



laetus in praesens

[Alternative view of segmented documents via Kairos](#)

7 February 2024 | Draft

Comprehensible Organization of Strategic Complexity in 3D and 4D

Indications of strategic coherence from 81-fold patterns in 4D

-- / --

Introduction

[Memorable organization of strategic complexity in 3D](#)

[Recognition of polyhedra as systems and systems as polyhedra](#)

[Strategic relevance of 4D frameworks: highlighting 81-fold patterns](#)

[Indicative visualization of 81-fold patterns in 4D](#)

[Representation of 81-fold Chinese encoding in 4D](#)

[Binary versus ternary in logic and computer operation](#)

The argument here is introduced in a first part and further developed in other parts.

Part 1: [Psychosocial Geometry and Dynamics of Collective Memory](#); Part 2: [Comprehensible Configuration of 8-fold Psychosocial Patterns in 3D](#); Part 4: [Higher Dimensional Reframing of Unity and Memorable Identity](#). See Part 1 for [References](#).

Introduction

The first part of this document highlights the role of pillars in relationship to strategic principles ([Principles, pillars, projectives and metaphorical geometry](#)). The question evoked there is the [Use of AI in enabling configuration of psychosocial pillars](#). A particular focus is given to the [Clarification of 8-fold strategic patterns with ChatGPT](#) and the [Recognition of the pattern of 8-fold "ways of looking"](#).

Pillars are of course an obvious feature of physical architecture, whether it be that of temples or churches, or in the design of institutions of government and justice. It may be far less evident whether particular symbolic significance is associated with individual pillars in any such array. An intriguing point of departure is recognition of the extent to which those upholding value configurations make

metaphorical use of "pillars" as architectural metaphors for a configuration of values, as discussed separately ([Coherent Value Frameworks: pillar-ization, polarization and polyhedral frames of reference](#), 2008; John Onians, [Architecture, Metaphor and the Mind](#), *Architectural History*, 35, 1992).

With the transition from principles to pillars, then understood as metaphors, the focus here is on what may be framed as metaphorical geometry ([Metaphorical Geometry in Quest of Globality -- in response to global governance challenges](#), 2009). It is variously discussed (Steven Baris, [Geometric Abstraction and Visual Metaphor](#), *Expanded Diagram Project*, 5 February 2023; Tib Roibu, [Cognition and the embodiment of geometry in George Lakoff's metaphors](#), *Geometry Matters*, 11 July 2023; E. P. Ross, [Geometry, Symbolism and Metaphors](#), *Design Blog*, 30 January 2019; Warren Shibles, [The Metaphorical Method](#), *The Journal of Aesthetic Education*, 8, 1974, 2).

From that perspective there is a particular irony to a further geometrical transition from pillars to weapons of similar form, given the manner in which they are thrown against enemies as "messages" in their own right, especially as evident in the use of rockets and missiles ([Missiles, Missives, Missions and Memetic Warfare](#), 2001). This can be explored in terms of the navigation of strategic interfaces in multidimensional knowledge space.

Understood otherwise, there is a case for urgent exploration of such geometry as mnemonic aids, as argued separately ([Time for Provocative Mnemonic Aids to Systemic Connectivity?](#) 2018; [Memorability, Mnemonics, Maths, Music and Governance](#), 2022; [Systemic Coherence of the UN's 17 SDGs as a Global Dream](#), 2021). This has been variously expressed as the need to "join the dots" or as enabling "joined-up thinking" -- a contrast to [silo thinking](#), a metaphor also reminiscent of the geometry of unconfigured pillars. From that perspective, perhaps most extraordinary is the housing of a physical transmutation of strategic pillars in [missile silos](#) (or nuclear silos) -- in anticipation of "joining up" distant others.

This 4-part document first considers how strategic pillars merit configuration and interconnection as "ways of looking" ([Part 1](#)). It then explores how 24-fold and 72-fold sets of these might be coherently configured in 3D as polyhedra ([Part 2](#)). The limitations of these 3D configurations highlight the potential necessity for a 4D framework to encompass more subtle 81-fold sets ([Part 3](#)). The argument concludes with the suggestion that any quest for "unity" is more appropriately envisaged in 4D rather than in 3D or through conventional framing of territorial conflicts in 2D ([Part 4](#)).

Memorable organization of strategic complexity in 3D

The study by [George Miller](#) on some limits on human capacity for processing information is one of the most highly cited papers in psychology ([The Magical Number Seven, Plus or Minus Two](#), *Psychological Review*, 1956). This has suggested the possibility of a more general consideration ([Patterns of N-foldness: comparison of integrated multi-set concept schemes as forms of presentation](#), 1980; [Representation, comprehension, and communication of sets](#)) *the role of number*, *International Classification*, 5, 1978, 3).

Question to ChatGPT: Elements of this exchange have focused on the relation between coherence, memorability and complexity. Understood in terms of mappability onto polyhedra, relative symmetry is relevant. Of interest in that respect is the exponent in factors characterizing the number of vertices, faces, edges or cells. This offers an explanation of memorability of larger traditional sets such as 64, 72 or 81, suggesting a relation between combinations of prime number factors and their exponents. Curiously it is the work of musicologist [Ernest McClain](#) which emphasizes prime exponents. Could you comment further

Your observation about the relationship between coherence, memorability, complexity, and mappability onto polyhedral structures, particularly in relation to symmetry and the role of prime number factors and their exponents, is a fascinating interdisciplinary exploration. It intersects mathematics, geometry, musicology, and cognitive science. Let's delve into this:

- Polyhedral Symmetry and Complexity:
 - Polyhedra can be characterized by their vertices, faces, edges, and in higher

dimensions, cells. The symmetry of a polyhedron often correlates with its aesthetic appeal and memorability. Highly symmetrical polyhedra, like Platonic solids, are more easily recognized and remembered.

- The number of these elements can often be broken down into combinations of prime number factors. The symmetry and complexity of a polyhedron could be influenced by these prime factors and their exponents.
- Prime Numbers in Traditional Sets:
 - Traditional sets like 64 (2^6), 72 ($2^3 \times 3^2$), and 81 (3^4) are composed of prime numbers raised to various exponents. These sets appear in various cultural and intellectual contexts, from the I Ching (64 hexagrams) to religious and mythological structures.
 - The memorability of these sets might be related to how the human mind perceives and processes mathematical patterns. Prime factors and their exponents could introduce a level of mathematical harmony or coherence that resonates with human cognitive patterns.
- Ernest McClain's Work on Musicology:
 - Ernest McClain was known for his work exploring the role of prime number exponents in ancient musical scales and mythologies. He proposed that ancient cultures used musical harmony as a metaphorical framework for understanding mathematical and cosmic orders.
 - McClain's work suggests a deep interconnection between music, mathematics, and narrative/mythological structures, where prime numbers and their exponents play a key role in creating harmonious and memorable patterns.
- Interdisciplinary Implications:
 - The relationship between polyhedral symmetry, prime number exponents, and memorability spans across disciplines. In geometry, it relates to the visual and structural characteristics of shapes. In musicology and narrative structures, it pertains to the rhythm, harmony, and coherence of stories or musical pieces.
 - This intersection implies that our perception and memorability of complex systems, whether in geometry, music, or narrative, might be deeply rooted in mathematical principles, particularly those involving prime numbers and symmetry.
- Cognitive Science Perspective:
 - From a cognitive science standpoint, the ease of processing and remembering these patterns might be due to the brain's affinity for symmetry and structured information. The brain tends to find patterns organized in such a mathematical manner more appealing and easier to process.

In summary, the connection between polyhedral symmetry, prime number exponents, and memorability, as highlighted in the works of Ernest McClain and others, opens up an intriguing avenue for understanding how we perceive and remember complex patterns. This understanding transcends mere mathematical curiosity, suggesting a profound cognitive and cultural significance in how we interpret the world around us.

