ball-games, conferences are not evaluated according to the quality of their dialogue? Are there any conferences of "5-star" quality offering models to which those engaged in dialogue of lower quality could aspire?

## Uniquely indicative encoding of processes of educational dialogue

There is no lack of research interest into [Spoken Language Understanding](#) (SLU) as a key component of goal oriented dialogue systems that would parse user utterances into semantic frame representations -- as reviewed by Ankur Bapna, et al. ([Sequential Dialogue Context Modeling for Spoken Language Understanding](#), *Google Research*, 2017). Of particular interest is that combining information technology and philosophy (Peter McBurney, David Hitchcock and Simon Parsons, [The Eightfold Way of Deliberation Dialogue](#), 2007).

Of potential relevance is the vision of the Intelekt, project as articulated by [Pierre Lévy](#) ([Vision: Semantic Interoperability and the Future of Artificial Intelligence](#)). This foresees how a common semantic code would make it possible to achieve a de-fragmentation of the global memory and an integration of symbolic and statistical AI. Noted is the lack of semantic interoperability between sectors relating to knowledge management.

Perhaps appropriately, the approach of most relevance to that explored below is that relating to codification of dialogue in classes of children, notably of a younger age. This is the result of the research of a cluster of related researchers, as described in the most recent overview by Sara Hennessy and colleagues ([Coding Classroom Dialogue: methodological considerations for researchers](#), *Learning, Culture and Social Interaction*, June 2020): The authors introduce their own adaptation of an earlier coding Scheme for Educational Dialogue Analysis or SEDA: (Sara Hennessy, Sylvia Rojas-Drummond, et al. [Developing a coding scheme for analysing classroom dialogue across educational contexts](#). *Learning, Culture and Social Interaction*, 9, 2016).

They then describe how SEDA was subsequently reformulated and condensed into a new version, the Cambridge Dialogue Analysis Scheme or CDAS (Maria Vrikki, Lisa Wheatley, Christine Howe, Sara Hennessy and Neil Mercer, [Dialogic practices in primary school classrooms](#), *Language and Education*, 33, 2019, 1) and used by some of its originators for a new research purpose: a large-scale investigation of the relationship between teacher-student dialogue and student learning and attitudinal outcomes (Christine Howe, et al, [Teacher-student dialogue during classroom teaching: Does it really impact upon student outcomes?](#) *Journal for the Learning Sciences*, 28, 2019, 4-5).

In their 2020 summary the authors share their approach to encourage others interested in developing, adapting or critiquing coding approaches, "with the aim not of offering any kind of recipe but of sharing reflections on the strengths and boundaries of approaches to coding in general and on the development of our own tools".

As presented there, SEDA contains 33 categories (in a condensed table subject to copyright), segmented at the communication event (meso-) level as well as the micro-level. The 33 codes are clustered into 8 groups:

Invite elaboration or reasoning (I1-I6)	Express or invite ideas (E1-E2)	Connect (C1-C4)
Positioning and coordination (P1-P3)	Make reasoning explicit (R1-R4)	Guide direction of dialogue or activity (G1-G5)
Reflect on dialogue or activity (RD1-RD3)	Build on ideas (B1-B2)	

A T-SEDA Collective elaborated a variant termed the T-SEDA dialogue coding scheme which it is claimed is now used globally by practitioners from pre-school years to higher education. It was the product of research involving: Ruth Kershner, Sara Hennessy, Elisa Calcagni, Farah Ahmed, Victoria Cook, Laura Kerslake, Lisa Lee and Maria Vrikki of the University of Cambridge Faculty of Education, and Nube Estrada and Flora Hernández of the National Autonomous University of Mexico ([Teacher Scheme for Educational Dialogue Analysis](#) (T-SEDA, v.8a resource pack, University of Cambridge, 2021). Use of this, together with its associated video and other resources, is not explicitly constrained by copyright.

An earlier study (Maria Vrikki, et al, [Dialogic practices in primary school classrooms](#), *Language and Education*, 33, 2019, 1) included a somewhat similar table ([Descriptions of dialogic moves codes and agreement levels](#)). However this took the form of a 12-category scheme and rating scales.

## Controversies and consensus in dialogue research

Controversies regarding the research on pedagogical dialogue (of which that cited is a primary example) have given rise to an initiative to map both minor as well as major points of contention, and to identify areas of consensus. This took the form of a multi-disciplinary discussion panel at the 2018 Biennial EARLI Conference on Argumentation, Dialogue and Reasoning (Jerusalem, 2018). Christa Asterhan formulated four provocative questions, each chosen to address a key issue (past, present or future) in research on dialogic education. Four leading scholars from different research disciplines, theoretical backgrounds and/or research communities were invited to react to these provocations (Christa Asterhan, Christine Howe, Adam Lefstein, Eugene Matusov, Alina Reznitskaya [Controversies and consensus in research on dialogic teaching and learning](#) *Dialogic Pedagogy*, 8, 2020).

As described in introducing the responses:

Scholarly interest in dialogic pedagogy and classroom dialogue is multi-disciplinary and draws on a variety of theoretical frameworks. On the positive side, this has produced a rich and varied body of research and evidence. However, in spite of a common interest in educational dialogue and learning through dialogue, cross-disciplinary engagement with each other's work is rare. Scholarly discussions and publications tend to be clustered in separate communities, each characterized by a particular type of research questions, aspects of dialogue they focus on, type of evidence they bring to bear, and ways in which standards for rigor are constructed. In the present contribution, we asked four leading scholars from different research traditions to react to four provocative statements that were deliberately designed to reveal areas of consensus and disagreement<sup>1</sup>. Topic-wise, the provocations related to theoretical foundations, methodological assumptions, the role of teachers, and issues of inclusion and social class, respectively.

Unfortunately those same arguments could be applied to the broader range of approaches to dialogue, of which some are indicated above. The four "provocative statements" (2018) were:

- **Provocation # 1:** The terms dialogic pedagogy and/or dialogic learning are used quite often and sometimes quite loosely in our field. It seems that different scholars do not necessarily mean the same thing.
- **Provocation # 2:** Dialogue is too complex to be genuinely understood and captured by quantitative statistical methods. Quantitative studies of dialogue and argumentation impress at first sight because of their numbers, tables and figures and the enormous amount of work that has been put into developing coding schemes, training coders, achieving reliability and analyzing hundreds of turns and features. But I have yet to meet a quantitative study that gave me real insight into the workings of learning through dialogue and/or argumentation
- **Provocation # 3:** Issues of social class, (in)equity, and accessibility to linguistic resources have been neglected in this research field. Enculturation into a particular, monolithic type of "ideal" academic discourse that is mainly based on rational reasoning and argumentation (typical of high SES families of particular cultures) disregards and disrespects the different registers of discourse minoritized students come to class with and alienates them from participation in classroom dialogue. To what extent does our work challenge or reify the implicit and explicit ways that schools fit minoritized students into singular or monolithic forms of thinking and discourse?
- **Provocation # 4:** In spite of extensive evidence about the benefits of dialogic pedagogy in classrooms, the impact of teacher professional development efforts to change classroom practices in this direction has overall been negligent. Why is it so hard? What, according to you, are the main obstacles or reasons for this failure?

It is appropriate to note that the SEDA coding was not applied to the panel discussion -- not being a dialogue in its own right. Additionally it is the case that whilst the the SEDA approach has indeed been extended to higher education, it has seemingly not been applied to dialogue between researchers in higher education.

There is some irony to the method employed by Christa Asterhan in that it recalls that devised by Gordon Pask (originator of conversation theory) and Stafford Beer (management cybernetician) on the occasion of the 1980 Conference of the Society for General Systems Research (*Metaconferencing Possibilities: discovering people / viewpoint networks in conferences*, 1980). There the participants were each invited to formulate one question which they believed would split the conference 50/50. A probable communication network of affinities of participants and topics was calculated from the results.

## Uniquely systematic approach of potential relevance to dialogue

Notably in the light of the questionable focus on dialogue through Western academic frameworks, there is a case for recognizing the possibility of considerable insight into forms of dialogue in other cultures and traditions.

For Kimberly Hutchings (*Dialogue between Whom? The Role of the West/Non-West Distinction in Promoting Global Dialogue in IR, Millennium: Journal of International Studies*, 39, 2011, 3):

There is a politics to the West/non-West distinction that is bound up with predominant models for dialogue in IR; rethinking these models of dialogue implies a new politics, and therefore also, I will suggest, a move away from the West/non-West binary as a way of characterising the participants in dialogic exchange oriented towards the expansive transformation of disciplinary imaginaries

For Kamila Pieczara (*Two Modes of Dialogue in IR: testing on Western versus Non-Western engagement with IR theory, Millennium: Journal of International Studies*, 2010, 9):

Dialogue in International Relations (IR) is neither a unitary concept nor an undifferentiated process; the aim of this paper is to separate out two principal modes of dialogue, their attributes and differing efficacy in contributing to IR theory (IRT)... The applied analysis indicates that dialogue can be most fruitful via engagement with existing theory from Asian perspective. This finding could not be achieved when treating dialogue as a uniform practice.

One highly systematic organization of relevant transformation processes is articulated in traditional Chinese culture as the *I Ching* (*Yi Jing*). This encodes 64 distinct modalities with which change can be associated in individuals or groups, with each of the 64 encoding 6 potential transformations to the other 63 -- making a total of 6x64 transformations.

The abstraction of the coding system and its distinctions is reframed through metaphor to render their implications comprehensible to a

degree -- given the elusive nature of what is described. There are many translations into English of the Chinese text. It is however possible, experimentally, to "interpret" or adapt the descriptions of what is described in order to apply it more specifically to themes which have become a preoccupation in Western civilization (*Transformation Metaphors derived experimentally from the Chinese Book of Changes (I Ching) for sustainable dialogue, vision, conferencing, policy, network, community and lifestyle*, 1997).

Of relevance to the argument here, the 64 distinctions have been traditionally clustered as 8 "houses" (*Organization of I Ching hexagrams in terms of traditional "houses"*, 1995). Links from each hexagram are provided to an [index](#) from which a range of commentaries may be selected with respect to: dialogue, vision, conferencing, policy, networking, community and lifestyle. In the table below the link gives direct access to the metaphorical description relating to dialogue. The set of 64 is presented separately in list form (*Documents relating to Sustainable Dialogue (adapted from I Ching)*, 2006).

