



# laetus in praesens

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## Memorable Packing of Global Strategies in a Polyhedral Rosetta Stone

### Correspondence of cognitive internalization with collective strategic articulation

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## Introduction

In a period of multiple global crises there is an extraordinary disconnect between strategic articulations -- such as the UN's [Sustainable Development Goals](#) -- and the remarkable media preoccupation with competitive ball games, such as the [FIFA World Cup](#) and the [Rugby World Cup](#). The disconnect is also evident with respect to the world tennis and golf championships in comparison with the relative neglect of strategic challenges faced by societies -- ranging from extreme hunger to disastrous climate change and systemic environmental degradation. Strangely the thrill of competition in sport echoes the attention accorded to global conflicts and the capacity to "crush" opponents -- currently evident in the Russia-Ukraine conflict. The disconnect is evident otherwise in the enthusiasm for music of every kind and for competitive electronic games ([esports](#)) -- too readily framed as a distraction from life-threatening challenges in a complex global system.

Arguably it could be asserted that strategic articulations and international declarations are now inherently boring in comparison with the dynamics of games and the uplifting qualities of music -- and hence the disconnect. That term is also applied to the disassociation of many from the natural environment -- and notably from others in straightened circumstances, exemplified by the increasing evidence of [compassion fatigue](#) and [psychic numbing](#). Much could be explained by the increasing complexity of the world in which people live -- with "everything connected to everything". The quest for competitive triumph then offers a unique sense of coherence -- a lens through which a degree of order can be perceived and assumed.

Some forms of philosophy have responded to the challenge by framing a quest for a so-called [philosophers' stone](#) -- itself elusive in nature although purporting to offer a higher order of coherence. Like the various religions, science has aspired to the formulation of a [Theory of Everything](#), although unable to address the radical differences between proponents. Success in these endeavours may itself be perceived as inherently boring and meaningless for most, especially given the language in which it is likely to be articulated. More realistic is the concluding assessment of [Nicholas Rescher](#):

For centuries, most philosophers who have reflected on the matter have been intimidated by the strife of systems. But the time

has come to put this behind us -- not the strife, that is, which is ineliminable, but the felt need to somehow end it rather than simply accept it and take it in stride. (*The Strife of Systems: an essay on the grounds and implications of philosophical diversity*, University of Pittsburg Press, 1985)

The following exploration imagines the possibility of a form of "cognitive container" -- a form of "arena" characterized by dynamics analogous to those in sport and other games -- effectively incorporating what renders them attractive. This contrasts with the stasis implied by the stone metaphor, or the closure implied by an ultimate theory. As understood in archetypal terms, the container could then be understood as one whose bounding walls can be dissolved by what it contains -- as featured in the toroidal design of nuclear fusion reactors (*Enactivating a Cognitive Fusion Reactor: Imaginal Transformation of Energy Resourcing (ITER-8)*, 2006).

Exploiting the stone metaphor, the question is whether the disparate interactions can be appropriately packed, succinctly and attractively, into a form of *Rosetta Stone* capable of paradoxically exemplifying the requisite dynamics. Beyond the necessary recourse to more obscure disciplines in its design, one requirement could be its significance in musical terms -- as a musical instrument of a higher order, as an invitation to play (*Memorability, Mnemonics, Maths, Music and Governance*, 2022). Engagement with it could then be understood as a form of memory enhancement ensuring strategic credibility.

The rapid evolution of artificial intelligence suggests the emergence of devices of this kind (*Envisaging a Comprehensible Global Brain - as a Playful Organ*, 2019; *Governance of Pandemic Response by Artificial Intelligence*, 2021). Such a device could also be understood in symbolic terms (*Reimagining Guernica to Engage the Antitheses of a Cancel Culture*, 2022; *Concordian Mandala as a Symbolic Nexus*, 2016).

The argument in what follows builds on the case previously made that there is an implicit collective recognition of the coherence of particular patterns (*Imagining Partnership of the SDG Goals as Phases of the Cross*, 2022). This is possibly to be understood as a characteristic of the collective unconscious (*Cognitive Embodiment of Patterns of Governance of Higher Order*, 2022). It is most evident in preferences for team sizes in rugby (15), football (11), and related games. It is a curious feature of the seam curve common to a baseball and a tennis ball. Such patterns are seemingly evident in global strategic articulation and fundamental declarations of principles. The concern here is whether there is some form of container capable of holding such disparate formulations -- and the potential dynamics between them.

The conclusion focuses on the possibility that competitive team sports may embody an intuitive understanding of the elusive abstractions of quantum theory and may well be effectively engaged in a form of quantum dynamics through their implicit polyhedral organization. As a response to any "disconnect", this suggests a mode of dynamic global strategic organization usefully illustrated by "cosmograms" in 3D as a form of Rosetta Stone,

## Polyhedral frameworks: following "the numbers" -- rather than "the money"?

**Following the numbers:** There is a curious neglect of what may be implied by the organization of sets of concepts, as discussed separately (Representation, *Comprehension and Communication of Sets: the role of number*, *International Classification*, 1978-1979; *Examples of Integrated, Multi-set Concept Schemes*, 1984). The value of polyhedral frameworks for the memorable visual representation of complex strategic articulations has been extensively argued separately (*Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors*, 2008). In contrast to "following the money" as widely affirmed in the clarification of strategic options, the numbers characteristic of the wide array of polyhedra suggest an unexplored approach to strategic coherence and comprehensibility -- and hence the suggestion to "follow the numbers".

Curiously the latter phrase is typically used misleadingly as a euphemism to refer to following the numbers in financial spreadsheets, especially in the detection of fraud (L. Christopher Knight, *Following the money isn't a cliché*, *Fraud Magazine*, January/February 2017; *Following the money: the drivers of fraud*, *Basset Telecom Report*, March 2012).

As misplaced concreteness, this could however be understood as a form of intuitive recognition of a neglected truth, as succinctly articulated by Pythagoras: *Number rules the universe*. That insight is to be distinguished from the excesses of *numerology*, the deprecation of which fails to recognize the quantity of "ore" it is necessary to process in data mining for the extraction of what is held to be valuable (Underwood Dudley, *Numerology: Or, What Pythagoras Wrought*, *Mathematical Association of America*, 1998). Does "following the money" merit a portion of the deprecation currently accorded to numerology? When is it to be recognized as "mere superstition"?

The speculative enthusiasm of physicists for the significance of constants, such as that indicated by "137", frames the matter otherwise (Arthur I. Miller, *137: Jung, Pauli, and the Pursuit of a Scientific Obsession*, 2010). How the significance of *sacred geometry* is to be appropriately recognized is another matter again -- confused as it is with *religious numerology* (*Mathematical Theology: future science of confidence in belief*, 2011).

**Numbers in sport:** In relation to football, "the numbers" are curiously evident in the traditional design of the association football as a truncated icosahedron (*Sustainability through Global Patterns of 60-fold Organization*, 2022). As the *fullerene* of greatest stability, that argument considered the psycho-social implications of fullerenes for coherence, integrity and identity of a higher order. A curious aesthetic counterpart is evident in another set of patterns of particular significance to poetry and oppositional logic (*Pattern of 14-foldness as an Implicit Organizing Principle for Governance?* 2021). Preferences are otherwise evident with respect to 12-fold patterns (*Checklist of 12-fold Principles, Plans, Symbols and Concepts*, 2011). Such indications suggested an earlier exercise (*Identifying Polyhedra Enabling Memorable Strategic Mapping*, 2020).

The implication to be stressed is that polyhedra offer a relatively neutral framework with which a variety of modes of organization can be

associated -- with respect to a wide variety of issues otherwise considered to be incommensurable. The symmetry offered by the simplest can be recognized as a primary factor in memorability -- if not in aesthetic attractiveness and "interestingness" (*Relative interestingness and boringness of forms of coherence*, 2022).

**Engaging with complexity:** Taking the argument further, the question is the nature of the requisite complexity which offers the possibility of comprehensibility. This is far from the extreme indicated by one of the highest orders of symmetry -- known to mathematicians as the **Monster Group** (*Potential Psychosocial Significance of Monstrous Moonshine*, 2007). This is an exceptional form of symmetry -- a challenge to any Rosetta stone for cognitive frameworks.

The ultimate focus in what follows is on one of the more complex Archimedean solids, namely the **truncated icosidodecahedron** (62 sides, 180 edges, 120 vertices), together with its dual the **disdyaktriacontrahedron** (120 sides, 180 edges, and 62 vertices). Both are significant as polyhedra in approximating most closely to a sphere. The latter has been termed the "120 Polyhedron", with vertices that can serve as a framework for multiple polyhedra, as noted in the conclusion.

## Metaphorical contrast between "experiential flesh" and "cognitive bones"

There is no lack of evocative images of complex polyhedra, potentially attractive in many ways -- rendered more so by new facilities in visualization technology. These derive from insights into levels of complexity which seemingly elude those without a mathematical background, and more specifically without a geometrical bias.