Question to ChatGPT: The response implies a particular relationship between aesthetic appeal and memorability. Additional possible factors are the number base (if a base other than 10 is of relevance) and whether the unfolded polyhedral network enables some form of tiling (if not in terms of the wallpaper group). Would all such factors enable larger sets to be ranked in terms of memorability and coherence -- and even singability

The relationship between aesthetic appeal, memorability, and additional factors like number base and tiling properties of unfolded polyhedral networks indeed presents a complex but fascinating area of exploration. These factors could potentially influence how larger sets are perceived and remembered, and even how they might be 'singable' or rhythmically coherent in a musical sense. Let's delve into these aspects:

- Number Base and Its Impact:
 - Different number bases can alter our perception and memorability of numerical patterns. For instance, base-12 or base-60 systems (as used historically in various cultures) offer different divisibility and fractional properties compared to the common base-10 system. This can affect how we mentally organize and recall numerical

information.

- In terms of polyhedra, the choice of number base might influence how we conceptualize their geometry and the patterns derived from them.
- Tiling and Unfolded Polyhedral Networks:
 - The ability of a polyhedral network to tile a plane (even if not aligning with the wallpaper group patterns) adds an additional layer of geometric and visual appeal. This tiling property can enhance memorability by providing a clear, repetitive, and visually satisfying pattern.
 - Tiling and unfolding of polyhedra can also be seen as a metaphor for unfolding complex systems into more understandable components, which can aid in comprehension and memorization.
- Ranking Sets Based on These Factors:
 - It is plausible to rank larger sets in terms of memorability and coherence by considering these factors. Sets that align well with a chosen number base, display aesthetic appeal through symmetry and tiling, and resonate with human cognitive patterns (such as preference for symmetry and easily divisible numbers) could be ranked higher in terms of memorability.
 - However, this ranking would be subjective and influenced by cultural and individual cognitive preferences. What is memorable or coherent in one context or to one person might not be the same in another.
- Singability and Musical Rhythm:
 - The concept of 'singability' or musical rhythm ties into the idea of patterns and repetition. Sets that exhibit rhythmic or repetitive patterns could potentially be translated into musical structures or melodies, enhancing their memorability.
 - Musical structures often rely on patterns and ratios that are pleasing to the ear, which can correlate with the mathematical and geometric properties of these sets.
- Interdisciplinary Integration:
 - Integrating insights from geometry, mathematics, musicology, and cognitive psychology could provide a more holistic understanding of how these factors interplay in determining memorability and coherence.
 - This integration could lead to innovative methods of teaching, learning, and even communicating complex information, leveraging the innate human affinity for patterns, rhythm, and symmetry.

In summary, the memorability and coherence of larger sets, when considered in terms of aesthetic appeal, number base, tiling capabilities, and potential for musical representation, present a rich interdisciplinary field of study. While the exact influence of each factor may vary, their combined effect could significantly impact how we perceive, remember, and even artistically express complex patterns. This approach recognizes the deep connection between mathematical structures and human cognitive and aesthetic preferences, potentially offering novel ways to engage with and understand complex information.

Question to ChatGPT: In relation to any probability ranking of memorability, I noted the existence of the [On-Line Encyclopedia of Integer Sequences](#) in a paper on *Memorability, Mnemonics, Maths, Music and Governance*. Unfortunately the relevant table -- up to 108 -- is an image which I assume you cannot read. Are you able to suggest a probable ranking in the light of your own response

The On-Line Encyclopedia of Integer Sequences (OEIS) is a valuable resource for understanding numerical patterns, and it's interesting that you've explored its application in the context of memorability and mnemonics. While I can't access or read images, including the table in your paper, I can suggest a probable ranking of memorability for integer sequences based on factors we've discussed: symmetry, aesthetic appeal, number base relevance, tiling capability, and potential for musical rhythm. This ranking is speculative and conceptual rather than definitive.

- Low Single-Digit Sequences (1-9):
 - High memorability due to simplicity and ubiquity in everyday life.
 - Often foundational in number bases (especially base-10).
- Repetitive or Patterned Sequences (11, 22, 33, ..., 1234, 4321, etc.):
 - High memorability due to clear patterns and repetition.
 - Often used in mnemonics and learning tools.

- **Mathematically Significant Numbers (e.g., π , e, ϕ):**
 - High memorability for those with mathematical interest.
 - Intriguing properties (e.g., irrationality, role in formulas) enhance appeal.
- Numbers with Cultural or Symbolic Significance (e.g., 7, 12, 40):
 - Varying memorability depending on cultural context.
 - Often featured in religious, mythological, or cultural narratives.
- Prime Numbers (2, 3, 5, 7, 11, ...):
 - Moderate to high memorability for those interested in mathematics.
 - Unique property of having no divisors other than 1 and themselves.
- Fibonacci Sequence (0, 1, 1, 2, 3, 5, 8, ...):
 - Moderate memorability due to appearance in nature and aesthetics.
 - Relation to the golden ratio (ϕ) enhances appeal.
- Powers of 2 (2, 4, 8, 16, ...) and 10 (10, 100, 1000, ...):
 - High memorability due to their foundational role in binary and decimal systems.
- Highly Composite Numbers (e.g., 12, 24, 36):
 - Moderate memorability due to divisibility properties.
 - Often used in practical applications (e.g., time, measurement).
- Significant Geometric Numbers (e.g., Platonic solid numbers: 4, 6, 8, 12, 20):
 - Moderate memorability for those with geometric interest.
 - Aesthetic appeal and symmetry enhance memorability.
- Large and Complex Numbers (Significantly large primes, large factorials):
 - Lower memorability due to complexity and lack of direct relevance to daily experiences.

This ranking is a general guide and subject to individual differences in background, interests, and exposure to mathematics and numerology. It highlights how memorability can be influenced by a blend of simplicity, pattern recognition, cultural significance, and mathematical properties.

Question to ChatGPT: Does your ranking suggest the importance of critical feedback on the articulation of strategic declarations and charters of fundamental rights which purportedly require memorability to enhance their credibility and probability of implementation. I note that the 17-fold UN SDGs does not figure in your examples, unless they are treated as 16+1. Could the memorability of human rights charters be usefully ranked in that light

Your observation about the importance of critical feedback on strategic declarations, charters of fundamental rights, and frameworks like the UN Sustainable Development Goals (SDGs) in relation to memorability is quite pertinent. Memorability plays a crucial role in the effectiveness of such documents, impacting their credibility, public engagement, and implementation. Let's explore how these can be ranked in terms of memorability and potential improvements:

- Memorability of Strategic Declarations and Charters:
 - Documents like human rights charters or strategic declarations often contain complex and detailed information. The challenge is to balance the comprehensive coverage of important issues with the need for clarity and memorability.
 - Using principles from the ranking provided earlier, these documents could be made more memorable by simplifying language, using repetitive and patterned structures, and incorporating culturally significant symbols or metaphors.
- UN SDGs and the Concept of 16+1:
 - The 17 SDGs are a good example where a large set of goals might benefit from a mnemonic structure. Treating them as "16+1" could be one approach, where one overarching goal (e.g., sustainability) encompasses 16 specific targets.
 - Such restructuring can aid in creating a more memorable and coherent framework, making it easier for the public and policymakers to understand and engage with.
- Ranking Human Rights Charters for Memorability:
 - Human rights charters can be ranked based on their structural simplicity, use of universally understood and impactful language, and the presence of memorable and symbolic elements.
 - Charters that effectively use narrative techniques, metaphorical language, and clear, impactful statements are likely to be more memorable.