Organization of transformations of dialogue clustered into 8 traditional houses -- adapted from the <i>I Ching</i> (links provide access to metaphorical interpretation of original commentary)							
House of creative	House of abysmal	House of keeping still	House of arousing	House of gentle	House of clinging	House of receptive	House of joyous
Possession in great measure [14]	Army [7]	Development [53]	Following [17]	Work on what has been spoiled [18]	Fellowship with men [13]	Holding together [8]	Marrying maiden [54]
Progress [35]	Darkening of the light [36]	Inner truth [61]	Preponderance of the great [28]	Corners of the mouth [27]	Conflict [6]	Waiting [5]	Preponderance of the small [62]
Splitting apart [23]	Abundance [55]	Treading [10]	Well [48]	Biting through [21]	Dispersion [59]	Break-through [43]	Modesty [15]
Contemplation [20]	Revolution [49]	Opposition [38]	Pushing upward [46]	Innocence [25]	Youthful folly [4]	Power of the great [34]	Obstruction [39]
Standstill [12]	After completion [63]	Decrease [41]	Duration [32]	Increase [42]	Before completion [64]	Peace [11]	Influence [31]
Retreat [33]	Difficulty at the beginning [3]	Taming power of the great [26]	Deliverance [40]	Family [37]	Caldron [50]	Approach [19]	Gathering together [45]
Coming to meet [44]	Limitation [60]	Grace [22]	Enthusiasm [16]	Taming power of the small [9]	Wanderer [56]	Return [24]	Oppression [47]
Creative [1]	Abysmal [29]	Keeping still [52]	Arousing [51]	Gentle [57]	Clinging [30]	Receptive [2]	Joyous [58]

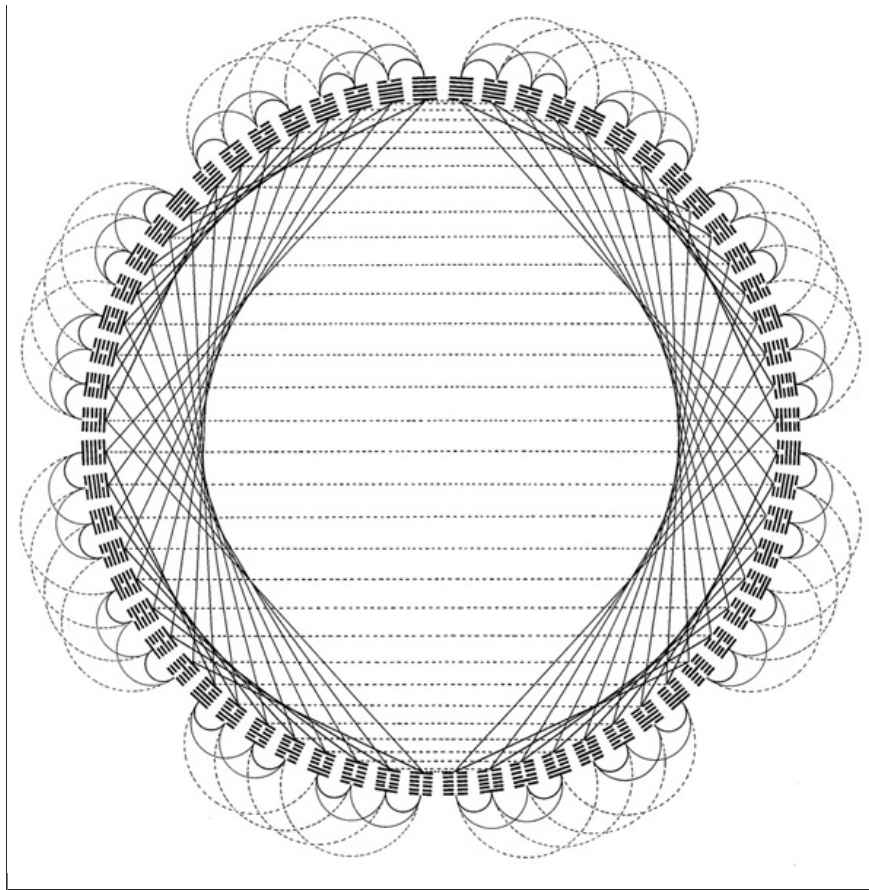
This eightfold clustering can be understood as potentially corresponding to the choice of an eightfold clustering in the SEDA initiative mentioned above.

The dynamics identified between the transformations derive from the systematic encoding of each hexagram which determines the change to an alternative condition. Historically it was this pattern of transformations which was influential in the original insight of [Gottfried Leibniz](#) that subsequently gave rise to the binary coding fundamental to modern computing.

In contrast to the more widely known tabular configurations of the hexagrams, the circle of [Shao Yong](#) (1011-1077) -- or the *I Ching* hexagram circle -- was an influential feature of the communication to Leibniz in 1701 (James A. Ryan, *Leibniz' Binary System and Shao Yong's "Yijing"*, *Philosophy East and West*, 46, 1996, 1). Features of the configuration are discussed separately (*Diagram of 384 Relationships between I Ching Hexagrams*, 1983; *Bagua and the sequence of 64 hexagrams*, *Shanghai Daily*, 20 December 2015).

Adapted to dialogue, the original circular pattern can be presented as follows (*Interrelationships between 64 Complementary Approaches to Dialogue*, 2007).

Circular pattern of <i>I Ching</i> hexagrams adapted to dialogue
Tentative names become evident if the cursor is placed over each; depending on the browser, clicking on them provides access to descriptions of them. (See also <a href="#">animated versions</a> ) 7 Equivalent maps are available for: <a href="#">Vision</a> / <a href="#">Conference</a> / <a href="#">Policy</a> / <a href="#">Network</a> / <a href="#">Community</a> / <a href="#">Lifestyle</a>



It could be argued that Western tradition developed a comparable articulation based on 72 distinctions. Unfortunately its credibility from a scientific perspective is not reinforced by a coding system equivalent to that which proved so influential in the development of binary logic and computers -- and is therefore deprecated to a far greater degree. It is discussed separately, but its implications are not developed here in relation to the following argument (*Variety of System Failures Engendered by Negligent Distinctions: mnemonic clues to 72 modes of viable system failure from a demonic pattern language*, 2016; *Engaging with Hyperreality through Demonique and Angelique? Mnemonic clues to global governance from mathematical theology and hyperbolic tessellation*, 2016; *Otherness, anti-otherness -- and elusiveness*, 2018).

## Metaphors of dialogue and dialogue through metaphor

In contrast to non-figurative discourse, use of metaphor in dialogue vokes the imagination to elicit a creative response (*Dreams that Dialogue is Made Of: radical reframing offering neglected degrees of freedom*, 2015), The role and potential of dialogue through metaphor is discussed separately (*Guidelines towards Dialogue through Metaphor*, 1993) in a collection of papers and notes, problems and possibilities on the new frontier of high-risk gatherings concerning social development (*Towards Transformative Conferencing and Dialogue*, 1991).

Other arguments include:

- David Demeritt Sarah Dyer: *Dialogue, metaphors of dialogue and understandings of geography* (*Royal Geographical Society*, 34, 2002, 3)
- Shirley Ann Freed: *Metaphors and Reflective Dialogue Online* (Andrews University)
- Sarah Davey Chesters *Developing the Socratic Classroom: metaphors of engagement in dialogue* (*The Socratic Classroom*, 2012)
- John H. Quirk, Jr: *Meaning through Metaphor: visual dialogic and the picturing of abstractions* (University of Massachusetts, 2008)
- Line Brandt: *Dance as Dialogue: metaphorical conceptualization and semantic domains* (*Signata: Annals of Semiotics*, 2015, 6)
- Anthony Judge: *Metaphor as fundamental to future discourse*, (*Futures*, 84, November 2016)

Individual metaphors may prove of less value when a set of contrasting metaphors offers a complementarity of perspective, as argued separately (*Metaphorizing Dialogue to Enact a Flow Culture: Transcending divisiveness by systematic embodiment of metaphor in discourse*, 2019). This includes sections on:

Metaphorizing -- beyond one-off usage  
Sustaining dialogue through metaphor?  
Indicative precedents of metaphorizing skills  
Discourse and debate reframed as cognitive combat through metaphor?  
Integrity of metaphorizing framed by complementarity between alternatives

Requisite metaphoric "circumlocution" avoiding disruptive disagreement  
Sustainable discourse framed metaphorically as "orbiting"  
Metaphorizing as artful indulgence in misplaced concreteness?  
Re-imagining: metaphorizing, metamorphizing and cognitive shapeshifting  
Sustainable discourse: longest conflict versus longest conversation?

## Re-cognition of N-fold sets of "modes", "ways", "moves" and "ploys"

The question underlying this exploration of dialogue is: **why is there a tendency to distinguish sets of particular size in organizing concepts, strategies and ways of doing anything?**?. Puzzling in this connection is how rare it is to see any explanation as to why information is clustered in a particular way -- rather than another. In the examples cited above with respect to pedagogical dialogue, one approach took the form of an 8-fold cluster, whilst another offered a 12-fold cluster. An earlier discussion of the phenomenon included many examples from a wide range of domains (*Patterns of N-foldness: comparison of integrated multi-set concept schemes as forms of presentation*, 1980; *Examples of Integrated, Multi-set Concept Schemes*, 1984)

It is in this sense that it is interesting to explore the contrasting implications of:

- 1-foldness: variously understood as the "one way" or the "only way", or some understanding of unity -- exemplified by the notorious **TINA** assertion of Margaret Thatcher. This could be caricatured by the strategic slogan: *If all you have is a hammer, then everything looks like a nail*
- 2-foldness: typically exemplified by "us versus them", the extensive commentary on binary and dualistic thinking, and the challenge of any alternatives
- 3-foldness: potentially recognized by contrast as a "third way", **triadic logic**, or through the problematic dynamics of the "eternal triangle" (*Triangulation of Incommensurable Concepts for Global Configuration*, 2011)
- 4-foldness: most obviously through the variety of 4-quadrant conceptual frameworks, including the **SWOT** strategic planning technique
- 5-foldness: highlighted traditionally by the Pythagoreans with respect to **Hygeia**, and by the *Wu Xing* model of Chinese philosophy (*Memorable dynamics of living and dying: Hygeia and Wu Xing*, 2014). It is of significance to navigation in terms of the mathematics of the **Pentagramma Mirificum** (*Global Psychosocial Implication in the Pentagramma Mirificum: clues from spherical geometry to "getting around" and circumnavigating imaginatively*, 2015), and of strategic significance by **Peter Senge** (*The Fifth Discipline: the art and practice of the learning organization*, 1990).