On the other hand such images may well appeal simply as aesthetic artefacts -- most notably as presented at the annual **Bridges Conferences** on mathematical connections in art, music, architecture, and culture. That environment, as its name implies, endeavours to enable "bridges" between mathematical and aesthetic insights -- and their implications for creative design.

**Relevance to governance?** Missing it would seem is the effort to "translate" such insights into the world of governance. This failure contrasts with the adaptation of graphic representations to the contextual effects of video gaming -- despite a degree of recognition that strategic governance has long been dependent on gaming insights, as exemplified by appreciation of chess. Given the worldwide preoccupation with games, it is appropriate to ask how graphic sophistication translates into the understanding and appreciation of games -- and most obviously ball games.

There is of course increasing recognition of the adaptation of technology to the analysis of games and most specifically to so-called "passing patterns" in ball games. There is of course a contrast between such analysis and how the game is appreciated by spectators and players, especially those enthralled by the "spirit of the game". This contrast suggests recognition of "cognitive bones" and "experiential flesh".

The suggestion here is that polyhedra can be understood as offering "cognitive bones" and skeletal frameworks. The question is how the "experiential flesh" can be recognized as connected to such frameworks. More specifically how does any judicious combination enable and enhance a deeper and more fruitful understanding of what may be "in play"?

However, rather than overemphasis on the subtle complexities of the cognitive support offered by polyhedra, there is a case for recognizing that the appreciation of insights into the "spirit of the game" may imply an intuition of higher patterns of order. It may well be the case that their representation in geometrical structures oversimplifies -- and even denatures -- such higher dimensionality.

**Neuroscience of 4 dimensions and more?** Geometry is somewhat helpful in this respect in recognizing the degree to which conventional polyhedra in three dimension have their analogues in 4D, 5D, and more -- justifying the more general terms of **polytopes** (or polychora) rather than polyhedra (*Comprehending the shapes of time through four-dimensional uniform polychora*, 2015). The focus on 3D polyhedra may then constitute a misleading simplification and an instance of misplaced concreteness.

This perspective has acquired a degree of justification through research in the neurosciences of the **Blue Brain Project**. This indicates the remarkable possibility of cognitive processes taking even up to 11-dimensional form in the light of emergent neuronal connectivity in the human brain:

Using mathematics in a novel way in neuroscience, the Blue Brain Project shows that the brain operates on many dimensions, not just the three dimensions that we are accustomed to. For most people, it is a stretch of the imagination to understand the world in four dimensions but a new study has discovered structures in the brain with up to eleven dimensions - ground-breaking work that is beginning to reveal the brain's deepest architectural secrets..... these structures arise when a group of neurons forms a clique: each neuron connects to every other neuron in the group in a very specific way that generates a precise geometric object. The more neurons there are in a clique, the higher the dimension of the geometric object. ...

The appearance of high-dimensional cavities when the brain is processing information means that the neurons in the network react to stimuli in an extremely organized manner. It is as if the brain reacts to a stimulus by building then razing a tower of multi-dimensional blocks, starting with rods (1D), then planks (2D), then cubes (3D), and then more complex geometries with 4D, 5D, etc. The progression of activity through the brain resembles a multi-dimensional sandcastle that materializes out of the sand and then disintegrates. (*Blue Brain Team Discovers a Multi-Dimensional Universe in Brain Networks*, *Frontiers Communications in Neuroscience*, 12 June 2017).

For Warren S. McCulloch and Walter Pitts:

Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more

complicated logical means for nets containing circles; and that for any logical expression satisfying certain conditions, one can find a net behaving in the fashion it describes. It is shown that many particular choices among possible neurophysiological assumptions are equivalent, in the sense that for every net behaving under one assumption, there exists another net which behaves under the other and gives the same results, although perhaps not in the same time. Various applications of the calculus are discussed. (*A Logical Calculus of the Ideas Immanent in Nervous Activity*, *Bulletin of Mathematical Biology*, 52, 1990, 1/2)

**Meaning of "dimension"?** One accessible clarification of the nature of higher-order dimensionality is offered in a [prize-winning article](#) by [Margaret Wertheim](#) (*Radical Dimensions*, *Aeon*, 10 January 2018) and introduced as follows:

... the notion that we inhabit a space with *any* mathematical structure is a radical innovation of Western culture, necessitating an overthrow of long-held beliefs about the nature of reality. Although the birth of modern science is often discussed as a transition to a mechanistic account of nature, arguably more important -- and certainly more enduring -- is the transformation it entrained in our conception of space as a geometrical construct.

Over the past century, the quest to describe the geometry of space has become a major project in theoretical physics.... While on the local level we are trained to think of space as having three dimensions, general relativity paints a picture of a four-dimensional universe, and string theory says it has 10 dimensions -- or 11 if you take an extended version known as M-Theory. There are variations of the theory in 26 dimensions, and recently pure mathematicians have been electrified by a [version](#) describing spaces of 24 dimensions. But what are these 'dimensions'? And what does it mean to talk about a 10-dimensional space of being?...

Wertheim's account seemingly avoids any clarification of what dimensions may mean -- other than in a mathematical sense -- noting merely that:

... mathematically the addition of another dimension is a legitimate move. "Legitimate" meaning there's nothing logically inconsistent about doing so -- there's no *reason* I can't.... From the perspective of mathematics, a "dimension" is nothing more than another coordinate axis (another degree of freedom), which ultimately becomes a purely symbolic concept not necessarily linked at all to the material world.

This leaves vague whatever constitutes a "symbolic concept" -- a challenge in its own right, given the importance attached to "symbols" in the psychosocial dynamics of the current civilization. \*\*\*\* symbol

## Unexplored correspondence between "experiential flesh" and "cognitive bones"?

There is a case for exploring whether the attractive coherence offered by particular sports reflects in some way an intuitive appreciation of the coherence of certain polyhedral frameworks -- whether in aesthetic terms or in the related role of symmetry which is a primary feature of their analysis ([Marcus du Sautoy](#), *Symmetry*, 2008).

**Team size and coherence:** A point of departure is the team size preferred in many ball games. Little seems to be said, or appreciated, as to why teams of a particular size "work" in exemplifying what is appreciated in the game in question. Rugby offers one example with 15 active players on each side. Why 15 in rugby union -- setting aside the question with respect to other variants? Association football offers another. Why 11 players a side?

The argument can then be understood as framing similar questions in relation to games of 2x11 players, as with association football, and the like. What polyhedral structure might these intuitively imply -- however unknowingly? Provocatively it could be argued that the failure of global governance is at least partially a consequence of a failure to discover the game-playing frameworks capable of holding and evoking the requisite coherence found elsewhere. Equally problematic is the failure to recognize any structure within which both 22-fold and 30-fold games can be played -- as a means of justifying the cultural preferences for each.

Of some relevance to the tetrahedral focus of the following argument are the guild sizes in some [multiplayer online games](#) and the team sizes in competitive gaming, beyond casual play. In the latter case, depending on the game, usually 3-6 players per team are required (Tristian de la Navarre, *eSports Teams and Players: how eSports teams work*, *Lineups*, 15 July 2020; *eSports: How can they keep growing in 2023?* *BBC News*). In the case of both guilds and teams, few operate successfully with numbers beyond six. Even six is challenging in the case of esports considering team salaries and attribution of prize money (*Are there esports titles with a team size of more than 6 players?* *r/esports*, 2020). Exceptions are cited with respect to the teams of 20-25 in *World of Warcraft*.

**Icosahedral dynamic?** To the extent that the argument has any merit, it could be considered remarkable that the icosahedron -- traditionally deemed one of the aesthetically attractive structures through its symmetry and other properties -- offers a relatively hidden 15-fold pattern. Is it the activation of that pattern through the game of rugby which is intuitively appreciated, especially since its coherence is rarely manifest in other forms? Does the much-valued symbolism with which the icosahedron has been widely associated in sacred geometry to be understood as derived from the capacity to celebrate it in the patterns of a 15-fold game -- to give it form?

The 15-fold pattern in the icosahedron is associated in part with the set of 15 golden rectangles that can be constructed within it. These are in turn related to a set of 15 great circles. As a design theorist, [Christopher Alexander](#) has related these to the complex traditional patterns in rugs from the Middle East (*Foreshadowing of 21st Century Art: the color and geometry of very early Turkish carpets*, 1993; *Harmony-Seeking Computations: a science of non-classical dynamics based on the progressive evolution of the larger whole*. *International Journal for Unconventional Computing (IJUC)*, 5, 2009), as discussed separately (*Magic Carpets as Psychoactive System Diagrams*, 2010; *Harmony-Comprehension and Wholeness-Engendering: eliciting psychosocial transformational principles from design*,

2010).

Particularly intriguing with respect to the 15 golden rectangles is that each necessarily implies 2 opposing edges (linked across the polyhedron, by the longer sides of each rectangle). With its 30 edges, the icosahedron then offers a means of mapping the two 15-player teams confronting each other in the game. The inadequacy of such a mapping is that it is essentially static and therefore of potentially limited value and interest. More intriguing is that the opposing edges of each rectangle -- as players marking their opponents -- are naturally engaged in a competitive dynamic in the game.