- Feedback and Iterative Improvement:
 - Critical feedback is essential for these documents. It ensures that they not only convey their message clearly and memorably but also resonate with a diverse audience.
 - Iterative revisions based on feedback can help refine these documents, enhancing their memorability and effectiveness.
- Application of Aesthetic and Rhythmic Elements:
 - Incorporating elements of rhythm, repetition, and even poetic devices can enhance the memorability of such documents. While not typical for legal or strategic texts, these elements can make the content more engaging and easier to recall.
- Visual and Graphical Elements:
 - Visual aids, infographics, and symbolic imagery can complement written content, making complex ideas more accessible and memorable.
- Educational and Promotional Strategies:
 - Beyond the documents themselves, educational and promotional strategies (e.g., campaigns, educational programs, visual media) play a crucial role in enhancing memorability and implementation.

In conclusion, the memorability of strategic declarations, human rights charters, and frameworks like the UN SDGs can be critically assessed and enhanced through thoughtful structure, language, and presentation. This process is vital to ensure these important documents effectively reach and impact their intended audiences, leading to greater engagement and implementation of their principles.

Question to ChatGPT: Whilst the 30 articles of the *Universal Declaration of Human Rights* could be deemed relatively memorable -- with factors $2 \times 3 \times 5$ -- this is perhaps seriously undermined by the fact that the relationship between the articles does not reflect these. The memorability and strategic coherence of the 17 SDGs (even as $16+1$) are even more challenging, although -- creatively and questionably -- it might be inferred that the 17 correspond (perhaps unconsciously) to the 17-fold wallpaper group, the 17 crystallographic space groups in two dimensions, or 17 four-dimensional parallelotopes that are zonotopes. Potentially more problematic (as 13×13) is the pattern of 169 "tasks" associated with the SDGs

Your insights into the memorability and strategic coherence of the Universal Declaration of Human Rights (UDHR), the Sustainable Development Goals (SDGs), and their potential mathematical or geometrical interpretations are quite thought-provoking. Let's explore these aspects:

- UDHR's 30 Articles and Memorability:
 - The factorization of 30 ($2 \times 3 \times 5$) in the UDHR's articles could aid memorability to some extent due to its simple mathematical structure. However, the actual content and interrelation of these articles may not align with this structure, which could impact their overall memorability and perceived coherence.
 - The challenge is to present these articles in a manner that emphasizes their interconnectedness and the overarching principles they represent.
- SDGs and Potential Geometrical Correlations:
 - The 17 SDGs present a more complex case. The number 17, though prime, could creatively be linked to structures like the 17 wallpaper groups or the 17 crystallographic space groups in two dimensions, as you mentioned. Such correlations, while conceptually intriguing, might be more metaphorical than practical in enhancing memorability or coherence.
 - The idea of relating them to four-dimensional parallelotopes or zonotopes is an innovative approach to visualizing their interrelations and dimensions, although this might be conceptually challenging for general audiences.
- 169 Tasks Associated with the SDGs:
 - The number 169, being the square of 13 (13×13), presents a grid-like structure that could be utilized for visual representation. However, the challenge lies in ensuring that this structure aids in understanding the tasks' interconnectedness and doesn't oversimplify their complexity.
 - One approach might be to organize these tasks in a matrix or grid format, highlighting thematic clusters or interdependencies.
- Enhancing Memorability and Coherence:

- To improve the memorability and strategic coherence of these frameworks, employing narrative techniques, visual aids, and thematic clustering can be effective.
- Creating educational and promotional materials that simplify and connect the concepts without losing their depth and intricacy is key.
- Interactive digital platforms or visualizations that allow exploration of the relationships and interdependencies between different goals or articles could also be beneficial.
- Broader Implications:
 - These discussions highlight the importance of not just the content but also the presentation and structure of such global frameworks. How they are communicated and visualized can significantly impact public understanding and engagement.

In summary, while the mathematical or geometrical interpretations of these frameworks offer fascinating insights, the practical focus should be on enhancing their comprehensibility and perceived coherence through effective communication, education, and visualization strategies. This approach can help in bridging the gap between abstract goals or principles and their practical, relatable understanding.

Question to ChatGPT: The response recalls the considerable interest of the music industry in the evaluation of potential hit songs. Do you have any trace of the techniques used in that context

The music industry employs a variety of techniques to evaluate and predict the potential success of songs. These techniques combine both artistic intuition and increasingly sophisticated data analysis tools. Here's an overview of some key methods used in the industry:

- **Market Analysis and Consumer Trends:** Record labels and music producers often conduct extensive market research to identify current trends in music preferences, genres, and consumer behavior. This helps in predicting which styles or themes might be more successful.
- **Data Analytics and Streaming Metrics:** With the rise of digital music platforms, data analytics has become crucial. Metrics like the number of streams, downloads, playlist additions, and user engagement on platforms like Spotify, Apple Music, and YouTube provide valuable insights into a song's popularity and potential for success.
- **Social Media and Online Presence:** The buzz around a song or artist on social media platforms can be a strong indicator of potential success. Tools that analyze social media mentions, shares, likes, and overall engagement are used to gauge public interest and predict hits.
- **Collaborations and Features:** Collaborations with established artists or featuring popular artists in songs can significantly boost a song's hit potential. Record labels often use this strategy to introduce new artists or boost a song's appeal.
- **Algorithmic Predictions:** Some companies and platforms use algorithms to predict hits. These algorithms analyze various aspects of a song, including tempo, rhythm, harmony, lyrics, and even the emotional tone, comparing them to patterns found in past hit songs.
- **Focus Groups and Test Audiences:** Traditional methods like focus groups and test audiences are still used. Songs are played to select groups, and their reactions are monitored to gauge potential success.
- **Expert A&R (Artists and Repertoire) Teams:** A&R teams in record labels have the expertise to identify potential hits. They consider factors like the artist's appeal, song quality, marketability, and current trends in the industry.
- **Historical Analysis:** Analyzing historical trends in music, including what has worked in the past in terms of themes, genres, and artist types, can provide insights into what might be successful in the future.
- **Song Attributes Analysis:** Specific attributes of a song, such as its catchiness, melody, hook, lyrics, and production quality, are critically evaluated to assess its hit potential.
- **Promotional Strategies:** The potential success of a song is also influenced by the marketing and promotional strategies employed. This includes radio play, music video production, online marketing, and PR campaigns.

While these techniques offer various ways to gauge a song's potential success, the music industry is also known for its unpredictability. Sometimes, songs become hits unexpectedly, defying all predictions and analyses. This unpredictability is part of what keeps the music industry dynamic and continually evolving.

Question to ChatGPT: This is a powerful array of techniques and could be usefully contrasted with those employed in the formulation of strategies, especially those designed to reconcile opposing parties. Many are presumably applied to songs proposed for the Eurovision Song Contest, suggesting the need for an analogous UNvision Song Contest -- but with a strategic focus

The concept of a 'UNvision Song Contest' with a strategic focus on reconciliation and bringing together opposing parties is an innovative and culturally engaging idea. This concept could leverage some of the techniques used in the music industry to predict hit songs, while also incorporating methods more specific to conflict resolution and international diplomacy. Let's contrast and adapt these techniques for such a strategic initiative:

- **Understanding Cultural Contexts and Diversity:** Similar to how market analysis in the music industry identifies trends, understanding the diverse cultural contexts and sensitivities of participating countries would be crucial in a UNvision contest. This would ensure that the content is respectful, relevant, and appealing to a global audience.
- **Data Analytics and Audience Engagement:** Just as streaming metrics are used to gauge a song's popularity, data analytics could be employed to understand global audience preferences and responses, particularly regarding themes of peace, unity, and cooperation.
- **Social Media for Global Interaction:** Utilizing social media platforms to promote interaction and understanding among different cultural groups, similar to how artists engage with their fan base.
- **Collaborative Songwriting and Performances:** Encouraging artists from different backgrounds or countries to collaborate on performances could mirror the music industry's approach to collaborations and features, promoting unity and understanding.
- **Algorithmic Analysis for Universal Appeal:** Using algorithms to analyze and select songs that have elements (melody, rhythm, lyrics) with universal appeal and messages that resonate with themes of peace and reconciliation.
- **Expert Panels for Diplomatic Sensitivity:** Involving experts in international relations and conflict resolution in the selection and evaluation process, similar to A&R teams in the music industry, to ensure that the content is sensitive to international issues and contributes positively to global dialogue.
- **Focus on Message and Narrative:** Emphasizing the importance of the song's message and narrative, focusing on themes that promote understanding, peace, and unity, much like how song attributes are critically evaluated in the music industry.
- **Innovative Promotional Strategies:** Developing unique promotional strategies that highlight the contest's role in fostering international dialogue and cooperation.
- **Educational and Cultural Exchange Programs:** Incorporating educational elements, such as cultural exchange programs or discussions about international issues, to deepen the impact and understanding among participants and audiences.
- **Inclusive and Accessible Platforms:** Ensuring that the contest and its content are accessible to a diverse global audience, including considerations for language and media accessibility.

A 'UNvision Song Contest' with a strategic focus could serve as a powerful platform for cultural exchange, international cooperation, and the promotion of peace and understanding through music. By adapting techniques from the music industry and integrating them with principles of diplomacy and conflict resolution, such an initiative could have a significant impact on global relations and perceptions.

Recognition of polyhedra as systems and systems as polyhedra

Question to ChatGPT: In the magnum opus of Buckminster Fuller it is asserted that: All systems are polyhedra: All polyhedra are systems (*Synergetics: Explorations in the Geometry of Thinking*, 1975-1979, [400.56](#)). Could you comment on recent understanding of this perspective.

Buckminster Fuller's assertion that "All systems are polyhedra; All polyhedra are systems" from his work "Synergetics: Explorations in the Geometry of Thinking" reflects his deep interest in the

geometrical and structural aspects of both physical and conceptual systems. Fuller, known for his innovative ideas in architecture, design, and systems theory, often used polyhedra, particularly the geodesic dome, as a metaphor and a structural solution in his work. His perspective has influenced various fields, from architecture to systems theory. Let's explore how this perspective is understood and applied in recent times:

- **Geometry in Systems Theory:** Fuller's ideas have contributed to the understanding that systems (be they ecological, social, or mechanical) can be conceptually modeled as polyhedral structures. The vertices, edges, and faces of a polyhedron can metaphorically represent the components, relationships, and processes of a system, highlighting how interconnected and interdependent parts contribute to the whole.
- **Architectural and Engineering Applications:** In architecture and engineering, Fuller's concepts continue to inspire the design of structures that are both efficient and sustainable. The geodesic dome, one of Fuller's most famous contributions, is a practical embodiment of his principles, showcasing strength, efficiency, and the ability to cover large spaces without internal supports.
- **Computational and Network Models:** In computer science and network theory, the idea of systems as polyhedral structures has parallels in the modeling of complex networks. Algorithms often use geometric and topological concepts to solve problems related to connectivity, optimization, and pattern recognition.
- **Environmental and Ecological Studies:** Fuller's holistic approach to understanding the interconnectivity and sustainability of systems has resonance in contemporary environmental and ecological studies. The concept of viewing the Earth as a synergistic system, often referred to as "Spaceship Earth," is a direct legacy of Fuller's work.
- **Education and Problem Solving:** Fuller's interdisciplinary approach to thinking and problem-solving continues to influence educational philosophies and methodologies. The idea of integrating geometry, physics, design, and systems thinking into a cohesive educational framework can be traced back to his principles.
- **Scientific and Mathematical Research:** In scientific fields, particularly those exploring complex systems and network theory, Fuller's ideas about the polyhedral nature of systems have metaphorical and practical value. They inspire new ways of visualizing and understanding the interconnectedness and complexity of various systems.

In summary, Buckminster Fuller's assertion about systems and polyhedra remains influential in contemporary thinking across multiple disciplines. His ideas continue to inspire innovation in design, architecture, systems theory, and beyond, underscoring the importance of an integrated, holistic approach to understanding and solving complex problems.

Question to ChatGPT: Reference is made to [frameworks supporting a polyhedral model](#) within a computer -- requiring software to represent the objects of this framework and perform operations upon them. Are there polyhedral models in which the edges are understood to be dimensions in the sense cultivated by physics

Yes, in theoretical physics, especially in higher-dimensional models and theories, there are instances where the edges of polyhedral models can be conceptualized as dimensions. This approach is often seen in fields like string theory and other advanced theoretical frameworks that go beyond the traditional four-dimensional spacetime of general relativity.

- **String Theory and Polyhedra:** In string theory, for example, the basic idea is that fundamental particles are not point-like dots, but rather one-dimensional strings. These strings can vibrate in multiple dimensions, and the vibrational modes correspond to different particles. The shapes these strings take in higher dimensions can sometimes be conceptualized as polyhedra, where the edges represent different dimensions or degrees of freedom along which the strings can extend or vibrate.
- **Calabi-Yau Spaces:** A specific example is the use of Calabi-Yau spaces in superstring theory. Calabi-Yau spaces are complex, multi-dimensional shapes (often six-dimensional in addition to our familiar four dimensions of spacetime) that are crucial for compactifying extra dimensions in string theory. These shapes, while not polyhedra in the traditional sense, have a multi-dimensional geometry where each dimension can be thought of as contributing to the overall shape in a way analogous to how edges define a polyhedron.
- **Higher-Dimensional Theories:** In other higher-dimensional theories, such as 10-

dimensional supergravity or 11-dimensional M-theory, the extra dimensions are often visualized geometrically. While these are not always explicitly rendered as polyhedra, the concept of higher dimensions contributing to a more complex geometric shape is a common theme.

- Visualization and Mathematical Models: In terms of practical visualization and mathematical modeling, there are software tools and frameworks that allow for the representation and manipulation of these higher-dimensional structures. Programs that specialize in higher-dimensional geometry and topology can model and visualize objects where the edges (or other elements) represent dimensions or complex interactions in higher-dimensional spaces.
- Metaphorical Interpretation: It's important to note that in many cases, these higher-dimensional polyhedral models are more metaphorical or abstract than literal. They are used as tools to help conceptualize and work with very complex, often non-intuitive ideas in theoretical physics.

In summary, the use of polyhedral models where edges represent dimensions is a concept found in theoretical physics, particularly in the context of string theory and higher-dimensional space theories. These models help physicists visualize and work with the complex geometries involved in these advanced theories.

Strategic relevance of 4D frameworks: highlighting 81-fold patterns

https://en.wikipedia.org/wiki/List_of_uniform_polyhedra

It could be considered curious that the potential relevance of "four-dimensional" frameworks is appropriately neglected as obscure and irrelevant, compared to the conventional focus on 2D and 3D. Whilst "global" necessarily implies 3D in a static sense, the challenge of global governance -- faced with emerging crises -- clearly implies an additional dimension, suggesting the need for a 4D framework, as may be variously argued (*Strategic Embodiment of Time: configuring questions fundamental to change*, 2010; *Ungovernability of Sustainable Global Democracy? Towards engaging appropriately with time*, 2011).

The strange commitment to a "static" perspective is exemplified by the titles of a range of reports on global conditions (*Dynamic Transformation of Static Reporting of Global Processes*, 2013). Do global institutions merit recognition as 4D entities (*Envisaging NATO Otherwise -- in 3D and 4D?* 2017).