Examples of unexplained preferences for other patterns include:

- 5-foldness: *Clues to systemic implications of 5-fold cognitive organization?* (2019)
- 8-foldness: notably including the Buddhist **Noble Eightfold Path**, and its "inspiration" for physics and policy analysis, as well as the **Millennium Development Goals** of the UN
- 10-foldness: *Habitual use of a 10-fold strategic framework?* (2020)
- 12-foldness: *Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources* (2011)
- 16-foldness: *Deprecation of potential correspondences: 16-fold patterns?* (2019); 16 global challenges identified by Policy Horizons Canada (*The Next Generation of Emerging Global Challenges, Horizons*, 19 October 2018).
- 17-foldness: **Sustainable Development Goals** of the UN
- 20-foldness: *Requisite 20-fold Articulation of Operative Insights? Checklist of web resources on 20 strategies, rules, methods and insights* (2018)
- 30-foldness: **Universal Declaration of Human Rights**

With respect to dialogue, this unexplained tendency is evident in assertions regarding dialogue, for example:

- *The Five Keys to Strategic Conversations* (Center for Coaching)
- *The 7 Habits of Highly Effective Dialogue* (ProjectSmart)
- *Mastering the Art of Conversation: 7 Steps to Being Smooth* (Time)
- *The Eightfold Way of Deliberation Dialogue*
- *Ten Characteristics of Effective Strategic Conversations* (Couravel)
- *10 Principles of Dialogue* (Scouts: World Organization of the Scout Movement)
- *10 ways to have a better conversation* (TED Talk)
- *12 Conversation Strategies worth Teaching* (ELT Planning)
- *The 12 Competencies of Mindful Compassionate Dialogue* (WiseHeart)

The implied question could be enriched by interpretation in terms of **arity** -- understood as the number of arguments or operands taken by a function or operation in logic, mathematics, and computer science. Examples distinguished include: **nullary**, **unary**, **binary**, **ternary**, **quaternary**, **quinary**, **senary**, **septenary**, **octonary**, **novenary**, and **denary**. These suggest a degree of relationship to the lemmas of increasing rarity by which discourse and governance may be challenged: **dilemma**, **trilemma**, **tetralemma** (or **quadrilemma**), **pentalemma** (or **quintilemma**), **hexalemma**, **heptalemma**, and **octalemma** (*Cognitive glass ceilings impeding integrative comprehension*, 2019).

Especially intriguing is any sense that any set of modalities is complete (and exhaustive) -- rather than incomplete. There is obviously little enthusiasm for extending a 10-fold set to 11, or a 12-fold set to 13 -- or reducing it to 10. It is in this sense that any 8-fold clustering of dialogic modalities merits a degree of challenge, especially when it clusters some 33 categories (rather than 32 or 30).

Of interest is then what might constrain the extension of a set and how the extension might reveal unsuspected constraints. Examples include:

- use of an 8x8 board game, as with chess, within which experts recognize how they are constrained to 20 opening moves. How many "opening moves" can be recognized within what kind of understanding of engaging in dialogue?
- much has been made of discovering the minimum number of moves required to solve a 3x3x3 Rubik Cube. However it is only recently that it has been determined that the minimum number of moves in which it can be solved is 20. The process has been described in the following terms by those involved: *God's Number is 20; Twenty as "God's number"?* (2018).

- constraints on jury size and the possibility of changing any commitment to a jury of 12 (*Fundamental operational concepts, jury size, financial ratios and "the Greeks"*, 2018).

With respect to dialogue, there is considerable familiarity with the lore of viable collective dialogue, namely that it should be limited to 7, as tends to be commonly accepted in terms of the need for requisite variety:

- Marcia Blenko, et al: *Decide and Deliver: 5 Steps to Breakthrough Performance in Your Organization* (Harvard Business Press, 2010), as reviewed by Sheila Margolis (*What Is the Optimal Group Size for Decision-making?* Workplace Culture Institute) and Sean Silverthorne (*Rule of 7: The Ideal Work Group Size*, CBS News, 28 September 2010)
- Bob Sutton: *Why Big Teams Suck: Seven (Plus or Minus Two) Is the Magical Number Once Again* (March 2014)
- Paul Axtell: *The Most Productive Meetings Have Fewer Than 8 People* (Harvard Business Review, 22 June 2018)
- Ritch Macefield: *How To Specify the Participant Group Size for Usability Studies: A Practitioner's Guide* (*Journal of Usability Studies*, 5, 1)
- Frank T. McAndrew: *What Is the Right Size for a Group Conversation?* (*Psychology Today*, 18 July 2017)
- Jaimie Arona, et al: *Something to Talk About: are conversation sizes constrained by mental modeling abilities?* (*Evolution and Human Behavior*, 37, 2016, 6)

The question can be explored more generally in terms of the constraints on the articulation of any viable set of strategies (*Eliciting a 12-fold Pattern of Generic Operational Insights: recognition of memory constraints on collective strategic comprehension*, 2011).

## Potential relevance of unrelated patterns of order to dialogue transformation

Whilst the traditional Chinese 64-fold pattern can be deprecated as artificial from the perspectives of many conventional academic disciplines, the quest in what follows is for systematic patterns of order which might indeed be understood as suggestive of such a pattern -- if only as cognitively constrained.

The bias cultivated here is the sense in which the theme of any dialogue, or indeed the individual or group central to the experience of that dialogue with any "other", is effectively positioned at some "virtual" centre of the potential pattern of transformations. The individual or the topic has the potential of being interactively transformed to any peripheral condition. Understood in terms of a ball game like tennis, that focus in variously passed across the net in engagement with otherness -- from which it may be trickily returned, transformed in some way.

A second bias is in favour of well-elaborated formalism as a means of limiting the tendency to unstructured modes of discourse in which any sense of coherence is elusive. Arguably there is a strong case for such a focus, given the anticipated role of AI in dialogue with humans (as noted above).

The approach here therefore relies to a degree on mathematics and the role it plays in geometry and topology in describing and transforming forms with a visible degree of comprehensible coherence. There is an extensive literature on the role of geometrical metaphors in discourse, although this tends to focus on the use of metaphor in mathematical education. This includes:

- Mathieu Aubry: *Metaphors in Mathematics* (2009)
- Maciej Rosiński: *Metaphor Activation in Multimodal Discourse: case studies on the emergence of geometrical concepts*. (University of Warsaw Repository, 2018)
- Gloriana González: *A geometry teacher's use of a metaphor in relation to a prototypical image to help students remember a set of theorems* (*The Journal of Mathematical Behavior*, 32, 2013, 3)
- Wajeeh Daher: *Students' Positioning and Emotions in Learning Geometric Definition* (*Journal on Mathematics Education*, 11, 2020, 1)
- Carlos Cornejo, et al: *The physiognomic and the geometrical apprehensions of metaphor* (*Culture and Psychology*, 19, 2013, 4)
- J. Rees: *Geometry and Rhetoric: Thinking about Thinking in Pictures* (*Nexus Network Journal*, 12, 2010, 3)
- Nathalie Sinclair et al: *To be or to become: how dynamic geometry changes discourse* (*Research in Mathematics Education*, 10, 2008, 2)
- Barry Stocker: *Pascal and Derrida: Geometry, Origin and Discourse* (*Symposium*, IV, 2000, 1)

The argument can be developed with respect to the simplest features of geometry employed as metaphors in dialogue, namely points, lines, volumes and holes (*Metaphorical Geometry in Quest of Globality -- in response to global governance challenges*, 2009),

An argument can be made for a radically recursive interpretation of such a geometric bias (*Radical recursivity of cognitive implications of topology and geometry*, 2021). Preference for that metaphor might indeed be fruitfully seen as one of a set of 8 "houses" of dialogue, raising the question as to the nature of insights into dialogue evoked by the others. **Given such recursion what indeed is the cognitive engagement with mathematical operations -- then to be understood as forms of cognitive displacement in their own right?**

How then does dialogue informed by one such metaphor engage with that informed by another? This can be explored in terms of requisite complementarity, but also merits exploration in terms of approaches to the encoding of cultural distinctions -- as highlighted in the work of a number of authors (*Systems of Categories Distinguishing Cultural Biases*, 1993). Potentially especially relevant in those studies is the work of W. T. Jones on axes of bias, as summarized separately (*Axes of Bias in Inter-Cultural Dialogue*, 1993).

## Re-cognition of clustering of fundamental N-fold sets

As noted below, it is curious that it is the 64-fold pattern of order which is so fundamental to both the organization of life and to the operation of computers as understood through the conventional academic disciplines (of the West). Ironically its potential implication for

psychosocial organization (as explored in an Eastern culture) is deprecated by disciplines which have demonstrated their inability to engender patterns of order of relevance to the governance of a global civilization in crisis.

Partly inspired by the manner in which the binary coding of the *I Ching* inspired the organization of computer memory and operations, one point of departure is therefore recognition of the fundamental role of 64-fold patterns of order:

- the array of genetic codons ([DNA and RNA codon tables](#))
- the curious fact that the comprehensive *Mathematics Subject Classification* is organized in terms of 64 categories, as discussed separately ([Configuring the 64 subjects of mathematics as a 64-edged drilled truncated cube](#), 2021).
- the set of there are 64 convex uniform 4-polytopes. A third lead to any intuited sense of 17-fold coherence in 4 dimensions is offered in by the 64 [convex uniform 4-polytopes](#) of which 5 are polyhedral prisms based on the Platonic solids and 13 are polyhedral prisms based on the Archimedean solids. One is however duplicated with the cubic hyperprism (namely a [tesseract](#)), reducing the set to 17. Known as [4-polytopes](#) (or polychora), there are 64 convex [uniform 4-polytopes](#). This recognized set includes 6 regular convex 4-polytopes, and excludes the infinite sets of the [duoprisms](#) and the [antiprismatic prisms](#). The set of 64 itself invites further reflection (as discussed below),
- the cultural framings traditionally offered by the *I Ching* and the *Kama Sutra*, as discussed separately ([Reframing the Dynamics of Engaging with Otherness: triadic correspondences between Topology, Kama Sutra and I Ching](#), 2011)

Given the complexity of such a pattern there is a fundamental challenge to comprehending the distinctions it makes ([Comprehension of Numbers Challenging Global Civilization](#), 2014). Of potential interest therefore is the sense in which the pattern of 64 may be understood as clustered into a more limited 20-fold pattern, also esteemed as fundamental in some manner, if only in its significance for design:

- as the array of amino acids encoded by 61 of the 64 genetic codons. Exceptionally, three of them (known as stop codons) do not code for an amino acid but instead signal the release of the nascent polypeptide from the ribosome.
- the 20 2-uniform tilings and their relation to the 17 plane symmetry groups ([Archetypal cognitive confusion with strategic implications?](#) 2021)
- the apparent preference for 20-fold arrays of strategic relevance ([Requisite 20-fold Articulation of Operative Insights? Checklist of web resources on 20 strategies, rules, methods and insights](#), 2018)
- the possibility of a 20-fold pattern of a psychosocial significance ([Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life?](#) 2015). Could life be understood as a form of "dialogue" between 20 complementary amino acids?