This suggests the merit of recognizing the icosahedron as a dynamic with cognitive implications -- with other polyhedra potentially having similar psychosocial significance. In that respect there is a degree of irony to the fact that the seam pattern of the [association football](#) is that of a [truncated icosahedron](#). The seams of the volleyball have [pyritohedral symmetry](#) (*Game ball design as holding insight of relevance to global governance?* 2020).

The icosahedron is then mistakenly understood as an essentially static form when it is better recognized as the resultant of the interplay of dynamic forces. This interplay has been explored from a cybernetic perspective by [Stafford Beer](#) (*Beyond Dispute: the invention of team synergy*, 1994). The icosahedron might then be more appropriately understood as a resonant hybrid.

**Stasis and "disconnect":** There is a naivety to the assumption that a simple list of principles enables comprehension, appreciation and memorability of what they are so vainly upheld to imply. In systemic terms, the principles imply functions partially recognized as a pattern of checks and balances -- however this is to be understood. Arguably the pattern can only be recognized through the paradoxical dynamics between principles which are effectively incommensurable.

Such considerations frame the question as to the relevance to the cognitive "disconnect" noted above. Granted enthusiasm for the experience of rugby by players and spectators, what unexplored implication do these have for issues of strategic governance? By contrast, the question could be provocatively focused on the relatively weak appreciation of the *Universal Declaration of Human Rights* (UNDHR) -- composed as it is of 30 Articles. Arguably few are able to recall the topics of all the articles, despite the fundamental importance purportedly attached to them. Little is recognized of how the topics in question play off against each other. The UNDHR is indeed an edifice, but it is provocatively questionable to whom it is attractive and meaningful -- in comparison with the dynamics of a rugby game.

The point to be explored is how the interaction between the 30 opposing players in the 15-fold teams engenders the attractiveness and interest of the game. Can two sets of 15 "opposing" (or "mutually contradictory") topics be recognized among the 30 Articles of the UNDHR? What might be the systemic relationship between them? How do they play off against each other? Is it this interplay which would reveal the underlying (hidden) coherence of the complex 30-fold structure -- thereby considerably enhancing its attraction and relevance? Would the reality of human rights in practice then invite fruitful comparison with rugby?

There are a number of international "declarations of human rights". Are they then to be understood as implying distinctive games? Or is the absence of matching games to be recognized as a reason for their limited appreciation -- except as "cognitive bones" from a legal perspective, with little "experiential flesh"?

**Jostle, struggle and jihad?** The coherence of the connectivity between seemingly incommensurable preoccupations emerges through the "struggle" between them. Stafford Beer offered the overly mild term of "jostle" (*Mini-Syntegeation: problem jostle; Problem jostle, DebateGraph*). The controversial understandings of "jihad" offer other insights, especially in the light of the challenging distinction between "inner jihad" and "external jihad". How is the struggle between opposing teams to be understood -- between its "inner" and "external" forms -- especially when framed by military metaphors, and despite purporting to be inspired by the "spirit of the game"?

It could then be asked whether the UNDHR is the consequence of some kind of "intelligent design" on the part of those who conceived it and agreed to its formulation. Or is it simply the consequence of political horse-trading, as has been claimed for the set of 17 Sustainable Development Goals? Alternatively are such articulations engendered by the collective unconscious in some manner -- in quest of "goodness of fit", as argued separately (*Systemic Coherence of the UN's 17 SDGs as a Global Dream*, 2021).

As a 16-fold pattern, is the set of SDGs to be usefully compared to the 8 opposing columns of pieces which the 2 players each control in chess -- a strategic game *par excellence*? (*Chess for Sustainable Development, International Chess Federation*, 17 July 2020; *Play Chess With Your Future, Sustainia*, 11 October 2018).

## Higher dimensional coherence intuitively implied by a tetrahedron

**Preoccupation with "holes" -- rather than "wholes"?** With respect to any correspondence between "experiential flesh" and "cognitive bones", it could be considered curious to note the degree to which ball sports share fundamental terminology with the abstractions of geometry. Both attach fundamental importance to points and lines, and the spaces (or fields) they frame. Their distinction is entangled with the process of decision-making, as discussed separately (*Decision-making capacity versus Distinction-making capacity*, 2015). There it is noted that, in the absence of distinction, no decision is required or possible. The issue is highlighted by [confirmation bias](#) (also known as "myside bias"), effectively obscuring the potential relevance of any distinctly contrasting perspective. It can be understood in terms of "conformity bias".

Also curiously shared is a degree of preoccupation with holes. In sports that is evident in understandings of a goal (and the attraction it exerts), as well as a sense of any "hole" in the defence of it by the opposing team -- and the need to do so at any cost. The subtle nature of a hole has been remarkably explored by [Roberto Casati](#) and [Achille C. Varzi](#) with respect to the borderlines of metaphysics, everyday geometry, and the theory of perception (*Holes and Other Superficialities*, 1994) -- summarized in the [entry on holes](#) in the *Stanford Encyclopedia of Philosophy*). They seek to answer two basic questions: Do holes really exist? And if so, what are they? Such questions could be asked of goals -- whether strategic or otherwise.

The primary markers for the distinction in question recall the language of the [calculus of indications](#) of [George Spencer-Brown](#) (*The Laws of Form*, 1969). This is renowned as a formal exercise to separate what are known as algebras of logic from the subject of logic, and to re-align them with mathematics. The result is the explicit, and extremely elegant logical re-integration of the observer -- effectively the decision-maker. His final chapter, entitled "reentry into the form" commences with: *The conception of the form lies in the desire to distinguish. Granted this desire, we cannot escape the form, although we can see it any way we please* (p. 69). It ends with:

An observer, since he distinguishes the space he occupies, is also a mark... In this conception a distinction drawn in any space is a mark distinguishing the space. Equally and conversely, any mark in a space draws a distinction. We see now that **the first distinction, the mark, and the observer are not only interchangeable, but, in the form, identical.** (p. 76) [*emphasis added*]

**Triangulation:** Rather than venture into such cognitive abstractions, more can be clarified by reference to what Buckminster Fuller has identified as the most fundamental system. His explorations of the matter have given rise to the widely recognized geodesic domes and derive from the insight that: *All systems are polyhedra: All polyhedra are systems* (*Synergetics: explorations in the geometry of thinking*, 1975/1979, 400.56). For Fuller, the fundamental system can be represented by a tetrahedron as the simplest polyhedron (as noted and illustrated previously, and below left). It is also the easiest to comprehend being akin to a pyramid.

In two dimensions the triangle is the simplest closed shape you can make with flat sides, and you need three line segments to do it. Understanding of triangles and triangulation has proved to be fundamental to the organization of space (*Triangulation, Trilateration, or Multilateration?*, *Circuit Cellar*, 19 March 2014; ). This can be understood more generally (*Triangulation of Incommensurable Concepts for Global Configuration?*, 2011). It is currently being presented as a technique of psychological manipulation (Arlin Cuncic, *What Is Triangulation in Psychology?* *VeryWell Mind*, 14 October 2022; Darius Cikanavicius, *Triangulation: the narcissists best play*, *PsychCentral*, 20 October 2019; Ahona Guha, *Understanding Triangulation: what to do when someone tries to draw you into a personal conflict*, *Psychology Today*, 4 October 2021).

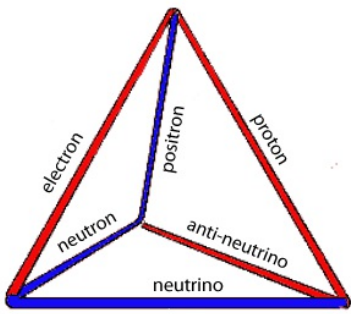
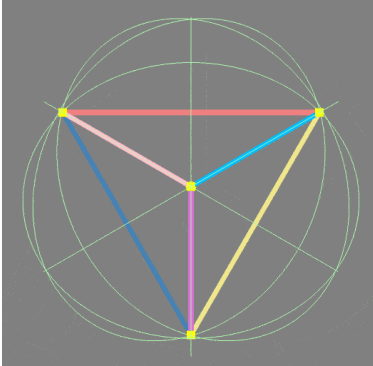
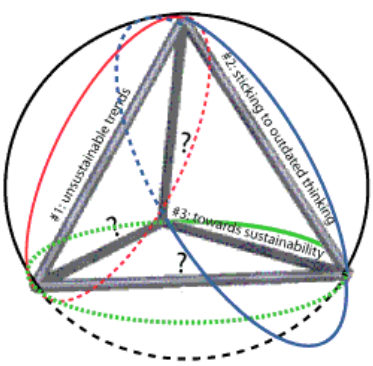
Triangulation has proved especially fundamental to navigation of the globe, as exemplified by the so-called [Pentagramma Myrificum](#) (*Global Psychosocial Implication in the Pentagramma Myrificum*, 2015). Spherical geometry can then be understood as offering clues to "getting around" and to circumnavigating imaginatively.