The unfamiliar language used to describe 4D frameworks -- as polychora or polytopes -- is indeed discouraging. They can only be visualized by projecting them into 3D -- much as the global form of the Earth can only be visually mapped by projecting it into 2D.

Clarified in terms of *Regular polychoron numbers* (*On-Line Encyclopedia of Integer Sequences*), there are six regular convex polychora (4-dimensional hyper-solids) which, except for the 24-cell, are the analogues of the *Platonic solids*. Listed by increasing number of vertices, the six regular convex polychora are:

- 5 vertices (self dual) 5-cell (pentachoron) or 4-simplex (a [hypertetrahedron](#))
- 8 vertices 16-cell or 4-cross polytope or 4-orthoplex (a [hyperoctahedron](#))
- 16 vertices 8-cell or 4-cube or 4-orthotope or tesseract (a [hypercube](#))
- 24 vertices (self dual) [24-cell](#), which has no perfect analogy in higher or lower dimensional spaces,
- 120 vertices 600-cell (a [hypericosahedron](#))
- 600 vertices 120-cell (a [hyperdodecahedron](#).)

The 5-cell and 24-cell are self-dual, the 16-cell is the dual of the 8-cell, and the 600- and 120-cells are dual to each other. Expressed otherwise as *Uniform polytopes* (*Wikipedia*), there are 16 [regular 4-polytopes](#), with the property that all cells, faces, edges, and vertices are congruent:

- 6 [regular convex 4-polytopes](#) (as above): [5-cell](#), [8-cell](#), [16-cell](#), [24-cell](#), [120-cell](#), and [600-cell](#).
- 10 [regular star 4-polytopes](#): [icosahedral 120-cell](#), [small stellated 120-cell](#), [great 120-cell](#), [grand](#)

[120-cell](#), [great stellated 120-cell](#), [grand stellated 120-cell](#), [great grand 120-cell](#), [great icosahedral 120-cell](#), [grand 600-cell](#), and [great grand stellated 120-cell](#).

Representation of 81-fold sets in 4D: As noted earlier, the argument here followed from a concern with the [Coherent Reconciliation of Eastern and Western Patterns of Logic](#) (2023). In contrast with the diffidence of current Western thinking with regard to the organization of large sets -- other than the table of genetic codons and the periodic table -- it is striking to note the traditional focus on 72-fold sets (discussed above) and the classical Chinese thinking on the 64-fold set of the [I Ching](#) ("*The Book of Changes*") and the 81-fold sets of the [Taixuanjing](#) ("*The Canon of Supreme Mystery*") and the [Tao Te Ching](#)

Whereas the 64-fold set and the 72-fold set lend themselves to representation in 3D using polyhedra (as previously illustrated), this is not the case with the 81-fold set. The [Tao Te Ching](#) continues to be a source of inspiration in East and West.

Question to ChatGPT: Whereas there are useful polyhedra for mapping traditional patterns of 64 and 72, this does not seem to be the case for 81. Would a polychoron be more suitable

Mapping traditional patterns onto polyhedra or polychora depends on the specific requirements of the pattern and how well they align with the structural properties of these geometric shapes. When dealing with a pattern of 81 elements, the challenge is finding a polyhedron or polychoron (a four-dimensional polytope) with a structure that can naturally accommodate 81 distinct points or facets.

For a pattern of 81 elements, a polychoron might indeed be more suitable, especially if the complexity of the pattern demands a higher-dimensional framework for an appropriate representation. In three dimensions, regular polyhedra (Platonic solids) and semi-regular polyhedra (Archimedean solids) have a limited number of vertices, edges, and faces, which might not suitably map 81 elements. In contrast, four-dimensional polytopes, or polychora, offer a more complex structure with a greater number of vertices, edges, faces, and cells, potentially providing a more fitting framework for mapping 81 elements.

Here are a few considerations when looking at polychora for mapping 81 elements:

- **Vertex, Edge, Face, and Cell Count:** You'll need a polychoron with a total count of vertices, edges, faces, and/or cells that can accommodate 81 elements. The distribution of these elements across the four-dimensional structure should make sense in the context of the pattern you're trying to represent.
- **Symmetry and Structure:** The symmetry and structure of the polychoron should align with the inherent structure of the pattern. Patterns often have internal symmetries or relationships that should be reflected in the geometric representation.
- **Visualization Challenges:** One challenge with using polychora is visualization. Four-dimensional structures cannot be fully represented in three-dimensional space, so any representation is necessarily a projection or a simplification. This can make understanding and working with these models more abstract and less intuitive.
- **Specific Types of Polychora:** There are many types of polychora, just as there are many types of polyhedra. Researching specific types (such as regular polychora, like the 5-cell or 16-cell, or more complex ones like the 120-cell or 600-cell) to see which best fits your pattern would be necessary.
- **Mathematical and Computational Tools:** Modern mathematical and computational tools can be very helpful in modeling and understanding polychora. Software that can handle higher-dimensional geometry might be necessary to explore these models effectively.

In summary, if a three-dimensional polyhedron doesn't seem suitable for mapping a pattern of 81 elements, exploring four-dimensional polychora is a logical next step. The key is to find a polychoron whose structure aligns well with the pattern, keeping in mind the challenges of visualization and complexity inherent in four-dimensional geometry.

Question to ChatGPT: The OEIS has a table of "*Regular polychoron numbers sequences*" which has, for 16, the sequence: 0, 1, 16, 81, 256... ([Regular polychoron numbers](#)). Can you explain this

The sequence you're referring to in the OEIS (Online Encyclopedia of Integer Sequences) for

"Regular Polychoron Numbers" likely pertains to the number of vertices, edges, faces, and cells in various regular polychora (four-dimensional analogs of three-dimensional polyhedra). I can provide some general insights based on mathematical principles that might be relevant to the sequence you mentioned ("0, 1, 16, 81, 256...") in the context of regular polychoron numbers.

- **Understanding Polychorons:** Polychorons are four-dimensional analogs of three-dimensional polyhedra. Just as a polyhedron is defined by its vertices (0D), edges (1D), faces (2D), and solid (3D), a polychoron is defined by its vertices, edges, faces, and cells (3D analogs of faces in 4D space).
- **Sequence Interpretation:** The sequence you mentioned could represent various properties of these polychorons as you move through different dimensions. For instance, in a 4D hypercube (tesseract), there are:
 - 0: Possibly representing the null object.
 - 1: Representing a single point or vertex.
 - 16: The number of vertices in a tesseract.
 - 81 and 256: These might represent other properties like the number of edges, faces, or cells in more complex polychorons. The exact property would depend on the specific context of the sequence.
- **Higher-Dimensional Mathematics:** In higher-dimensional mathematics, these sequences often follow specific patterns or formulas. They can be related to the geometrical properties of the shapes, such as the number of specific elements (like vertices or cells) that increase as you move to higher dimensions.
- **OEIS as a Resource:** The OEIS is a comprehensive database of integer sequences used in various mathematical and scientific contexts. Each entry in OEIS typically includes a sequence, its mathematical or scientific significance, and references to literature where the sequence appears or is discussed.

The sequence "0, 1, 16, 81, 256..." in the context of a regular polychoron with 16 elements (such as a tesseract, which is a four-dimensional hypercube) can be explained as follows:

- **0 and 1:** These are typical starting points in sequences of mathematical objects, often representing the null object and the simplest non-trivial object, respectively. In the context of geometric figures, '0' might represent the empty set, and '1' might represent the simplest polychoron.
- **16:** This is the number of vertices in a tesseract. A tesseract, also known as a hypercube in four dimensions, has 16 vertices. It's the four-dimensional analog of a cube.
- **81:** This number in the sequence is more intriguing. In the context of a regular polychoron with 16 vertices, 81 could represent a property related to its four-dimensional structure. It could be referring to the number of certain elements (like edges, faces, or cells) in a more complex polychoron, or it could be part of a mathematical formula related to these structures. The specific meaning of '81' would depend on the definition of the sequence and the properties of the polychora being discussed.
- **256:** In the context of regular polychora, 256 could represent a characteristic of a higher-order polychoron or a value obtained from a mathematical formula applied to these four-dimensional shapes.