Clearly of related interest are the preferences under different conditions for 12-fold, 10-fold and 8-fold patterns, for example. More mysterious is how a 17-fold pattern emerged as a curious global consensus fundamental to the challenge of governance at this time through the UN's 17 Sustainable Development Goals, as discussed separately in terms of 4-dimensionality ([Systemic Coherence of the UN's 17 SDGs as a Global Dream -- rather than merely an arbitrary outcome of political horse-trading](#), 2021). There the pattern was noted:

- as a subset of the 64 convex uniform 4-polytopes (as noted above)
- its relation to the so-called 17-fold [wallpaper group](#), and its role in design, music and dance (as noted below)

It should be stressed that it is the potential complementarity of disparate frameworks -- if understood in metaphorical terms -- which merits appreciation. A case has been separately made for the requisite variety of any such set of metaphors in framing the potential for transformation of the central entity -- whether theme or discussant. ([Framing Cognitive Space for Higher Order Coherence: toroidal interweaving from I Ching to supercomputers and back?](#) 2019)

Of some relevance is how an 8-fold dialogue (as an archetypal "eightfold way") might be transformed into a 12-fold, 16-fold, 20-fold, or 64-fold dialogue -- and how any such pattern can be transformed to one of lower complexity.

The question is necessarily fundamental to the **questionable global preference for dysfunctional binary dialogue between "Us" and "Them"** ([Destabilizing Multipolar Society through Binary Decision-making: alternatives to "2-stroke democracy" suggested by 4-sided ball games](#), 2016; [Us and Them: Relating to Challenging Others patterns in the shadow dance between "good" and "evil"](#), 2009).

There is clearly very limited enthusiasm or competence in dialogue among 3, 4 or more parties. It is also questionable under what conditions interaction skills and thinking transcend the "glass ceiling" of binary thinking. Such limitations help to frame the question of the memorability of the set of 17 SDGs of the UN or to comprehend their systemic relationships -- just as it is problematic to distinguish and name the 17 tiling patterns so fundamental to design.

## "Cognitive tiling" or "conceptual tiling"?

**Tiling operations:** As noted in the previous discussion ([Cognitive operations potentially analogous to generation of tiling patterns](#), 2021), the unusually limited set of 17 tiling patterns (the wallpaper group) is generated by distinctive combinations of so-called Euclidean plane isometries. With respect to the geometry of the patterns, these operations are described in terms of the 4-fold set of **translation, rotation, reflection and glide reflection**. **To what degree do these operations bear significant resemblance to cognitive operations**, most obviously as might be perceptible in the discourse by which strategies are engendered? Understood metaphorically, **what role might they play in dialogue in general?**

In considering that question, it is necessary to recognize how the 4-fold set is more complex than the 2-fold set so characteristic of humanity's cognitive glass ceiling. This would tend to imply that the four operations are only comprehended in other ways -- as embodied in skills for example, rather than understood and organized rationally. Thus they could well feature in dialogue but with limited capacity "to get one's head around them" in rational terms.

**Satisfactory closure:** What could be recognized as the central mystery of any recognized tiling pattern is how a sense of satisfaction is associated with the closure of pattern recognition. More generally this is the satisfaction of resolving a riddle or completing, any puzzle (crossword, sudoku, Rubik cube, etc), or solving a problem of mathematics or design. How is such closure most fruitfully understood in generic terms ([Hilary Lawson: Closure: A Story of Everything](#), 2001). In such terms, how is the "closure" to be understood in making for a dialogue deemed satisfactory -- beyond its articulation in feel-good terms or in the conclusion of a deal?

Mathematics frames the matter in terms of symmetry in relation to closure, with an extensive literature on [closure in set theory](#) and on [symmetric closure](#) -- expressed formally through requisite abstractions (Marcin J. Schroeder, *Concept of Symmetry in Closure Spaces as a Tool for Naturalization of Information* 2008). In general, the closure of a relation is the smallest extension of the relation that has a certain specific property such as the reflexivity, symmetry or transitivity. Distinctions are made between difunctional closure, contact closure, reflexive transitive closure, reflexive transitive symmetric closure, equivalence closure and congruence closure.

A seemingly quite unrelated approach to the experience of closure has been developed by [Gestalt psychology](#) which offers a number of principles (termed "laws") now influential in design, including: [Law of Closure](#), [Law of Symmetry](#), [Law of Similarity](#), [Law of Proximity](#). Thus the Law of Symmetry is the gestalt grouping law that states that elements that are symmetrical to each other tend to be perceived as a unified group; with closure, parts are combined to form a simpler whole (Steven Bradley, *Design Principles: Visual Perception and The Principles of Gestalt*, *Smashing Magazine*, 29 March 2014). These insights have been developed in relation to problem solving.

Arguably, in the following focus on polyhedra, is the contrast between the more "diluted" form of closure associated with a tiling pattern in 2D (tiling an "infinite plane") and the closure represented by a spherical polyhedral form, whether in 3D, 4D. or more.

**Metaphorical implications from tiling operations:** It is curious to note, although consistent with the possibility argued here, that the geometrical terms by which the tiling patterns are distinguished are far from unrelated to those descriptive of discourse and debate -- if only metaphorically and/or via synonyms. Potentially more relevant, however, is the manner in which metaphorical uses of the geometrical terms are employed far more loosely in discourse than they are with respect to the transformations associated with those operations. This may indeed have implications for distinctive formulation of strategies.

Exploration of cognitive implications of operations engendering distinctive "tiles"		
operation	synonym of cognitive significance	metaphors
translation	movement; shift framework; change language; change of perspective	Jessie Chaffee, <i>36 Metaphors for Translation, Words without Borders</i>
rotation	reorientation; change of orientation; rotation of office	<i>Rotation and Other Metaphors; Multidimensional Word and Sentence Rotation; Rotation as Metaphor; Crop Rotation as a Metaphor for Interdisciplinary Software Work; What Is an Orientational Metaphor?; Orientational Metaphors</i>
reflection	reversal, opposite perspective; mirroring	<i>Reflecting on the Metaphor and Practice of Reflection in Education; A critical and functional analysis of the mirror metaphor with reference to the media's responsibility towards society; Mirror as a metaphor; Human Mirrors: metaphors of intersubjectivity</i>
glide reflection	segue; nimble?; "revolving door"? "fast footwork"?	<i>The Revolving Door Metaphor</i>

Given the common interpretations of the terms describing the operations in the Euclidean plane, it is to be expected that the metaphors would be suggestive of dynamics that are well recognized in discourse, group dynamics, negotiation and politics. It is however possible that their generic significance would be even more familiar as articulated in dance -- notably through the manner in which dialogue and group dynamics are frequently framed in musical terms or as a "dance":

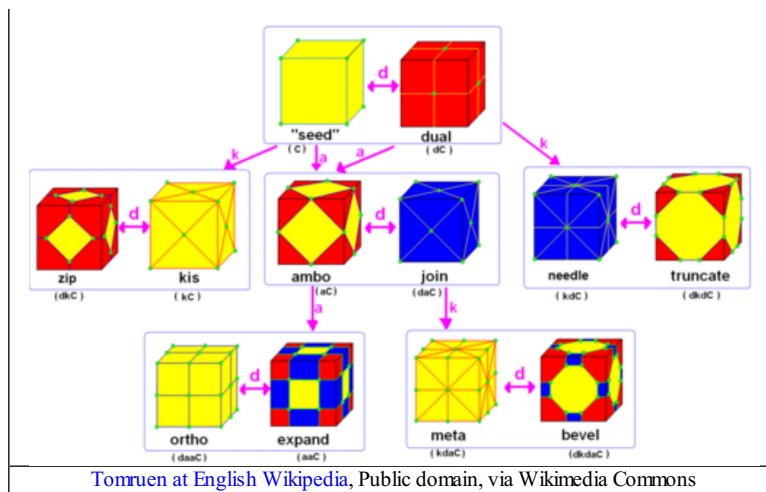
- Dana Mills: *Dance and Politics: moving beyond boundaries* (2016)
- Anca Giurchescu: *The Power of Dance and Its Social and Political Uses* (*Yearbook for Traditional Music*, 33, 2001)
- Susan A. Reed: *The Politics and Poetics of Dance* (*Annual Review of Anthropology*, 27, 1998),
- Siobhan Burke: *Making (and Seeing) Dance in the Politicized World* (*The New York Times*, 17 November 2017)

Do the dynamics of pattern transformation between "left" and "right" in politics call for reframing in terms of dance? More fundamental are then the cognitive implications of any such dance, as suggested by the work of [Mark Johnson](#) (*The Meaning of the Body: aesthetics of human understanding*, 2007) and Maxine Sheets-Johnstone (*The Primacy of Movement*, 1999). One exploration of potential relevance is that of [Paris Arnopoulos](#) (*Natural Energy and Social Power: metaphors from physics to politics*, Gamma Institute, 1985).

## Cognitive implications of operational modification of polyhedra -- "global tiling"

Distinctive polyhedra may be created through modification of a seed polyhedron by various prefix [operations](#), as described by the [Conway polyhedron notation](#), and discussed separately (*Topological operations on polyhedra as indicative of cognitive operations*, 2021). The following example shows how 11 new forms can be derived from the cube using 3 operations (named **dual**, **ambo** and **kis**). The new polyhedra are shown as maps on the surface of the cube so that the topological changes are more apparent.

Conway relational chart Showing 12 forms created by 3 operations on the cube



The three basic operations (when used successively) are sufficient for generation of the 5 Platonic and the 13 Archimedean polyhedra are:

- **dual**: replaces each face with a vertex, and each vertex with a face
- **ambo**: creates degree-4 vertices (otherwise known as rectification)
- **kis**: raises a pyramid on each face

Other operations have been distinguished, together constituting a more conventional total of 13 and denoted by the letters *abdegikmoprst* (see table below). These have been extended in the Antiprism application of Adrian Rossiter to a further set of 18 -- although how they might together be understood as a set remains unclear (*Conway Notation Transformation, Antiprism; Wythoff-style constructions, Antiprism*).

As developed by Rossiter to produce weave patterns on tiled surfaces, the "Wythoff constructive notation" can be used to define "Conway operations" more precisely. According to Rossiter, the number of Conway operator type patterns is unlimited. raising the question as to whether they can be meaningfully ordered. This would probably depend on the definition of "Conway operation" pattern, and in particular the definition of when the pattern of tiles close.

Of primary relevance to this argument is how such operations might be distinguished in cognitive terms with regard to the transformation of any ordered *memplex* in the course of discussion -- or in the development of a strategy. **What indeed are the distinctive "operations" in discourse?** Again some terms identified with respect to transformation of polyhedra offer clues when metaphorically understood: *truncate, expand, join*. Valuable indications of potential relevance to discourse analysis are to be found in the work of B. R. S. Recht (*Operations on Polyhedra, GitHub*, 15 July 2019).

## Dialogue coherence through formalization of local symmetry-preserving operations

An obvious clue to the coherence and memorability of any sense of closure is provided by the degree to which the factors of the number of set elements facilitate memorability, as with 6 (2x3), 10 (2x5), 12 (3x4), and 20 (4x5). This may well be reinforced through rhyme in poetry, rhythm in song and music, and symmetry effects in any visualization or design. In terms of the use of polyhedra to map sets of topics in memorable discourse, such patterns can be explored to identify polyhedra for maps of particular complexity (*Identifying Polyhedra Enabling Memorable Strategic Mapping*, 2020). Any such quest in 3D makes apparent that the coherence of a 17-fold pattern calls for representation in 4D.