**Tetrahedron:** Corresponding to the triangle in three dimensions is the tetrahedron. It is the simplest closed shape that can be made with flat sides, and it takes four triangular faces to do it. It is the simplest of all the ordinary convex polyhedra and the only one that has fewer than 5 faces.

Experientially it can then be asked how much can be meaningfully associated with the bones of a tetrahedral framework. The question follows from an earlier exercise in [metaphorical geometry in quest of globality](#) -- in response to global governance challenges (*Engaging with Globality -- through cognitive lines, circlelets, crowns or holes*, 2009).

The tetrahedron is a strange reflection of the ways in which dimensionality can be variously understood. Its form most clearly reflects the four large-scale dimensions of spacetime (described by general relativity). However it then renders explicit, by its edges (and their associated great circles), a 6-fold pattern (as depicted below). This could be understood (if only mnemonically) as the configuration of 5 nuclear forces, plus one for electromagnetism (as noted by Wertheim, 2018). Together the 4-fold and 6-fold patterns could then be recognized as a reflection of the 10 dimensions of string theory. A simplification of the 10-fold pattern is offered by an 11th dimensional perspective -- that of M theory -- suggestively indicated at the centre of the tetrahedral animation (below centre).

The articulation by Wertheim is confused by the description in physics of 4 [fundamental interactions](#) (fundamental forces), namely the interactions that do not appear to be reducible to more basic interactions. These are held to be the [gravitational](#) and [electromagnetic](#) interactions (producing long-range forces whose effects can be seen directly in everyday life), and the [strong](#) and [weak interactions](#) (which produce forces at minuscule, subatomic distances and govern nuclear interactions). A [fifth force](#) has been hypothesized to explain various anomalous observations that do not fit existing theories, notably those relating to [dark matter](#).

Tentative configuration of strategic significance in the light of the quantum speculation of Buckminster Fuller		
Fundamental tetrahedral configuration proposed by Buckminster Fuller	Animation of tetrahedron with great circles	Adaption of Fuller configuration (on left)
		
Redrawn from Buckminster Fuller, <i>Synergetics: explorations in the geometry of thinking</i> , 1975		Reproduced from discussion in <a href="#">Towards a geometry of systemic thinking and</a>

## From strategic distinctions to viable operating models

It is curious to explore the transition from the distinctions -- characteristic of sets of principles and embodied in declarations -- to viable models. Distinctions can be emphatically asserted but with seemingly little need to clarify the relationships between them, except as loosely indicated by "checks and balances". Through the elaboration of models, such relationships are rendered explicit to a degree. Difficulties arise from distinctions that are neglected in the first place, and relationships vital to viability whose relevance is subsequently discovered to have been ignored (*Misleading Modelling of Global Crises*, 2021; *Transcending One-eyed Global Modelling Perspectives*, 2010).

The transition can be explored metaphorically with the first stage being the elaboration of what amounts to a "to-do list", or a checklist of beliefs. This can be upheld as a form of vehicle enabling a group to navigate the complexities of the world. Are the distinctions then to be understood as the wheels of the vehicle or the cylinders of engine? However those implications for viability are necessarily somewhat optimistic in the absence of systemic connectivity between the distinctions made. This can be seen as a fundamental problem for declarations of human rights.

A group of people can embody the principles into a strategic plan or a constitution, clearly to be understood as a relatively carefully engineered vehicle. Whether the vehicle is capable of navigating multiple domains is another matter. Single domain vehicles are necessarily distinct from "all-terrain" vehicles -- or those adapted for the astronautics of communication space. Curiously however such plans or constitutions are understood as inherently static rather than dynamic, even though they may be designed to enable movement or change (*Dynamic Transformation of Static Reporting of Global Processes*, 2013)

Elaborated as a carefully designed operating model, possibly subject to extensive testing, another perspective is offered. In the case of belief systems, the operability may be ensured by ritual practices and taboos. The domains in which the vehicle can be operated may be carefully circumscribed -- with condemnation of attempts at their use in other arenas.

It is however appropriate to recognize that a vehicle may be assumed to be viable in the absence of any such elaboration. Societies may infer viability in terms of sets of values whose nature is elusive, however frequently reference is made to them (*Values, Virtues and Sins of a Viable Democratic Civilization*, 2022). It could be asked how some models are equivalent in viability to toy models with which children play.

It would appear that there are circumstances in which belief in the viability of any set of principles (or practices) ensures a degree of viability -- or its perception -- obviating the need for any more detailed design or engineering. Crucial to the possibility and need for "global coherence" is the degree to which any particular articulation of principles (whether articulated in a model or not) is experienced as "credible" to some and a matter of indifference to others. The issue is fundamental to the conflict between religions, disciplines, ideologies and cultures.

The issue is clearly fundamental to the challenges of global governance -- problematically reframed by efforts to impose particular narratives as "universal" -- to the exclusion of others, with implications for biodiversity, human equality and anti-otherness (*Application of Universal Vaccination Narrative to Climate Change*, 2021)

## From triangulation for confirmation to tetrahedralization for coherence?

**Triangulation:** As noted above, [triangulation](#) is the process of determining the location of a point by forming triangles to that point from other known points. It has long been recognized as basic to the process of [surveying](#) any terrain. It is of particular importance to navigation, metrology, binocular vision, model rocketry and gun direction of weapons. As a branch of astronomy, astrometry makes use of triangulation -- exemplified by [very long baseline interferometry](#). The process of triangulation has acquired widespread recognition through its use in locating mobile telephone usage in relation to radio signal towers.

Triangulation primarily enables the confirmation of location within a 2-dimensional framework. This frames the question of the relevance of any analogous process with respect to a multidimensional context -- 3 dimensions or more. It is indeed the case that this has engendered a largely unrecognized focus on "tetrahedralization" on which there is an extensive specialized literature (Boting Yang, *Studies of several tetrahedralization problems*, Memorial University of Newfoundland, 2002). As introduced by Hugo Ledoux:

It is notoriously difficult to visualise trivariate fields, even if a three-dimensional computer environment offering translation, rotation and zoom is available. The major problem is that unlike bivariate fields, where the attribute  $a$  can be treated as another dimension, we can not 'lift' every location  $\mathbf{x}$  by its value  $a_i$  to create a surface in one more dimension and visualise/analyse it -- we can not see in four dimensions! (*Modelling Three-dimensional Fields in Geoscience with the Voronoi Diagram and its Dual*, University of Glamorgan, November 2006)

**Tetrahedralization:** Whilst tetrahedralization is especially relevant to 3-dimensionality in the geosciences, it clearly has implications for [space warfare](#) -- and analogously to [memetic warfare](#) in cognitive and communications spaces (*Missiles, Missives, Missions and Memetic Warfare*, 2001; Brian J. Hancock, *Memetic Warfare: the future of war*, *Military Intelligence Professional Bulletin*, 2010). Arguably triangulation has a qualitative form through which information from different sources is validated by cross-referencing between the variety of sources. This is a feature of investigative reporting and forensic analysis. Such triangulation is most readily recognized as 2-dimensional and may well be presented on 2-dimensional maps of relationships between perspectives variously upheld to be "facts". Of relevance to this argument however is the challenge of information into which other "dimensions" are effectively introduced -- exemplified by the challenge of conflicting beliefs and fake news.

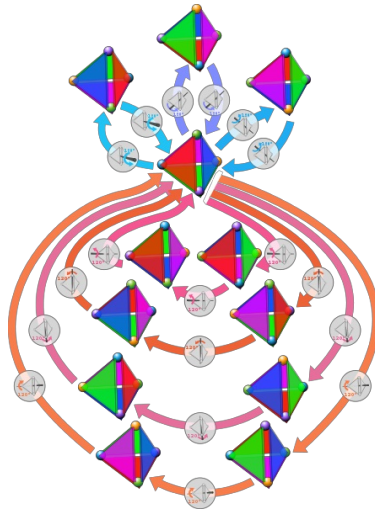
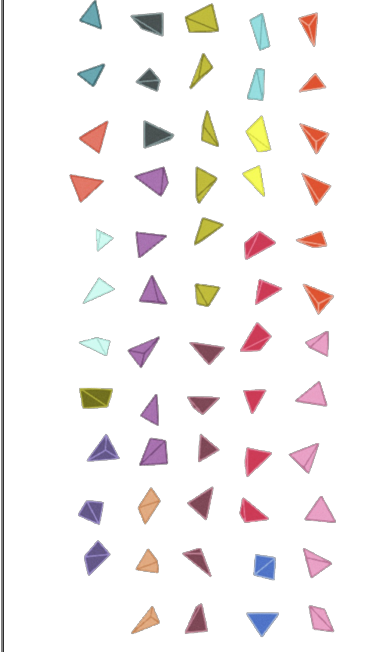
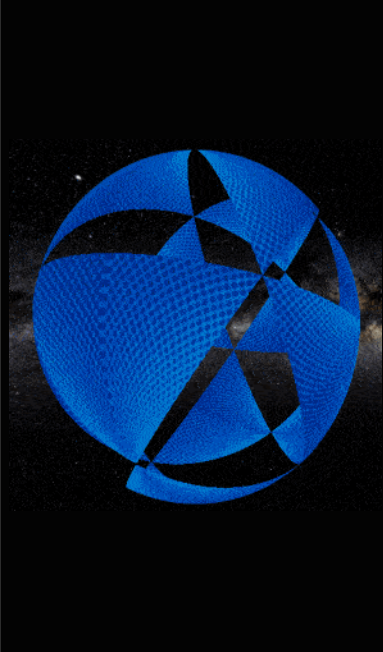
One approach to "multidimensional information" is to acquire information from disparate, even incommensurable, sources and to configure it in an array (\*\* disparate). This may both offer a degree of coherence regarding the domain and thereby framing a central point of reference. The advantages of doing so are readily recognized in a highly politicized context -- although they also engender the efforts to suppress information from particular sources, deemed especially inappropriate or irrelevant (Thierry Meyssan, *Only the plurality of information can prevent war, Voltairenet*). This process is a feature of the criticism of efforts to craft a mainstream narrative excluding alternative perspectives.