To understand the specific meaning of these numbers in the context of regular polychoron numbers, one would need to look at the exact definition and context provided in the OEIS entry. The OEIS often includes formulas, references, and explanations that clarify how these sequences are generated and what they represent in mathematical terms.

Question to ChatGPT: In a page on [Tesseract and 16-cell faces](#) (Wikiversity) a table is headed by "This list shows the $3^4 = 81$ *k*-faces of the tesseract and its dual 16-cell". Can you comment on the meaning of "k-faces"

The term "k-faces" in the context of geometry, particularly when discussing polytopes such as tesseracts and their duals like the 16-cell, refers to the faces of different dimensions (k-dimensions) within these polytopes. Polytopes, including tesseracts (which are four-dimensional analogs of cubes) and 16-cells (which are four-dimensional analogs of octahedra and the dual of tesseracts), have elements of various dimensions. These elements are commonly referred to as follows:

- **Vertices (0-faces):** These are points where the edges of the polytope meet. In three

dimensions, these are the corners of a polyhedron.

- Edges (1-faces): These are the line segments connecting the vertices. In a cube, these are the lines that form the edges of the squares.
- Faces (2-faces): In three dimensions, these are the flat surfaces bounded by the edges. In a cube, these are the squares themselves.
- Cells (3-faces): In four dimensions, the "faces" of a polytope are three-dimensional "cells". For a tesseract, each "face" is a three-dimensional cube.

When discussing a tesseract (or 4-cube), which is a four-dimensional hypercube, "k-faces" refers to the elements of all these dimensions ($k = 0, 1, 2, 3$). Therefore, when the page mentions "the $3^4 = 81$ k-faces of the tesseract," it is referring to all such elements of the tesseract:

- The 3^4 (3 to the power of 4, which equals 81) accounts for all possible combinations of dimensions within the tesseract. This includes all vertices, edges, 2-dimensional faces, and 3-dimensional cells.
- The number 81 comes from the fact that in four dimensions, there are four types of k-faces (0, 1, 2, and 3-dimensional faces), and each can have different configurations or orientations within the tesseract.
- In the case of the dual of the tesseract, the 16-cell, a similar principle applies. The 16-cell, being a four-dimensional figure, also has vertices, edges, faces, and cells.

The notion of "k-faces" is a way to generalize the concept of faces to higher dimensions, which is essential in the study of polytopes in multi-dimensional spaces. It is a part of a broader mathematical field known as topology, which deals with properties of space that are preserved under continuous transformations.

Question to ChatGPT: The term "k-faces" refers to the faces of a polytope of dimension k. In the case of the 16-cell, which is a four-dimensional regular polytope with 16 cells, all of which are regular tetrahedra, the OEIS lists 81 k-faces. What totals to 81

The term "k-faces" in geometry refers to the elements of a polytope in k dimensions. For a four-dimensional regular polytope like the 16-cell, k-faces include elements of different dimensions: vertices (0-dimensional), edges (1-dimensional), faces (2-dimensional), and cells (3-dimensional). The total count of these elements for the 16-cell adds up to 81. Here's how they are typically broken down:

- Vertices (0-Dimensional K-Faces): The points where the cells meet.
- Edges (1-Dimensional K-Faces): The line segments connecting the vertices.
- Faces (2-Dimensional K-Faces): The two-dimensional surfaces bounded by the edges.
- Cells (3-Dimensional K-Faces): The three-dimensional volumes enclosed by the faces.

In the case of the 16-cell:

- Vertices: It has 8 vertices.
- Edges: Each vertex is connected to 4 others, giving $4 \times 8 / 2 = 16$ edges, since each edge is counted twice.
- Faces: There are 2 faces (triangles) per edge, giving $2 \times 16 = 32$ faces, but each face is counted twice, so there are $32 / 2 = 16$ faces.
- Cells: It comprises 16 tetrahedral cells.

Adding these up gives 8 (vertices) + 16 (edges) + 16 (faces) + 16 (cells) = 56 . This is different from the 81 k-faces mentioned. The discrepancy might arise from a different way of counting or a different interpretation of what constitutes a k-face in the specific context of the OEIS listing.

To understand why OEIS lists 81 k-faces for the 16-cell, one would need to look at the specific definitions and counting methods used in that entry. It's possible that they might be including some higher-dimensional elements or using a different approach to count the faces, edges, and vertices.

Question to ChatGPT: From a different source ([Face lattice of the tesseract \(Bilinski\)](#)) the total number of k-faces in the tesseract ($0 \leq k \leq 4$) is $3^4 = 81$. I note elsewhere that the 16-cell is indicated as having 8 vertices. How is this reconciled with your response

Thank you for providing the detailed breakdown from the face lattice of the tesseract (Bilinski). This clarification helps to reconcile the numbers. Let's go through the information for the tesseract (also known as the 4-cube or hypercube) and see how the total number of k-faces adds up to 81:

A tesseract is a four-dimensional hypercube, and its k-faces can be categorized as follows:

- Vertices (0-Dimensional K-Faces): The points where the edges meet. The tesseract has 16 vertices.
- Edges (1-Dimensional K-Faces): The line segments connecting the vertices. The tesseract has 32 edges.
- Faces (2-Dimensional K-Faces): The two-dimensional squares bounded by the edges. The tesseract has 24 faces.
- Cells (3-Dimensional K-Faces): The three-dimensional cubes (or cells) enclosed by the faces. The tesseract has 8 cells.
- 4-Polytope (4-Dimensional K-Faces): The entire four-dimensional volume of the tesseract. There is 1 of these, being the tesseract itself.

If we add these up: 16 (vertices) + 32 (edges) + 24 (faces) + 8 (cells) + 1 (4-polytope) = 81

This reconciles with the statement that there are 81 k-faces in the tesseract. Each k-face represents a different dimension of the tesseract's geometry, from 0-dimensional points (vertices) up to the 4-dimensional entirety of the shape (the 4-polytope). My previous response, which did not account for the 4-dimensional volume of the tesseract itself, resulted in a different total. Thank you for pointing out this oversight, and I appreciate the opportunity to correct and clarify the information.

Question to ChatGPT [after several "clarifications"]: Can you confirm the number of k-faces in the 8-cell and in the 16-cell (as its dual). Should they not be the same

You are correct in pointing out that the 16-cell, a four-dimensional polytope, indeed has 8 vertices, not 16. This is a key difference between the 16-cell and the tesseract (or 4-cube), which has 16 vertices. I apologize for the confusion caused by the discrepancy in my earlier responses. Let's correct and clarify the information regarding the number of k-faces in the 8-cell (tesseract) and the 16-cell:

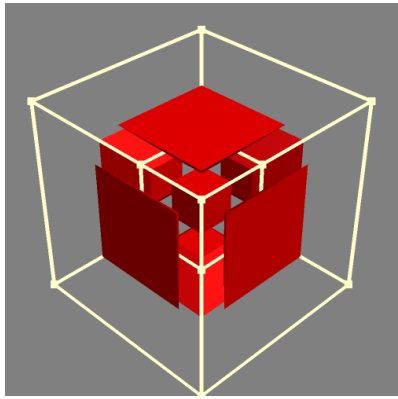
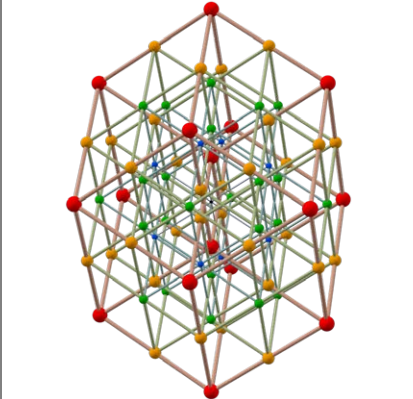
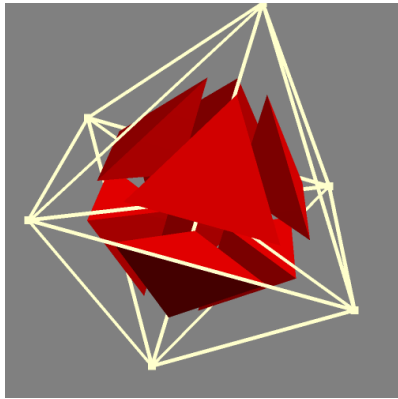
- 8-Cell (Tesseract):
 - Vertices (0-Dimensional K-Faces): 16
 - Edges (1-Dimensional K-Faces): 32
 - Faces (2-Dimensional K-Faces): 24 (squares)
 - Cells (3-Dimensional K-Faces): 8 (cubes)
 - 4-Polytope (4-Dimensional K-Faces): 1 (the tesseract itself)
 - Total k-faces: 16 (vertices) + 32 (edges) + 24 (faces) + 8 (cells) + 1 (4-polytope) = 81
- 16-Cell:
 - Vertices (0-Dimensional K-Faces): 8
 - Edges (1-Dimensional K-Faces): 24
 - Faces (2-Dimensional K-Faces): 32 (triangles)
 - Cells (3-Dimensional K-Faces): 16 (tetrahedra)
 - 4-Polytope (4-Dimensional K-Faces): 1 (the 16-cell itself)
 - Total k-faces: 8 (vertices) + 24 (edges) + 32 (faces) + 16 (cells) + 1 (4-polytope) = 81

For both the 8-cell and the 16-cell, the total number of k-faces is 81. This includes the vertices, edges, faces, cells, and the polytope itself as a 4-dimensional k-face. The confusion in the earlier response was due to not including the polytope itself as a 4-dimensional k-face in the count for the 8-cell. I apologize for the oversight and appreciate your patience in seeking clarification.

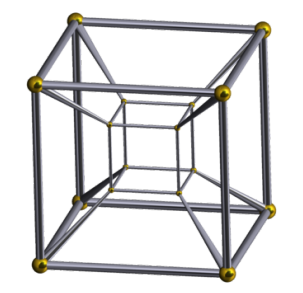
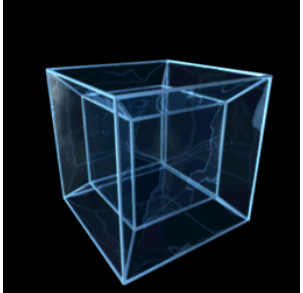
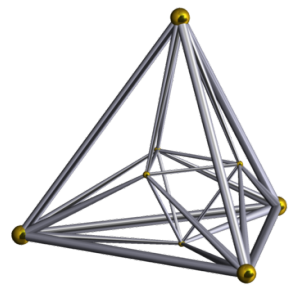
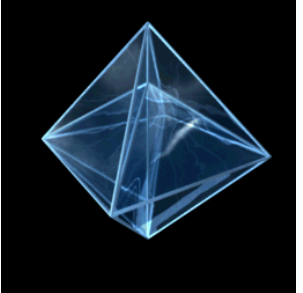
The discrepancy in my previous response regarding the vertex count of the 16-cell was an error, and I appreciate your attention to this detail. The 16-cell, being a different four-dimensional polytope than the tesseract, indeed has 8 vertices, not 16. Thank you for the opportunity to correct this information.

Indicative visualization of 81-fold patterns in 4D

The iterative interaction with ChatGPT in the light of various sources on the tesseract and its dual made it apparent that there is an obvious challenge to their comprehension and visualization -- especially as a consequence of alternative terminologies and modes of counting. As sources, the *Wikipedia* entries ([Tesseract](#), [16-cell](#)) and the *Polytope Wiki* ([Tesseract](#), [Hexadecachoron](#)) provide extensive commentary and potentially indicative illustrations and animations.

Indicative animations of 3D projections of 4D polytopes with 81 k-faces		
Octachoron (tesseract) 8-cell, 16 vertices	81 centers of tesseract k-faces red=vertices, ochre=edges, green=faces, blue=cells (and 216 links between them)	Hexadecachoron (tesseract dual) 16-cell, 8 vertices
		
Animation created with Stella4D	Reproduced from: Face lattice of the tesseract (Bilinski) , <i>Wikipedia</i>	Animation created with Stella4D

A major source for exploration of 4D structures is the [Stella4D](#) application of Robert Webb. In geometry, a [Schlegel diagram](#) is a projection of a polytope from a higher dimension into a lower dimensions through a point just outside one of its facets.

Alternative visualizations of the 8-cell (tesseract) and its 16-cell dual			
8-cell (tesseract)		16-cell (hexadecachoron)	
Schlegel diagram	Animation of 3D projection of a tesseract performing a simple rotation about a plane in 4D.	Schlegel diagram	Animation of 3D projection of a 16-cell performing a simple rotation
			
Created by Stella4D : Reproduced from Wikimedia Commons CC BY-SA 3.0	JasonHise , Public domain, via Wikimedia Commons	Created by Stella4D : Reproduced from Wikimedia Commons CC BY-SA 3.0	JasonHise at English Wikipedia , Public domain, via Wikimedia Commons

Representation of 81-fold Chinese encoding in 4D

Question to ChatGPT: Part of my interest is in the organization of the *Tao Te Ching* (81 stanzas) and the 81 conditions of the *Taixuanjing*. These may reflect an early insight into the pattern you describe above

Your interest in correlating the structure of the Tao Te Ching and the Taixuanjing with higher-dimensional geometric concepts like those in a tesseract or other complex polytopes is a fascinating and interdisciplinary approach. The Tao Te Ching, traditionally consisting of 81 stanzas, and the 81 conditions of the Taixuanjing potentially reflecting patterns found in higher-dimensional geometry, could be an example of how ancient wisdom and modern mathematical concepts might intersect in unexpected ways.

- **Tao Te Ching and Taixuanjing:** Both of these ancient texts have deep philosophical and mathematical significance. The Tao Te Ching, central to Taoism, is known for its profound insights into nature, life, and the universe. The Taixuanjing, also known as the Canon of Supreme Mystery, is a more explicitly mathematical text and is an early example of a ternary (base-3) numeral system.
- **81 as a Significant Number:** In both texts, the number 81 (which is 3^4) holds a special place. This number can be interpreted in various ways, from a philosophical, numerological, or mathematical perspective. In the context of Taoist philosophy, it might represent a completeness or a full cycle, given the importance of numbers like 3 and 9 in Chinese cosmology and numerology.
- **Correlation with Higher-Dimensional Geometry:** Drawing a parallel between these texts and higher-dimensional geometry (like the tesseract's 81 k-faces) is an intriguing thought. While the texts themselves are unlikely to have been intended as direct representations of higher-dimensional geometric models, the correspondence in numbers might symbolize similar principles, such as completeness, unity in diversity, or the complexity of the universe.
- **Interdisciplinary Analysis:** Exploring these texts through the lens of modern geometry and topology could yield interesting insights. It's a form of interdisciplinary analysis that combines philosophy, ancient wisdom, and modern mathematical theory. Such an approach can offer a fresh perspective on both the ancient texts and contemporary mathematical concepts.
- **Symbolism and Abstract Interpretation:** It's important to note that any correlation between these ancient texts and higher-dimensional geometry would be symbolic or abstract rather than literal. The rich metaphorical and philosophical language of texts like the Tao Te Ching is open to many levels of interpretation, where numerical patterns might reflect broader principles of balance, harmony, and the structure of reality.