**LSPs:** A systematic approach to a formalization, which may be of relevance to dialogue, is through the language of "local symmetry-preserving operations" (LSPs) as recognized with respect to tiling and the formation of polyhedra (Gunnar Brinkmann, Pieter Goetschalckx and Stan Schein, *Comparing the constructions of Goldberg, Fuller, Caspar, Klug and Coxeter, and a general approach to local symmetry-preserving operations, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 25 October 2017).

The use of operations on polyhedra possibly dates back to the ancient Greeks, who were the first to describe the Archimedean solids, which can be constructed from the Platonic solids by local symmetry-preserving operations (e.g. truncation) on the solid. By contrast, the results of decorations of polyhedra, e.g. by Islamic artists and by Escher, have been interpreted as decorated polyhedra -- and not as new and different polyhedra. Only by interpreting decorations as combinatorial operations does it become clear how closely these two approaches are connected

Of more general relevance is the variety of domains in which there is a preoccupation with "local symmetry" (Edward Betts, *Local symmetry, Find Llink*).

**Polyhedral notation:** The operations on polyhedra have acquired a degree of recognition through the development of the *Conway polyhedron notation* (*Wikipedia*) and its promotion by *George Hart*. These are extensively described in that *Wikipedia* document.

However, as noted by B R S Recht (*Notes on operations on polyhedra, Antitile*, 2017), initially there was not much theory supporting operations on polyhedra. That study cites -- as providing a framework -- what is seemingly an earlier version of the Royal Society paper

quoted above (Gunnar Brinkmann, Pieter Goetschalckx, Stan Schein, [Goldberg, Fuller, Caspar, Klug and Coxeter and a general approach to local symmetry-preserving operations](#), *arXiv:1705.02848*, 8 May 2017). For Recht:

This text is an attempt to use Brinkmann's work to find ways to quantify, analyze, and expand these operators. In particular, it focuses on operators that can be described in terms of a linear operator on the counts of vertices, edges, and faces. These linear operators can be used to examine the composition and decomposition of operations on polyhedra. Such operators do not constitute all possible operations on polyhedra, or even all those that can be represented by *Wythoff-style constructions* [*Antiprism*], but they are an interesting subset of those operators with many nice aesthetic and geometric qualities.

*Antitile* is a package for manipulation of polyhedra and tilings using Python. It is designed to work with *Antiprism Polyhedron Modelling Software*, but can be used on its own.

Subsequent to Recht's clarification, the arguments of Brinkman (2017) have been further developed (Pieter Goetschalckx, Kris Coolsaet, Nico Van Cleemput, [Generation of Local Symmetry-Preserving Operations](#), *arXiv:1908.11622*, 12 April 2020). This is described as:

We introduce a new practical and more general definition of local symmetry-preserving operations on polyhedra. These can be applied to arbitrary plane graphs and result in plane graphs with the same symmetry. With some additional properties we can restrict the connectivity, e.g. when we only want to consider polyhedra. Using some base structures and a list of 10 extensions, we can generate all possible local symmetry-preserving operations isomorph-free... In this paper we shall slightly extend the definition of LSP operation so it can be applied to any graph embedded on a compact closed surface, and at the same time provide a reformulation of these operations as decorations, which will turn out to be easier to use in practice.

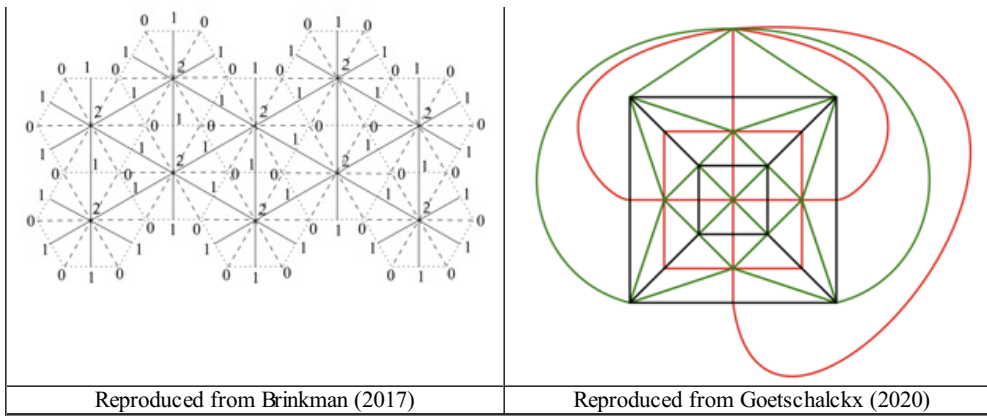
**Graph representation of dialogue:** In considering the relevance to dialogue, it is appropriate to recall that patterns of dialogue and social interaction have long been represented as graphs. This approach extends to the relationship between topics as implied by any sense of a semantic web. Recent references include:

- Michael Galkin:
  - [Building a Knowledge Graph-based Dialogue System at the 2nd ConvAI Summer School](#) (*DeepPavlov.ai*, 16 July 2019)
  - [Mushroom Effect or Why You Need Knowledge Graphs for Dialogue Systems](#) (*SlideShare*, 26 June 2019 )
- Milan Gritta, Gerasimos Lampouras, Ignacio Iacobacci: [Conversation Graph: Data Augmentation, Training, and Evaluation for Non-Deterministic Dialogue Management](#) (*Transactions of the Association for Computational Linguistics*, 9, 2021)
- Debanjan Chaudhuri, Md Rashad, Al Hasan Rony, Jens Lehmann: [Grounding Dialogue Systems via Knowledge Graph Aware Decoding with Pre-trained Transformers](#) (*PapersWithCode.com*. 30 Mar 2021)
- Wenge Lium JianhengTang, Xiaodan Liang abd Qingling Cai: [Heterogeneous graph reasoning for knowledge-grounded medical dialogue system](#) (*Neurocomputing*, 442, 28 June 2021)
- Jiaying Shen, Hendrik Harkema, et al: [Layered Semantic Graphs for Dialogue Management](#) (*Nuance Communications*)
- Hongcai Xu, Junpeng Bao and Junqing Wang: [Knowledge-graph based Proactive Dialogue Generation with Improved Meta-learning](#) (*Proceedings IPMV*, 2020)
- Svitlana Vakulenko, Svitlana Vakulenko, Maarten de Rijke and Michael Cochez: [Measuring Semantic Coherence of a Conversation](#) (*CommuniData Project*, June 2018)
- Jiaqi Li, Ming Liu, Zihao Zheng et al.: [DADgraph: A Discourse-aware Dialogue Graph Neural Network for Multiparty Dialogue Machine Reading Comprehension](#) (Harbin Institute of Technology, China, 2021)
- Siru Ouyang, Zhuosheng Zhang, Hai Zhao: [Dialogue Graph Modeling for Conversational Machine Reading](#) (*arXiv:2012.14827v1*, 2012)
- David Price: [DebateGraph](#)
- Lei Qiao Zhi: [Branching Dialogue System for Unity](#)
- [Dialogues Graph Insight](#) (*BlueConic*, 2021)

**Chambers:** It is argued by Goetschalckx (2020) that every embedded graph has an associated chamber system. This chamber system is obtained by constructing a barycentric subdivision of the graph by adding one vertex in the centre of each edge and face of the graph, and edges from each centre of a face to its vertices and centres of edges. These vertices can be chosen invariant under the symmetries of the graph. To avoid confusion, the tiles are named as chambers.

Obviously, the barycentric subdivision is a triangulation. Each chamber has three types of vertices, namely one being an original vertex, one chosen inside an edge and one chosen inside a tile. This language invites reinterpretation in terms of dialogue.

Barycentric subdivision of a graph into chambers -- with implications for dialogue	
Chamber system of a hexagonal tiling.	Barycentric subdivision of the plane graph of a cube
Dotted lines are edges of the original tiling (not containing the vertex labelled 2), full lines are connections between face centres and edge centres (not containing vertex 0) and dashed lines are connections between face centres and vertices (not containing vertex 1)	Edges of type 0 are red, edges of type 1 are green and edges of type 2 are black.



**LSP operations as decorations:** As noted, the definition of an LSP operation is slightly extended by Goetschalckx (2020) so it can be applied to any graph embedded on a compact closed surface, whilst at the same time providing a reformulation of these operations as "decorations". It is claimed that these are easier to use in practice. Such terminology raises the provocative question as to whether modalities in dialogue are typically understood as "decorative" -- or could be so understood given the mathematical interpretation.

Each triangular face of a chamber system can be filled with a decoration, by distinguishing different vertices. This results in a new chamber system of a new graph. This is very similar to the LSP operations. Graphs are constructed by subdividing the chambers of the chamber system. One key difference is that no restrictions are imposed on the connectivity. This means that decorations can be applied to arbitrary embedded graphs, but when applied to a polyhedron – i.e. a 3-connected plane graph – it is possible that the result has a lower connectivity. Additional restrictions can be applied to decorations.

**Inflation rate:** As indicated by Goetschalckx (2020), the impact of an operation on the size of a polyhedron can be measured by the inflation rate. This is the ratio of the number of edges before and after the operation, and is equal to the number of chambers in the decoration. As clarified in the table below, the number of LSPs for each inflation rate is 2,2,4,6,6,20,28,58,82... starting with inflation rate 1. However, not all LSPs necessarily produce a polyhedron whose edges and vertices form a 3-connected graph, and as a consequence of Steinitz's theorem do not necessarily produce a convex polyhedron from a convex seed.