The studies of tetrahedralization suggest the possibility of richer insights into qualitative analogues to their quantitative focus -- as in the case of triangulation. The question is the nature of such "experiential flesh" in contrast to the "cognitive bones". Ironically one such 4-fold disparate configuration is indicated by a traditional recommendation to brides (Quinn Fish, *Why Do Brides Need Something Old, New, Borrowed, and Blue?* *Reader's Digest*, 21 October 2022; Francesca Cocchi, *Origin of Something Old, Something New, Something Borrowed, Something Blue for Brides*, *The Pioneer Woman*, 7 May 2021).

**Tetrahedral symmetry:** Although seemingly simple, considerable attention is devoted to the complexities of *tetrahedral symmetry*. A regular tetrahedron has 12 rotational (or **orientation-preserving**) symmetries. Difficulties in recognizing all 12 are usefully addressed separately (*Symmetries of Tetrahedron*, *Mathematics Stack Exchange*, 2022). A tetrahedron is also recognized as having 6 planes of symmetry.

A tetrahedron can be placed in 12 distinct positions by rotation alone. These are illustrated in the **cycle graph** format (below left), along with the 180° edge (blue arrows) and 120° vertex (reddish arrows) rotations that permute the tetrahedron through those positions. (This is variously identified as one of the finest images on the English *Wikipedia*). A tetrahedron has a **symmetry order** of 24 including transformations that combine a reflection and a rotation. This is partially illustrated by the animation below right adapted from a more complex animation and discussion by Robert Woodley (*Tetrahedral Symmetry*, 25 April 2017).

Particular interest has recently focused on the set of 59 tetrahedra with rational dihedral angles (Olena Shmahalo (*59 Tetrahedra*, *Quanta Magazine*, 2020; Kiran S. Kedlaya, et al, *Space Vectors Forming Rational Angles*, *arxiv.org.2011.14232*, 28 November 2020). These can be readily generated by the Antiprism application (Roger Kaufman, *59 tetrahedra with rational dihedral angles*, *Antiprism*).

Indications of degrees of symmetry of the tetrahedron		
12 Rotations of the tetrahedron in cycle graph format	Screenshot of animation of 59 tetrahedra with rational dihedral angles	Animation of tiling of 24 triangles on the Riemann Sphere
		
Reproduced from Wikipedia. Made by <a href="#">Debivort</a>	Adaptation from animation by <a href="#">Olena Shmahalo</a> ( <i>59 Tetrahedra</i> , <i>Quanta Magazine</i> , 2020)	Adaptation of animation by Robert Woodley ( <i>Tetrahedral Symmetry</i> , 25 April 2017)

## Minimal systemic framing offered by a tetrahedron

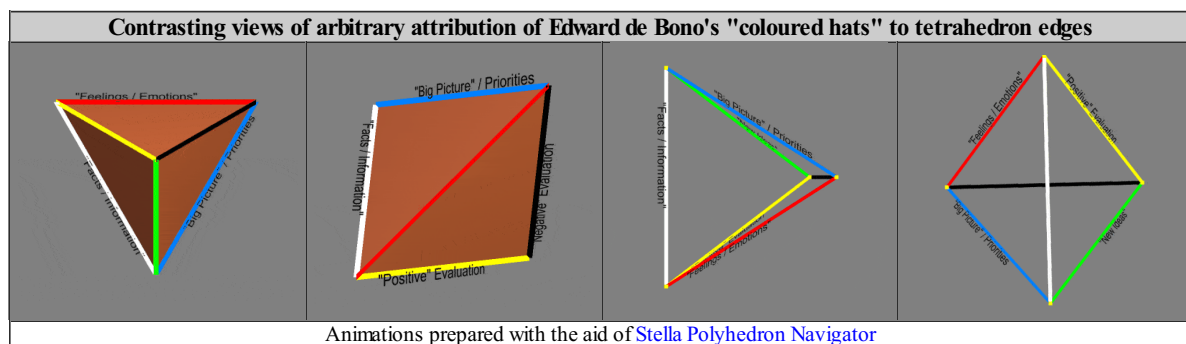
A point of departure is offered by the 6 edges that are configured to form a tetrahedron. Particular significance is associated with a 6-fold pattern by [Raymond Abelio](#) (*La Structure Absolue*, 1965) and the hexagrammatic encoding of the Yi Jing (Larry Schulz, *Hexagrammatics: Rules and Properties in Binary Sequences*, 2016). Far less abstract are the arguments of [Edward de Bono](#) (*Six Frames For Thinking About Information*, 2008) which summarized his widely promoted work (*Six Thinking Hats*, 1985; *Six Action Shoes*, 1991; *Six Value Medals*, 2005).

De Bono's basic argument made with respect to any group process is the need to enable the expression of six contrasting styles of thought (the "hats") in order to encompass a topic appropriately. The argument is extended in later work to action and to the expression of values. No reference is made to a tetrahedron, but the distinctions made can clearly be associated with the six edges of a tetrahedron to clarify further some fundamental challenges of discourse usefully clarified separately by Edward de Bono (*Water Logic: The Alternative to I am Right You are Wrong*, 1993).

De Bono identifies six distinct "directions" in which the brain can be challenged. In each of these directions the brain will identify and bring into conscious thought certain aspects of issues being considered (e.g. gut instinct, pessimistic judgement, neutral facts). Colored hats are used as metaphors for each direction. Switching to another direction is symbolized by the act of putting on a colored hat, either literally or metaphorically. Clearly any such application is far from the preoccupations of mathematics despite the clarification offered by the tetrahedron.

As implied by "*I Am Right Your Are Wrong*", society is faced with a fundamental challenge of how to engage with the disagreement which undermines the approaches to coherence, whether in governance, religion, or between religions or cultures. Conflict resolution has been developed into a set of skills and practices but has had relatively little impact on the differences from which extreme violence continues to arise. Less evident is how conflict is to usefully represented to enable more fruitful reflection.

In the light of de Bono's argument, a tetrahedron of directions offers a point of departure, as illustrated below. It is however useful to stress that, although a 6-fold pattern of distinctions can be ordered in this way, there is something of a mystery as to what is ordered and the nature of that order. As with the example of "something old, something new", arguably it is an elusive pattern of thinking which underlies any articulation -- whether in geometry, management or sport. As a framework, the tetrahedron (merely) offers the indication of the nature of a container for a complex of "directions" or "dimensions" -- a "container" which frames the question as to what it is that can be coherently contained.



## Requisite dimensionality: coherence in 4-fold, 5-fold or 6-fold terms?

Beyond the well-recognized challenge of the 3-fold in a world preoccupied by binary decision-making, the framework offered by de Bono offers the implication that 6 "dimensions" are required to elicit coherence in fruitful group discourse. Coherence of other kinds is however implied by 4-fold and 5-fold frameworks.

Examples of 4-fold articulations are presented separately (*Quaternity, quaternions and the unifying goal of partnership?* 2022). Of particular interest are:

- 4-fold pattern of Eastern logic, as extensively promoted by Johan Galtung with respect to conflict resolution (\*\*\*) Musha
- 4-fold pattern of "knowns", most notably associated with Donald Rumsfeld as US Secretary of Defense. It has given rise to extensive commentary both as a 3-fold pattern (Paul Austin Murphy, *Rumsfeld's Logic of Known Knowns, Known Unknowns and Unknown Unknowns, A Philosopher's Stone*) and a 4-fold pattern (Michael O'Donnell, *Rumsfeld Matrix: theory and application*, 4 August 2022). Discussed separately with respect to systemic neglect (*Unknown Undoing: challenge of incomprehensibility of systemic neglect*, 2008).
- 4-fold pattern of sides or vertices, as with any quaternity (*Quaternity, quaternions and the unifying goal of partnership?* 2022). The four constituents of a potential fire in any situation are variously described in such terms (*Fire Tetrahedron, Safeopedia*).