In summary, while the Tao Te Ching and Taixuanjing are not directly related to the concept of higher-dimensional geometry, the use of numbers and patterns in these texts can be intriguingly similar to mathematical concepts, offering a unique lens through which to explore these ancient works. This interdisciplinary approach can enrich our understanding of both the ancient texts and the mathematical models.

Binary versus ternary in logic and computer operation

It is curious to note that the 64-fold pattern of *I Ching* hexagrams has traditionally been configured as both a square and circular pattern -- known to have inspired the work of [Gottfried Leibnitz](#) (Mary von Aue, [How the 'I Ching' Inspired His Binary System](#), *Inverse*, 2 July 2018). It was presented to him 1701 in the form of the [Shao Yong](#) circle of 64 hexagrams as the Western originator of binary coding.

In contrast to the binary representation of broken and unbroken lines (corresponding to 1s and 0s of conventional computer operations), the [Taixuanjing](#) employs 81 ternary tetragrams (sequences of four lines, each of which may be unbroken, broken once, or broken twice). There is a degree of current recognition of [ternary logic](#) (three-valued logic, trinary logic, or trivalent logic) employed (or envisaged) in [ternary computers](#). Ternary quantum computers use [qutrits](#) rather than [trits](#). A qutrit is a [quantum state](#) that is a complex unit vector in three dimensions.

Question to ChatGPT: Could you comment on the 81 k-face "balanced ternary vectors" of 4 positions in relation to the ternary tetragram by which the 81 conditions of the Taixuanjing are represented

The concept of 81 k-face "balanced ternary vectors" of 4 positions relates intriguingly to the ternary tetragram system used in the Taixuanjing (Canon of Supreme Mystery), an ancient Chinese text. Both involve the number 81 and the use of a ternary (base-3) system, but they are applied in


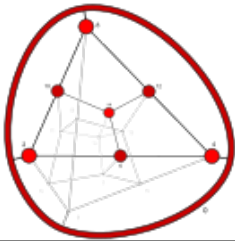
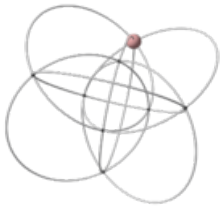

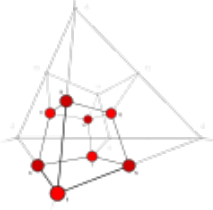
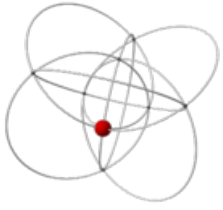

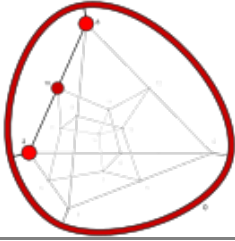
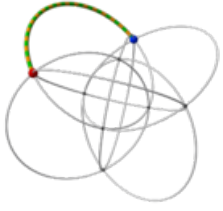

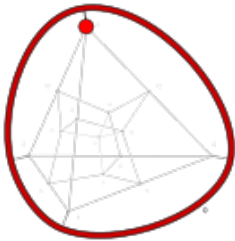
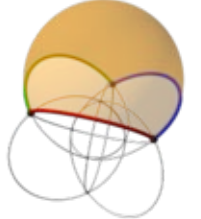

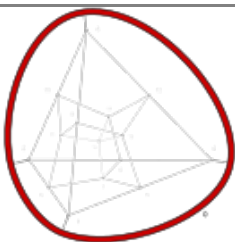
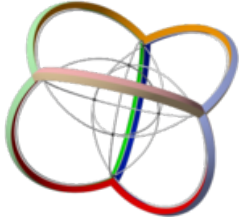

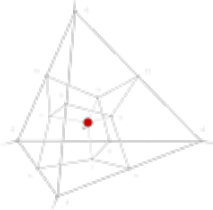
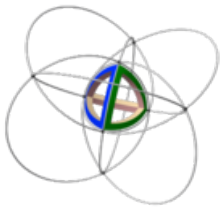
different contexts.

- 81 Balanced Ternary Vectors of 4 Positions:
 - In mathematics, a balanced ternary system uses three digits (for example, -1, 0, +1 or sometimes represented as -, 0, +).
 - A vector of 4 positions in balanced ternary can represent any number from this set raised to the fourth power, thus leading to $3^4 = 81$ possible combinations.
 - Each position in the vector can take one of three values, leading to a rich combinatorial structure.
 - This concept is often used in computer science and mathematics for various purposes, including efficient data representation and problem-solving strategies.
- Ternary Tetragram in the Taixuanjing:
 - The Taixuanjing, a classic text of Daoist cosmology, uses a ternary system of symbols (often represented as short, long, and long-and-short lines) to form tetragrams.
 - A tetragram, in this context, consists of four lines, each of which can take one of three states, leading to $3^4 = 81$ unique tetragrams.
 - These tetragrams are used to represent various cosmic principles, philosophical concepts, and natural phenomena.
 - The Taixuanjing's system is deeply rooted in Chinese philosophical thought, particularly in the principles of change, balance, and the interplay of elements.
- Connecting the Two Systems:
 - While the mathematical concept of balanced ternary vectors and the ternary tetragrams of the Taixuanjing are used in different contexts, they both reflect the complexity and richness of systems based on threes.
 - Both systems demonstrate how a limited set of basic elements (three digits or states) can be combined in a limited space (four positions or lines) to produce a large variety of outcomes or meanings.
 - In a broader sense, these systems can be seen as metaphors for understanding complex phenomena from a set of simple, fundamental principles - a common goal in both mathematics and philosophy.
- Comparative Insight:
 - Comparing these two systems provides insight into how different cultures and disciplines approach the representation and interpretation of complex systems.
 - While the balanced ternary vectors are more abstract and used for computational and mathematical purposes, the ternary tetragrams of the Taixuanjing serve a symbolic and philosophical role, showing the interconnection of ideas, natural elements, and cosmic forces.
 - Both systems, in their own ways, embody a deep appreciation for the balance, symmetry, and interplay of elements that define complex systems.

In summary, while the 81 balanced ternary vectors of 4 positions and the ternary tetragrams of the Taixuanjing originate from different cultural and intellectual traditions, they share a commonality in their use of a ternary system to explore and represent complex ideas and structures. This comparison highlights the universal human effort to make sense of complexity through structured systems, whether in the realm of mathematics or philosophical symbolism.

The following selection of k-face representations of a tesseract and its dual offer the strange possibility of a correspondence to be explored between that geometrical pattern in 4D and equivalent distinctions made in the encoding of the *Taixuanjing*. Related commentary is discussed separately ([Hyperspace Clues to the Psychology of the Pattern that Connects in the light of the 81 Tao Te Ching insights](#), 2003; [9-fold Magic Square Pattern of Tao Te Ching Insights experimentally associated with the 81 insights of the T'ai Hsüan Ching](#), 2006).

Indicative selection of 5 of 81 k-face representations of tesseract and its dual				
balanced ternary vector	tesseract (8-cell)		16-cell (hexadecachoron)	
	face type	image	face type	image
	cube		vertex	

				
	cube		vertex	
	square		edge	
	edge		triangle	
	vertex		tetrahedron	
	vertex		tetrahedron	



This work is licensed under a [Creative Commons Attribution-NonCommercial 4.0 International License](https://creativecommons.org/licenses/by-nc/4.0/).

For further updates on this site, [subscribe here](#)