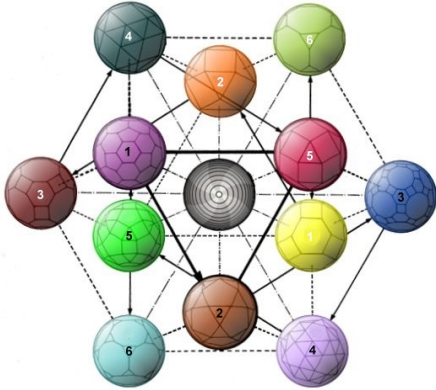
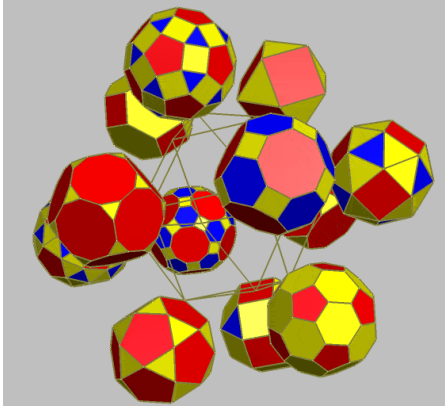
Indication of increasing complexity of operations (suggesting corresponding implications for dialogue)					
Number of k-connected decorations up to inflation rate 10 (extracted from a table up to a rate of 40)					LSP operations (see below)
inflation rate	k-connected decorations			predecorations	
	k=1	k=2	k=3		
1	2	2	2	1	identity and dual
2	2	2	2	1	ambo and join
3	4	4	4	1	truncate, zip, needle, kiss
4	6	6	6	2	chamfer, subdivide, ortho, expand (+2)
5	6	6	4	2	gyro, snub, propeller, loft (+5)
6	20	20	20	4	meta, bevel, quinto (+3)
7	28	28	20	7	lace, stake, whirl, volute (+8)
8	58	58	54	8	join-kis-kis (+3)
9	82	82	64	7	waffle
10	170	168	144	19	cross (+3)

With respect to dialogue, it is intriguing to note a degree of correspondence between the chambered graph language above, as it relates to LSPs, and the illustration offered by Ronald Atkin through his development of q-analysis (*Multidimensional Man; can man live in 3-dimensional space?*, 1981), as separately summarized (*Comprehension: Social organization determined by incommunicability of insights*). Atkin illustrates the challenge of comprehension in relation to experience "within" the geometry of a triangle.

Contrasting interpretations of a graph			
Formation of chambers (as indicated by Goetschalckx, 2020)		Comprehension through transcending the plane (as indicated by the q-analysis of Ron Atkin)	
Vertex coding convention	Decoration <i>ambo</i> applied to the cube; resulting graph is the one in black.	Conjection as recognizing "white"	Relative orders of comprehension
			<p><b>0-dimension:</b> Red, Green or Blue</p> <p><b>1-dimension:</b> Yellow (=Red/Green) Purple (=Red/Blue); or Turquoise (=Blue/Green)</p> <p><b>2-dimension:</b> White (=Red/Green/Blue)</p>

# Operations transforming polyhedra as modelling transformations of dialogue coherence

A point of departure is the closest packing configuration of 12 Archimedean polyhedra around the 13th (image below left), namely the truncated tetrahedron as clarified by [Keith Critchlow](#) (*Order in Space: a design source book*, 1969), discussed separately in terms of *12-fold Modalities for "heavy duty" global governance?* (2008) with regard to *Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors* (2008). The cuboctahedral array below can be partially represented by the animation on the right, as previously discussed (*Packing and unpacking of 12 semi-regular Archimedean polyhedra*, 2015)

Arrangement of the 12 Archimedean polyhedra in their most regular pattern, a cuboctahedron as presented by Keith Critchlow	
Successive truncations of <b>octahedron</b> 2, 3, 4-fold symmetry	Successive truncations of <b>icosahedron</b> 2, 3, 5-fold symmetry
<ol style="list-style-type: none"> <li>1. <b>truncated octahedron</b> (14 polygons: 4 / 6 sided)</li> <li>2. <b>cuboctahedron</b> / vector equilibrium (14: 3 / 4)</li> <li>3. <b>truncated cuboctahedron</b> (26: 4 / 6 / 8)</li> <li>4. <b>snub cube</b> (38: 3 / 4)</li> <li>5. <b>rhombicuboctahedron</b> (26: 3 / 4)</li> <li>6. <b>truncated cube / hexahedron</b> (14: 3 / 8)</li> </ol>	<ol style="list-style-type: none"> <li>1. <b>truncated icosahedron</b> (32 polygons: 5 / 6 sided)</li> <li>2. <b>icosidodecahedron</b> (32: 3 / 5)</li> <li>3. <b>truncated icosidodecahedron</b> (62: 4 / 5 / 10)</li> <li>4. <b>snub dodecahedron</b> (92: 3 / 5)</li> <li>5. <b>rhombicosidodecahedron</b> (62: 3 / 4 / 5)</li> <li>6. <b>truncated dodecahedron</b> (32: 3 / 10)</li> </ol>
With inclusion of truncated tetrahedron at centre	Rotation array 12 Archimedean polyhedra (around an omitted 13th at the centre; totalling 984 edges, 558 vertices, 452 faces)
	
Adaptation from Critchlow ( <i>Order in Space</i> , 1969)	Animation prepared with the aid of <a href="#">Stella Polyhedron Navigator</a> ;

As noted above, the 17 4D polyhedra presented there are based on the 5 Platonic polyhedra and the 13 Archimedean polyhedra. It could then be assumed that the 3D-projections of the 4D variants could potentially be configured in a similar manner -- if only for mnemonic purposes.

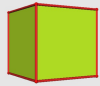

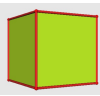
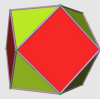
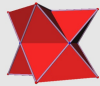
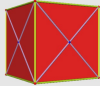
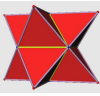
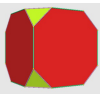

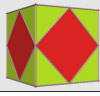


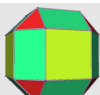
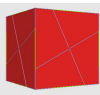

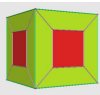
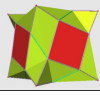
The local symmetry operations (as currently defined) originate from those required to create the Archimedean solids and their duals from the Platonic solids. Currently the 13 operations *abdegjkmoprst* (see table) are defined in terms of the Conway polyhedron notation. They have been variously extended, notably by Antiprism (*Conway Notation transformations / Wythoff-style constructions*). They can be variously combined and applied successively to the product of any transformation.

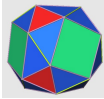

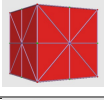
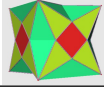

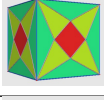
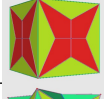
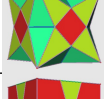
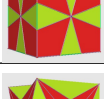
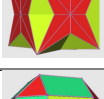
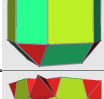
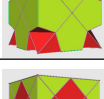
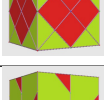

The following table is a combination of information about the operations in *Wikipedia (Conway polyhedron notation)*, *Antiprism (Conway Notation transformations / Wythoff-style constructions)* and *Antitile (Listing of operators and transforms)* -- in the light of the table above by Goetschalckx (2020). Much more information is available in each source, including polyhedral illustrations, graphs and the characteristic 3x3 matrices. The table below is therefore a simplification to clarify the argument with respect to dialogue. Experimental visualizations of the polyhedra resulting from application of the operations to a cube are presented below (as generated by the *Antiprism application*).

Note that parameters (not indicated) may be added to the operators. Such parameters are devised from recognition of a sequence pattern based on "one" named operator, so while the named operator will be in the sequence, there may be more than one sequence that could reasonably be associated with a parameter for that operator. Operators can be clustered into groups of four (or fewer if some forms are the same) by identifying the operators *x*, *xd* (operator of dual), *dx* (dual of operator), and *dx* (conjugate of operator).

The following list of operators has been sorted in ascending order of complexity of transformation, namely to indicate how much greater is the complexity of the polyhedron after the transformation. The table makes provision for indication of the degree to which such a transformation might apply in a dialogue mapped as a graph. The terms used to describe the operations are potentially somewhat suggestive of this in metaphorical terms.

Examples of local symmetry operations on a cube ordered by increasing inflation rate (images generated by the Antiprism application)						
inflation rate	operation (Conway)	operation (Wythoff)	chiral	screen shot of	highly oversimplified technical description of operation	operation understood metaphorically as transformation of

	notation)		notation)			polyhedron		graphical mapping of a dialogue on a polyhedron
1	S	identity (seed)			No		No change	
1	d	dual			No		The <i>dual</i> of a polyhedron has a vertex for each face, and a face for each vertex, of the original polyhedron.	
1	r	reflect			n/a		Changes a left-handed solid to right handed, or vice versa (where chiral is Yes), but has no effect on a reflexible solid (like a cube).	
2	a	ambo (rectification)			No		The <i>ambo</i> operation can be thought of as truncating to the edge midpoints. There is one face for each face of X and one face for each vertex of X.	
2	i	join	j	join	No		Produces a polyhedron with one 4-sided face for each edge of X. Dual to <i>ambo</i> .	
3	k	kis	k	kis	No		Divides each n-sided face into n triangles. A new vertex is added in the center of each face. Dual to <i>truncate</i>	
3			n	needle	No		Triangulates with 2 triangles across every edge. This bisects faces across all vertices and edges, while removing original edges. Dual of <i>truncate</i>	
3	t	truncate	t	truncate	No		Cuts off each vertex, producing a new n-sided face for each n-fold vertex. The faces of the original polyhedron still appear, but have twice as many sides	
3			z	zip (bi-truncation)	No		Creates new edges perpendicular to original edges, a truncation beyond " <i>ambo</i> " with new edges " <i>zipped</i> " between original faces. Dual of <i>kis</i> or <i>truncation</i> of the dual.	
4			u	sub-divide	No		<i>Ambo</i> while retaining original vertices. Similar to Loop subdivision surface for triangle face	
4	o	ortho	o	ortho	No		Has the effect of putting new vertices in the middle of each face of X and connecting them, with new edges, to the edge midpoints of X. Dual to <i>expand</i>	
4			c	chamfer	No		New hexagonal faces are added in place of edges	
4	e	expand (cantellation)	e	expand	No		Each face of X is separated from all its neighbors and reconnected with a new 4-sided face, corresponding to an edge of X. An n-gon is then added to connect the 4-sided faces at each n-fold vertex.	
5	g	gyro	g	gyro	Yes		Like <i>kis</i> but with the new edges connecting the face centers to the 1/3 points on the edges rather than the vertices. Dual operation to <i>snub</i>	
6	b	bevel (omni-truncation)	b	bevel	No		Adds faces at the center, seed vertices, and along the edges. Produced by <i>truncation</i> after <i>ambo</i>	
5			l	loft	No		Augmentation of each face by prism, adding a smaller copy of each face with trapezoids between the inner and outer ones	
5	p	propeller	p	propellor	Yes		Makes each n-gon face into a "propeller" of an n-gon surrounded by n quadrilaterals	