Examples of 5-fold articulations are also presented in that context as instances of *Five Principles of Strategic Communication*. They include the 5 Turnarounds of *Earth4All* (to create wellbeing for all), 5 Dimensions of *Inner Development Goals*, 5-fold *Viable System Model*, Chinese 5-phase *Wuxing* cycle, and the *Hygieia* Pentagram of Pythagoreans.

With respect to both the 4-fold and 5-fold articulations, the question here is the nature of the "experiential flesh" associated with such "cognitive bones" -- however remarkably and appropriately articulated for strategic purposes. De Bono's framework is a reminder of the need to emphasize that the instances of "experiential flesh" associated with the "cognitive bones" of any polyhedral configuration are indicative of a form of systemic experience for which no particular set of terms is adequate. The experience in question is implicit in a variety of understandings of any given term.

The argument can be usefully reinforced with respect to the variety of configurations in 3D forming tetrahedra, especially in the light of a recent proof:

The new proof identifies all the different ways of configuring a tetrahedron so that all six dihedral angles have rational values, meaning each can be written neatly as a fraction. It establishes that there are exactly 59 isolated examples plus two infinite families of tetrahedra that meet this condition. (*Tetrahedron Solutions Finally Proved Decades After Computer Search*, *Quanta Magazine*, 2 February 2021)

Whether it is the colours with which de Bono associates the cognitive "hats" distinguished, or the terms used, the clarification extends to the lengths of the edges of the tetrahedra -- where these are explored in terms of the "magical" coherence they may together offer (*Magic Triangular Pyramids, Magic Squares, Spheres and Tori*). In each case these are particular instances of a pattern -- potentially to

be understood in general systems terms -- for which cognitive articulations are misleading in relation to the "experiential flesh".

The animation on the left below emphasizes the possibility that the arbitrary attribution of distinctive cognitive dimensions ("coloured hats") to the tetrahedron may imply a form of resonance between alternative configurations. Rather than any particular configuration being "correct", ironically they are all subject to the 4-fold category schema advocated in the light of an Eastern perspective by [Johan Galtung](#): *correct, not-correct, both correct and not-correct, and neither correct nor not-correct* (*Toward a Conflictology: the quest for transdisciplinarity, Handbook of Conflict Analysis and Resolution, 2008*).

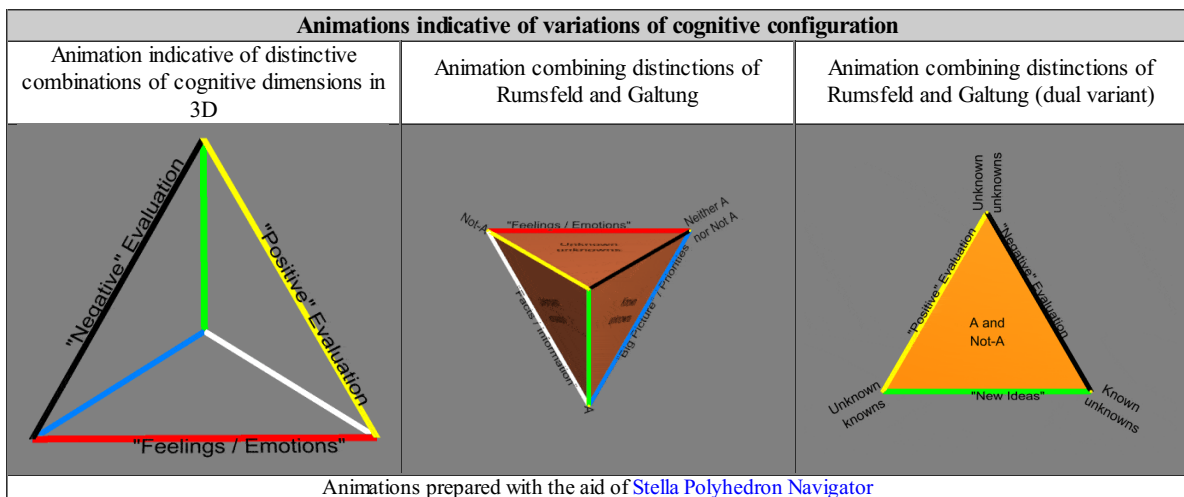
The 4-fold pattern is presented otherwise by [Kinhide Mushakoji](#) (*Towards a Multi-Cultural Modernity: beyond neo-liberal/neo-conservative global hegemony, UNESCO, 2005*). Problematic instances of the "both" condition are offered with respect to the Russia-Ukraine conflict by [Caitlin Johnstone](#) ("*US Interests*" in "*Unprovoked War*", *Consortium News*, 28 December 2022). Arguably the neither/nor condition is best characterized by the current extent of "post-truth" presentations of information (*Governance beyond the logical focus on true vs false?* 2019; *Towards articulation of a "post-truth table"?* 2016).

That 4-fold pattern may be fruitfully interpreted as indicative of the following conditions:

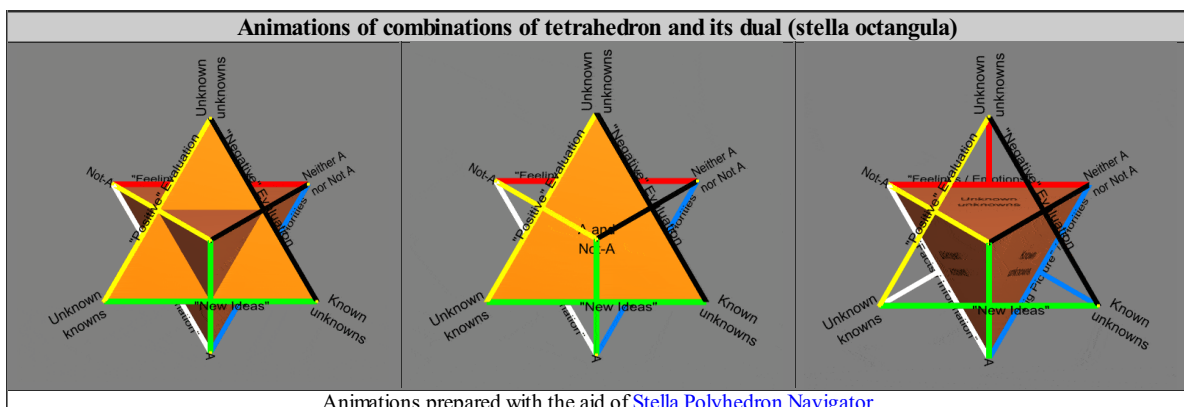
- I/We are all right -- potentially suggesting that the perspective of those who disagree needs to be understood otherwise
- I/We are all wrong -- potentially suggesting the need for reframing our perspective
- We are both right and wrong -- thereby calling for a more complex framework justifying both perspectives
- We are neither right nor wrong -- thereby implying that the categories of "right" and "wrong" are inappropriate to the circumstance

Whilst there are 720 arrangements of 6 distinctions (factorial 6), a much more limited number results from the possibility of their tetrahedral configuration as edges in 3D -- without duplication. [Adrian Rossiter](#) of [Antiprism](#) suggested that the problem can be clarified using [Burnside's lemma](#). Although subject to verification, this seemingly indicates that the number of distinct configurations including mirror forms is  $6!/12$  (namely 60), further reduced to  $6!/24$  (namely 30) through exclusion of such forms. The variety of coloured configurations can be presented using the Antiprism application.

The form of the tetrahedron suggests the possibility of configuring together the 4-fold patterns associated with Rumsfeld and Galtung, together with the 6-fold pattern advocated by de Bono. Presented together in this way the configuration serves as a provocative mnemonic -- evoking questions as to exactly how the different categories might be most insightfully positioned in relation to one another.



Of related interest, as illustrated below, are the cognitive implications of a dual configuration of tetrahedra and the patterns of dialogue which may be related to it. Rendering the sides of one of the tetrahedra transparent helps to clarify the constraint on representation in 3D and any sense of one configuration being explicit whilst the other is implicit. The transformation of the tetrahedron into its dual form (another tetrahedron) also evokes the question of whether and how "category vertices" are best related to "category sides" -- assuming a fixed configuration of "category edges". The question relates to the distinction in argumentation between "making a point" and "being on a side".



The dynamics of transformation between contrasting dual configurations are usefully illustrated by the animation below left. That on the right indicates the possibility of attributing to the dual configuration the contrasting 8-fold distinctions made by Edward Haskell through his generalization of the periodic table, and those of Arthur Young in his exploration of the geometry of meaning (as clarified below). Clearly the animation could be further enhanced by use of the 4-fold distinctions made above -- to the vertices for example.

Animations indicative of greater cognitive complexity using double tetrahedron			
Morphing between tetrahedral configurations	Variants of 12-fold pattern of Arthur Young		Combination of patterns of Haskell (8-fold) and Young (12-fold)
Animations prepared with the aid of Stella Polyhedron Navigator			

As presented above the categories and their attributions are primarily indicative of the challenge to the enhancement of the quality of dialogue. In this respect, metaphorical reference can be usefully made to diamond cutting and polishing, or to the enhancement of "precious stones" more generally. As in some traditions, a diamond can be compared to a configuration of categories which need to be suitably juxtaposed ("cut") and clarified ("polished") to enable the maximum collection, reflection and transmission of light -- as discussed separately (*Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue*, 2002).

Asked simplistically, how many "facets" are required to enable the most fruitful dialogue on matters of fundamental importance? Does the language of crystal cutting clarify distinctions between minimal dimensions and the optimum number and disposition of facets? (*Summary of Gemstone Faceting and Crystals*, 2002).

Rather than the insights offered by gemstones (and the value associated with them), clarification of the challenge is surprisingly available from the experience of helicopter navigation. These have emerged from the work of Arthur Young as the developer of the Bell helicopter (*Geometry of Meaning*, 1976), as discussed separately (*Engaging with tendencies to twisting movement -- insights from helicopter control*, 2004; *Existential dynamic in a "cognitive helicopter"* 2004)

Coaction cardioid (Haskell / Cassidy) Geometric representation of conditions in 8-fold [see also articulations by Wilken, pp. 157-161)	Four triangular patterns (triplicities) (one triangle per table row)	Three square patterns (quadruplicities) (one square per table column)
Reproduced from Edward Haskell (Full Circle: The Moral Force of Unified Science, 1972)	Reproduced from Arthur Young ( <i>Geometry of Meaning</i> , 1976)	

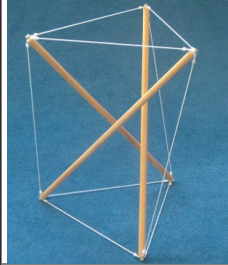
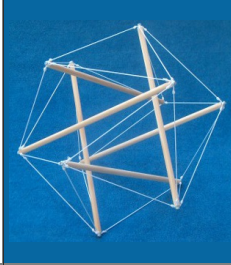
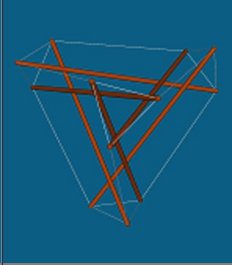
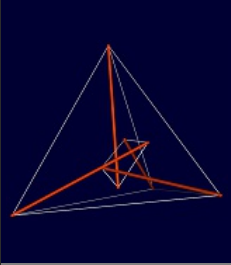
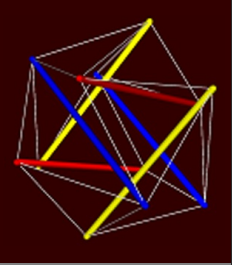
## Psychosocial implications of tensegrity as minimal coherent design

The role of tensegrity structures in architecture has been promoted by Buckminster Fuller with respect to the design of geodesic domes (*Synergetics: explorations in the geometry of thinking*, 1975/1979). Implications of tensegrity for strategic communication feature prominently in the cybernetic study by Stafford Beer (*Beyond Dispute: the invention of team synteegrity*, 1994) as noted above, and subsequently developed (Allenna D. Leonard, *Team Synteegrity Background*. 2002; J. Truss, et al. *The Coherent Architecture of Team Synteegrity: from small to mega forms*, 2003). The process has been franchised as Synteegration<sup>®</sup>. Tensegrity has been explored as a fundamental architectural principle in biological cell structure (Donald E. Ingber, *Tensegrity: cell structure and hierarchical systems biology*, *Journal of Cell Science*, 116, 2003, 7).

Far less attention has been given to its cognitive implications, despite the subtitle of Fuller's study, as argued separately (*Geometry of Thinking for Sustainable Global Governance: cognitive implication of synergetics*, 2009), notably with respect to psychosocial organization (*From Networking to Tensegrity Organization*, 1984). Fuller's articulation of tensegrity offers insights into how disagreement might be configured, namely how perspectives of different orientation might be configured without seeking to ensure that there is only one orientation (*Using Disagreements for Superordinate Frame Configuration*, 1992).

With respect to the polyhedral argument here, it is intriguing to note the structure of tensegrities as remarkably illustrated by an extensive

range of images on the [Tensegrity](#) website of [Marcelo Pars](#) (including [3D animations](#)) -- of which the simplest are reproduced below.

Images of tensegrity structures indicative of minimal design				
Simplest tensegrity	Icosahedral tensegrity	Screen shots of <a href="#">interactive 3D animations</a>		
		Truncated tetrahedron	Tetrahedral variant	Icosahedron
				
		3D version ( <a href="#">wrl</a> )	3D version ( <a href="#">wrl</a> )	3D version ( <a href="#">wrl</a> )
Reproduced from the <a href="#">Tensegrity</a> site of <a href="#">Marcelo Pars</a> (with permission)				

The psychosocial implications of such structures are discussed separately (*Transcending Psychosocial Polarization with Tensegrity: biomimetic clues to collective resilience and unshackling knowledge*, 2021). This includes the following sections:

- [Eliciting provocative clues for psychosocial challenges](#)
- [Tensegrity form-finding of relevance to integrative configuration of polarities](#)
- [Psychosocial tensegrity as a necessarily mysterious collective blindspot?](#)
- [Towards polarity containment -- psychosocial tensegrity in practice](#)
- [Predetermination of tensegrity forms of relevance to integrative configuration of polarities](#)
- [Matching sets of psychosocial polarities to tensegrities: case of the 10 Commandments?](#)
- [Spherical tensegrity as "container" for polarization dynamics of global civilization?](#)
- [Tensegrity torus as complementary framing of integrative psychosocial structure?](#)
- [Matching sets of psychosocial polarities to tensegrities: case of Axes of Bias?](#)
- [Matching sets of psychosocial polarities to tensegrities: case of Sustainable Development Goals?](#)
- [Multi-polar homeostasis, sustainability and transcendence?](#)
- [Tensegrity as a key to psychosocial polarity reversal in practice?](#)

As implied above, tensegrity evokes interest in the minimal design which may be especially relevant to sustainable initiatives, whether in the form of team building or otherwise ([Minimal Tensegrity](#), *Pretentst*, 25 January 2021; Muhao Chen and Robert E. Skelton, *A general approach to minimal mass tensegrity*, *Composite Structures*, 248, 15 September 2020; K. Nagase and Robert E. Skelton, *Minimal mass design of tensegrity structures*, *Smart Structures* 8 March 2014). Robert Burkhardt has presented *A Practical Guide to Tensegrity Design* (2008).

## Reframing conflict through tetrahedralization of dialogue

**Tetrahedralization?** There is no lack of indications regarding the inadequacy of dialogue to the current crises at every level of society. Reference continues however to be made to "dialogue" as the ideal panacea -- especially in contrast to violent conflict. Game-playing, especially in competitive sports, serves as a substitute for chaotic violence. Such violence could however be understood as "dialogue by other means" -- adapting the insight of Carl von Clausewitz: *Politics is war by other means*. Understood in this light might dialogue be fruitfully explored otherwise?

Stafford Beer endeavoured to reframe dispute from the perspective of the cybernetics of management by reframing it within a tensegrity context, making particular use of the icosahedron, as noted above (*Beyond Dispute*, 1994). To the extent that the tetrahedron is a geometrical or topological "prelude" to the icosahedron, there is then a case for identifying potentially neglected insights associated with that form.

Is there a possibility of reframing conflict through the tetrahedralization of dialogue? This would contrast with the overly optimistic implication of the potential of a "third way" in dialogue as currently associated with "mediation" -- especially given the manipulative dynamics of triangulation (as noted above). Tetrahedralization would then call for a higher order of vigilance to counteract a higher order of manipulation.

**Systemic functional challenges?** The 6-fold articulation of Edward de Bono is clearly one point of departure. As noted above, it is typically illustrated with 2-dimensional images only. As a point of departure, how might the adequacy of the terms suggested by de Bono be challenged from a systemic perspective? What is it that they imply from the more fundamental cognitive perspectives by which conflict is typically engendered? Do they reflect a minimal set of "dimensions" which can be understood in terms of knowledge cybernetics, for example (\*\*\*)). Are there other "languages" in which they can be more meaningfully articulated -- possibly in response to contrasting cognitive preferences? What dynamics are called into play in any effort to reconcile distinct 6-fold patterns variously deemed to be preferable?

Understood as a minimal pattern of checks and balances -- well illustrated by the simplest tensegrities -- what contrasting perspectives, styles and orientations merit recognition to ensure the coherence of the pattern as a whole? As in competitive team sports, a recognition of how the excesses of each perspective have to be vigilantly constrained is then required.