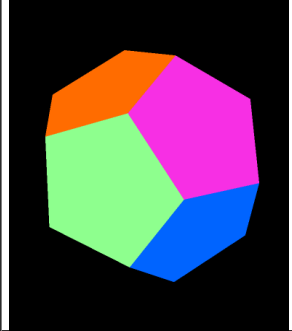



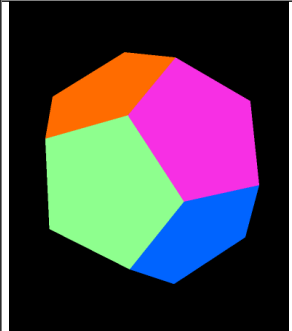

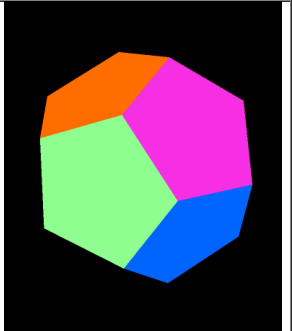

5	s	snub	s	snub	No		Produces the snub cube. It can be thought of as eC followed by the operation of slicing each of the new 4-fold faces along a diagonal into two triangles.
6			q	quinto (pental)	No		<i>Ortho</i> followed by <i>truncation</i> of vertices centered on original faces. This create 2 new pentagons for every original edge it effectively lines the original faces with pentagons
6	m	meta	m	meta	No		New edges connect new vertices at the face centers to the old vertices and new vertices at the edge midpoints. Dual to <i>bevel</i>
6			L_0	joined-lace (join-lace)	No		Similar to <i>lace</i> , except with new quad faces produced in L_1 are not split
7			w	whirl	Yes		<i>Gyro</i> followed by <i>truncation</i> of vertices centered on original faces. This creates 2 new hexagons for every original edge it effectively lines the original faces with hexagons
7			L	lace	No		Augmentation of each face by an antiprism, adding a twist smaller copy of each face, and triangles between
7			K	stake	No		<i>Subdivides</i> faces with central quads, and triangles
7			G	opposite-lace	No		Similar to <i>lace</i> , except with new quad faces split opposite L_1.
7			M	edge-medial	No		Similar to <i>meta</i> except no diagonal edges added, creating quad faces.
8			J	joined-medial (join-kis-kis)	No		Like <i>medial</i> but new rhombic faces in place of original edges
8			E	ethel	Yes		Like <i>expand</i> but triangles are divided into 3 kites
9			W	waffle	No		<i>Truncation</i> on all vertices and then all faces split into sections
10			X	cross	No		Combination of <i>kis</i> and <i>subdivide</i> operation. Original edges are divided in half, with triangle and quad faces.
10			B	bowtie	Yes		Bowtie-like triangles divide pentagonal faces

## Rendering a 64-fold pattern dynamically comprehensible via 20-fold and 12-fold patterns

Examples were cited above of the manner in which psychosocial organization is apprehended through 5-fold, 8-fold, 10-fold, 12-fold, 20-fold and 30-fold sets. The question was raised as to: **how do these constitute efforts to comprehend the complexity of a 64-fold pattern -- and the challenge of engaging with it**, if it implied distinct modalities of dialogue? One way to approach this is through two of the five simplest regular polyhedra, namely the [dodecahedron](#) and the [icosahedron](#) -- both possibly reflective of limits to memorable recognition and comprehensibility, and widely valued for that reason in symbolic form.

**Encoding 64-foldness by implication in morphing dynamics:** Arguably the properties of the two polyhedra offer a clue -- one being the dual of the other -- since the total number of the faces and vertices of each is 32 (namely 12 + 20), totalling 64 for both together. The faces and vertices alternate when subject to the dual operation indicated above -- one of the simplest. The 64-fold pattern could then be said to be "hidden" within the dynamic of that alternation, just as that pattern could only be expressed in 4D as the set of 64 convex uniform 4-polytopes -- which can only be partially visualized in 3D, as previously illustrated through animations ([Higher dimensional coherence of SDGs implied by a set of 17 4-dimensional polyhedra?](#) 2021).

It is therefore intriguing to visualize the morphing from dodecahedron to icosahedron (and back) through an 8-fold set of operations variously implied by a selection of those in the table above. This can be achieved through features of the *Stella Polyhedron Navigator* as shown below.

8 Animations of the morphing dynamic between the dodecahedron and the icosahedron (between a polyhedron with 12 faces and 20 vertices and one of 20 faces with 12 vertices; each has 30 edges)			
By truncation	By sizing	By expansion	By augmentation
			
By tilting triangles	By tilting quadrilaterals	By tilting to rectify	By tilting to compound
			

Animations made with *Stella Polyhedron Navigator*

As remarked with regards to the relationship to the operators above, by Robert Webb (developer of the Stella application):

Correspondence between Conway operators and morphing dynamics (above)	
<ul style="list-style-type: none"> <li>• expansion is related to the <i>expand</i> operator</li> <li>• augmentation relates to <i>kis</i></li> <li>• truncation it relates amore complex, to the <i>truncate</i> operator (via <i>ambo</i>)</li> <li>• sizing just keeps the two polyhedra separate, and does not have a Conway equivalent (exceptionally)</li> </ul>	<ul style="list-style-type: none"> <li>• tilting quadrilaterals relates to <i>ortho</i></li> <li>• tilting triangles relates to <i>meta</i></li> <li>• tilting to rectify (a combination of operations)</li> <li>• and tilting to compound (a combination of operations)</li> </ul>

**Encoding 64-foldness explicitly in the drilled truncated cube:** Whereas the 64-fold pattern is implied through the dynamics of each animation above, a different approach can be taken through use of a unique polyhedron with 64 edges. That is the drilled truncated cube, as explored separately as a means of mapping the genetic codons or the Chinese hexagrams -- especially when the latter are indicative of dialogue modalities (*Proof of concept: use of drilled truncated cube as a mapping framework for 64 elements*, 2015). In this case however the 20-fold and 12-fold patterns are otherwise "hidden". They can be rendered visible by dynamically highlighting particular sets of edge or face types. An 8-fold pattern is hoever explicit in the cubic organization of the polyhedron -- potentially with a 16-fold pattern through the nesting of one cube within another. That nesting recalls the 4D form of the tesseract of significance to oppositional logic and its geometrical representation of the 16 **Boolean connectives**.

Note that both approaches could make use of the axes between opposing faces or vertices to highlight 5-fold or 10-fold patterns. Approaches dependent on dynamics in this way could be said to be consistent with the distinction made by David Bohm between an explicate order and an implicate order (*Wholeness and the Implicate Order*, 1980).

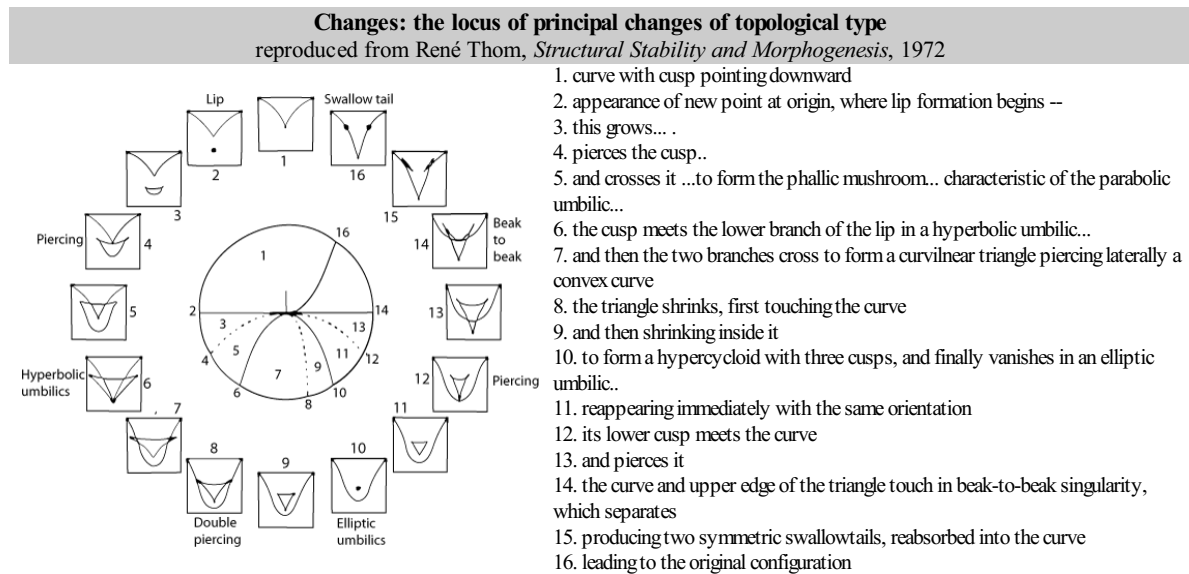
The edges of the polyhedron can be explored as a mapping surface for the genetic codons (left below). Hexagram metaphorical labels can be similarly associated with the edges (right below), as discussed separately with respect to "memetic codons" (*Relating configurative mappings of 64 I Ching conditions and 48 koans*, 2012).

Drilled truncated cube -- a polyhedron with 64 edges, approximating a torus (indicative assignment of labels, with faces coloured, or not)		
Codons assigned as labels to edges	Chinese hexagram metaphoric labels assigned to edges	Edges coloured and numbered by type



*Islamic patterns*, *International Journal of Architectural Computing*, 16, 2018,1). This found that such designs correlate to each other beyond just the formal dimension and that deep, morphological connections exist between them. The study identifies these connections and presents a categorization system that groups designs together based on their "morphogenetic" characteristics. As indicated there, determining the wallpaper group is important for identifying the "fundamental unit" representing the minimum geometric composition that is being systematically replicated.

From this perspective it is interesting to recognize the "suggestive" nature of the following sequence of 16 images -- possibly to be understood as "visual primitives" characteristic of the attraction dynamics of intercourse, in its specific and general senses (*Reframing the Dynamics of Engaging with Otherness Triadic: correspondences between Topology, Kama Sutra and I Ching*, 2011). From a cognitive perspective, the question might be asked how Thom's 16-fold identification relates both to the set of Conway operations (above) and to the 16+1 set of SDGs (discussed below).



Of interest with respect to dialogue, is the degree to which Thom's approach is associated with chaos theory -- notably giving rise to recognition of a set of elementary catastrophes. Given the extent to which dialogue processes -- and the outcomes of dialogue in this period -- merit recognition as catastrophic, there is a case for exploring dialogue in terms of catastrophe. Any initiative in dialogue could indeed be understood as engendering a form of catastrophe for the other -- just as any initiative by the other may well be experienced as catastrophic (as in competitive "verbal tennis").

With respect to dialogue, the approach could recognize the manner in which a question is experienced as catastrophic, whether in challenging the other or being challenged by the other. This approach was considered separately:

- *Conformality of 7 WH-questions to 7 Elementary Catastrophes: an exploration of potential psychosocial implications* (2006)
- *Interrelating Cognitive Catastrophes in a Grail-chalice Proto-model: Implications of WH-questions for self-reflexivity and dialogue* (2006)

Any understanding of sustainable development could be explored in terms of the pattern of catastrophic questions it poses. Fruitful dialogue could be framed by the "deadly question" by which the integrity of any framework is threatened, as explored with respect to communication in a conference (*World Futures Conference as Catastrophic Question: from performance to morphogenesis and transformation*, 2013). This included the following sections:

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>Conferencing: Outside Inside or Inside-Outside?</li> <li>Conferencing of a higher order: a Quest or an Inquest?</li> <li>Deconstructing conference communication processes to elicit meta-discourse</li> <li>Questioning as cognitive portal to the future</li> </ul> | <ul style="list-style-type: none"> <li>In quest of the most deadly question</li> <li>Enabling morphogenesis and transformation through catastrophic questioning</li> <li>Conferencing as putting identity to the question</li> </ul> |
|--|--|

## Dialogue as a collective navigational challenge in knowledge space

In a period of preoccupation with "sustainable development", it can be usefully asked how any requirement for "sustainable dialogue" might be understood (*Sustainable Dialogue as a Necessary Template for sustainable global community*, 1995). In the light of the focus above on polyhedra, especially intriguing is the possibility that some polyhedral forms are of particular relevance -- notably the so-called fullerenes (*Understanding Sustainable Dialogue: the Secret within Bucky's Ball?* 1996).

Irrespective of the role of Pentagramma Mirificum (as mentioned above), of particular interest to navigation in 3D are the reflections of Arthur Young as designer of the first *Bell Helicopter* (*The Bell Notes: A Journey from Physics to Metaphysics*, 1979; *Geometry of Meaning*, 1976). In contrast to the 16-fold articulation of René Thom (above), Young's 12-fold framework invites adaptation to the distinctive forms of dialogue required for collective navigation of knowledge space (*Typology of 12 complementary dialogue modes essential to sustainable dialogue*, 1998).

The limited or extended list of linear symmetry-preserving operators (LSPs), as presented above, invites further reflection as to whether

and how those listed could be clustered more meaningfully -- given their potential implications for dialogue. There is a sense in which they could be configured as a periodic table corresponding to the pattern so fundamental to the organization of the [periodic table of chemical elements](#). This is all the more intriguing in that the mathematics governing that order is a matter for continuing investigation -- inspiring the many [experimental representations](#) (D. H. Rouvray and R. Bruce King, *The Mathematics of the Periodic Table*, 2005).

Could the modalities of dialogue be ordered in a periodic table? This is suggested by the possibility of such a correspondence (*Periodic Pattern of Human Knowing: implication of the Periodic Table as metaphor of elementary order*, 2009; *Towards a Periodic Table of Ways of Knowing in the light of metaphors of mathematics*, 2009). In the light of the above argument, it is curious to note the manner in which the chemical elements are commonly perceived as clustered into oddly related 8-fold, 10-fold and 14-fold patterns, with exceptional handling of a 2-fold and a 1-fold pattern -- an occasional extension to a [32-fold pattern](#). It could be assumed that, as with the chemical elements, a periodic table of dialogue is in its early stages of being populated by "modalities" as yet to be clearly distinguished.

## Global dialogue via a 17-fold pattern of Sustainable Development Goals?

In contrast to the relatively comprehensible 12-fold and 16-fold articulations through which necessary dialogue modalities might be understood, there is a major challenge to any understanding of the nature of global dialogue as framed by the 17-fold patterns of the UN's Sustainable Development Goals. This was explored in the paper which preceded this argument (*Systemic Coherence of the UN's 17 SDGs as a Global Dream -- rather than merely an arbitrary outcome of political horse-trading*, 2021).

That exploration highlighted the possibility that any coherence to such dialogue could best be understood in 4-dimensional terms, as discussed in the following sections:

[Higher dimensional coherence of SDGs implied by a set of 17 4-dimensional polyhedra?](#)      [Distinguishing a domain of hyperreality as the appropriate SDG context?](#)  
[Interrelationship of 17 SDGs modelled by 17 regular polyhedra in 4D](#)

By contrast, the alternatives in 2D exemplify what can be recognized as the lack of functional relationship between the 17 Goals, as in the case of the::

- the [17-fold wallpaper group](#)
- the [17 uniform tilings with star polygons](#)

## Embodiment of dialogue operations in crafts and skills

**Metaphors of dialogue?** The following comment is a development of an earlier focus on aesthetics (*Coherence of hyperreality through aesthetic intuition and embodied cognition?* 2021). There it was argued that the fundamental cognitive operations in dialogue were variously implied in aesthetic practice and performance, most obviously dance and music. The suggestion was made that the operations on polyhedra invited metaphorical interpretation to elicit their significance for dialogue -- without aspiring to closure on the matter.

That argument can be further developed by recognizing the extent to which a number of crafts can be understood as "shaping", "moulding" or "crafting" a whole of some kind. Understanding of cognitive engagement in the practice of the craft can then be usefully compared to the operations on polyhedra of which some of the descriptors are suggestive, as with "bevel" (a term from carpentry). The correspondence may be all the greater in the light of mathematical insights into its operations, as in the case of knotting. This has notably featured in the psychoanalytic work of [Jacques Lacan](#) (*Borromean Knot*, *Encyclopedia of Psychoanalysis*) and [R. D. Laing](#) (*Knots*, 1972).

Similar arguments can be explored in relation to juggling (*Governance as "juggling" -- Juggling as "governance": dynamics of braiding incommensurable insights for sustainable governance*, 2018). This included sections on:

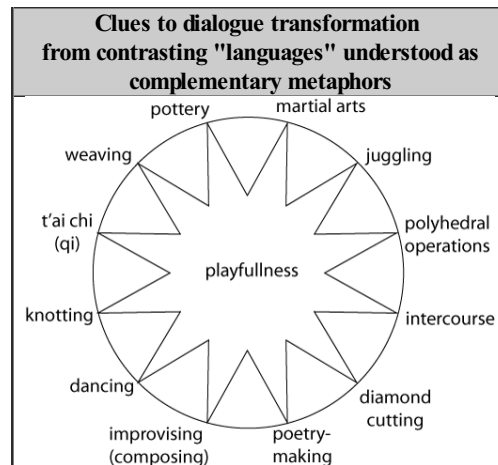
[Juggling as a metaphor in governance and policy implementation](#)      [Juggling highly controversial metaphors as a key to higher orders of governance?](#)  
[Recognition of juggling in practice in relation to cognitive development and creativity](#)      [Correspondences between juggling and governance](#)  
[Juggling of concepts, theories and topics](#)      [Determining the requisite number of patterns, partners and "balls" in governance](#)  
[Juggling of metaphors as a dynamic key to elusive challenges of governance](#)      [Acquiring a "sense" of 12 modalities for viable system awareness?](#)

Possibly to a greater degree than carpentry, potentially a rich source of clarification of cognitive operations -- via metaphor -- are disciplines such as pottery, sculpture, weaving and knitting (*Interweaving Thematic Threads and Learning Pathways*, 2010). The possibility is reinforced by recognition of the importance of such disciplines to spiritual practice (Laurie Erdman, *Pottery as Spiritual Practice -- Centering, Spirit of Clay*, 1 February 2010; Marjory Zoet Bankson, *Soulwork of Clay: a hands-on approach to spirituality*, 2008).

The focus on "shaping", and its cognitive implications, has notably been celebrated in the cutting and polishing of precious stones in order to enhance their collection and transmission of light, as noted separately (*Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue*, 2002). This includes sections on:

[Metaphoric examples of sustainable, dynamic coherence](#)      [Current metaphoric applications of gemstones](#)  
[Gemstones as an accessible metaphoric exemplar of the dynamics of coherence](#)      [Enlightening dialogue dynamics](#)

**Complementarity of metaphors indicative of dialogue?** The argument here is the value of disparate sources of metaphor to be understood as complementary sources of insight, as previously argued (*Complementary Metaphors of Discourse: towards transformative conferencing and dialogue*, 1984). This might be illustrated schematically as in the following.



**Dialogue as higher-order playfulness?** The schematic suggests the central role of a form of playfulness inherent in the complementarity between different modalities and their implications for dialogue of a higher order, as argued separately (*Playfully Changing the Prevailing Climate of Opinion*, 2005). This explored the possibility through the following:

- |  |  |
|--|--|
| Contrasting conventional foci: technology vs gardening | Games: prime focus of finite players                                 |
| Bridging metaphoric focus -- the nature of engagement  | Play: mediatisation of playfulness -- envirotainment? climatainment? |
| "Playing" with interrelated metaphors                  | Higher dimensions of "game-play space"                               |
| Developing playful insight                             | Meta-games   |
| Vital distinction: gaming vs playing                   |  |

The implied understanding of game-playing in relation to dialogue follows from the arguments of James Carse (*Finite and Infinite Games: a vision of life as play and possibility*, 1987).

**Shaping and reshaping as primary processes in dialogue?** The implication of the various metaphors cited is that dialogue involves a process of shaping, more or less evident in each case. This is consistent with understandings of cognitive embodiment, as argued separately (*En-minding the Extended Body: Enactive engagement in conceptual shapeshifting and deep ecology*, 2003).

Andy Fisher (*Radical Ecopsychology: psychology in the service of life*, 2002) specifically explores the psychological roots of the current ecological crisis. This perspective is endorsed by David Abram (*The Spell of the Sensuous*, 1997) who argues that:

The body is the location of all knowledge. What we "know" comes to us through our senses, through our contact with physical, earthly experience. That experience invariably shapes what we perceive. Perception is inherently participatory. To the body, the world is not "object". There is no "me" apart from an "other". Everything is animate for the sensing body. Touch a tree and the tree is touching you back. That we no longer know this is part of the tragedy. [\[more\]](#)

Given a tendency to frame dialogue mechanistically, more akin to conventional preoccupation with operations on polyhedra as graphs, curiously the notion of shaping is central to an unusual understanding of mechanism, as argued by Horea T. Ilieş and Vadim Shapiro (*On Shaping With Motion*, *Journal of Mechanical Design*, 122, 2000, 4):

Mechanical parts are modeled as (predominantly rigid) solid shapes that may move in space in order to function, be manufactured (for example, machine or be machined), and be assembled or disassembled. While it is clear that such mechanical shapes are greatly influenced by collision, interference, containment, and contact constraints through prescribed motions, the motion itself is usually not part of these shape models. This in turn leads to proliferation of computational methods for modeling and analysis of various motion-related constraints. We show that **all motion-related constraints can be formulated and applied within the same computational framework that treats motion as an integral part of the model**. Our approach relies on two computational utilities. The first one is the unsweep operation which, given an arbitrary n-dimensional subset of Euclidean space E and a general motion M, returns the largest subset of E that remains inside E under M. The second modeling utility is a disjoint decomposition of space induced by the operations of unsweep and the standard set operations. The proposed approach subsumes and unifies the traditional sweep-based modeling of moving parts, and provides improved computational support for mechanical shape design. *[emphasis added]*

**Unexplored metaphors?** Arguably other activities in which people assiduously engage are themselves suggestive of other insights into dialogue. Notable examples include the engagement in the selection and appreciation of wines and beers, as tentatively explored (*Global Quality Navigation System (GQS): participative enhancement of aesthetic discovery*, 2008).

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