There is a lack of transparency to the manner in which dialogues between conflicting parties at the highest level are organized. A number of mediators may be considered necessary. Less evident is the range of skills they represent and what potentially vital skills could be deemed to have been neglected in their selection. Any evaluation of such intractable dialogues is typically presented simplistically (if at all)

-- with no assessment in technical terms. The comparison with the degree of insight brought to bear on competitive team sports is laughable. It is questionable how one such dialogue engenders learnings for any subsequent dialogue. The pattern of interfaith dialogues over decades offers cases worthy of exploration. The "dialogue" over decades between North and South Korea offers an extreme case, as does that regarding Jerusalem.

**Identifying requisite variety:** In the quest for requisite variety in dialogue, one hypothetical case for consideration is that with extraterrestrials -- especially given the degree to which the intractability of dialogue now readily has one side framing the perspective of the other as "alien" (if not "evil"). As an exercise, contact with aliens readily evokes imagination regarding the variety of human perspectives which might enhance a fruitful outcome (*Designing a Team for Alien Encounter*, 2000).

That tentative earlier exercise cited a possible team of 12, understood to be composed of archetypal styles such as the following:

<b>Speculative set of team roles for alien encounter</b>	
<ul style="list-style-type: none"> <li>• Martial arts (aikido, etc) -- vigilance, preparedness, respect for opponent</li> <li>• Performer, aesthete -- responsiveness, reframing, expression</li> <li>• Communicator, facilitator, empath, humorist -- (Peter Ustinov)</li> <li>• Biologist, species empath -- deep ecologist</li> <li>• Conspiracy theorist -- <i>advocatus diaboli</i></li> <li>• Anthropologist, linguist, protocol</li> </ul>	<ul style="list-style-type: none"> <li>• Crazy wisdom response to the moment (Mullah Nasrudin, Taoist)</li> <li>• Theoretician, physics, mathematics (Richard Feynman)</li> <li>• Strategic game player -- chess, go</li> <li>• Philosopher</li> <li>• Lawyer</li> <li>• 'Operator', trader, con-man -- opportunism, vigilance ("it takes one to know one")</li> </ul>
<p>Reproduced from <i>Designing a Team for Alien Encounter</i> (2000).</p>	

An obvious difficulty in designing any such team is that intractable differences exist between the proponents of each such category. As a team, further difficulties arise from the limited available (collective) skills to engender a sustainable self-organizing process (*Enabling Creative Response to Extraordinary Crises*, 2001). The US Senate inquiry into 9/11 determined that "failure of imagination" was a factor in enabling the disaster. There is also the question of how the team of 12 might be reduced to one of 6 -- corresponding to de Bono's "hats".

The exercise is however misleading in focusing to an unnecessary degree on professions and disciplines rather than on functions and cognitive competences. How many cognitive functions merit recognition? The difficulty has been usefully highlighted by the epic movies *Avatar* (2009) and *The Way of Water* (2022) -- with the latter title corresponding ironically to that of Edward de Bono's *Water Logic: an alternative to I am Right, You are Wrong* (1987). Any team organized by technocrats or the military is clearly not fit for purpose in failing to reflect the spectrum of functions required -- and in remarkably lacking the imagination to recognize this.

What functions are recognized to be necessary in "intelligence agencies" tasked with anticipating crisis? Especially intriguing is the challenge of integrating the functions of the nonconformist and contrarian into the design of any team as argued by Costica Bradatan (*The Herd in the Head, Aeon*, 13 December 2022). The tendency to "demean" such perspectives is evident, even to the point of associating them with subverting a desirable "positive" dynamic and their exemplification of a hostile threat. One corrective is the use of counteracting teams as with Red Team / Blue Team exercises to test strategic assumptions (especially in military contexts), as notably dramatised in science fiction by M. A. Foster (*The Game Players of Zan*, 1977).

The challenge can be explored as the integration of the "disparate", curiously exemplified by the organization of computer memory (*Dynamics of N-fold Integration of Disparate Cognitive Modalities*, 2021; *Framing Cognitive Space for Higher Order Coherence*, 2019), as discussed separately (*Suggestive "con-vergence" of "dis-parate" indications meriting "re-cognition"?*, 2019). The challenge is otherwise explored in terms of the adequate diversity of a gene pool -- suggesting the need for an analogous focus on that of a **meme pool** (Andrew Chesterman, *The Memetics of Knowledge, Knowledge Systems and Translation*, 2005; *Strategy and Memes, Fractal Consulting*, 17 November 2016).

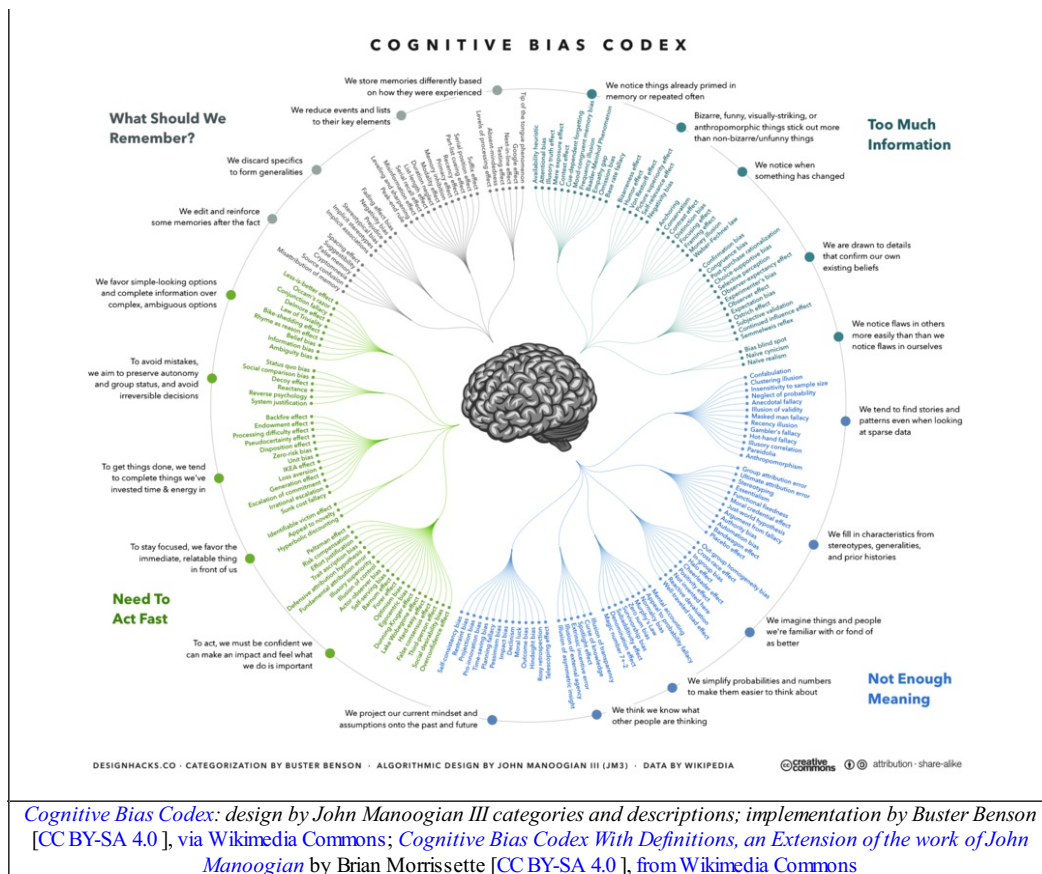
Such considerations frame questions regarding the viability of a political system within which particular memes are deprecated or suppressed -- only too evident in the questionable process of demonisation.

## Appreciating the coherence of patterns of cognitive biases

As discussed separately, particular personality types tend to be understood as associated with particular **cognitive biases** -- or vulnerable to them (*Interrelating Multiple Ways of Looking at a Crisis*, 2021). Much more challenging is then the representation of a comprehensive set of such biases, as ways of thinking, as indicated in the schema below -- variously perceived as requisite (however framed as "distorted") sources of alternative perspective. This configuration of 180-plus biases is reproduced from the **checklist** in the relevant *Wikipedia* entry.

Centered on the depiction of a human brain, the configuration raises the question as to how this pattern might be understood in terms of any integrative "global brain" and the responsibility of global governance of a crisis (such as a pandemic). Of particular interest, in the light of the extensive role envisaged for artificial intelligence, is the extent to which such biases may be inadvertently embodied in algorithms with implications for the dominant strategic response.





The above schema was previously discussed in relation to the [Group of Seven](#) (*Group of 7 Dwarfs: Future-blind and Warning-deaf: self-righteous immoral imperative enabling future human sacrifice*, 2018). There a specific concern was whether the circular array might be rendered more comprehensible through representation in 3D, with examples of how this might be achieved (*Global configuration of cognitive biases: towards mapping G7 susceptibility*).

**Engaging systemically with probability of terminological confusion?** The array above can be recognized as an example of a circular [dendrogram](#) -- widely used in hierarchical cluster analysis. The question here is how such an array is suggestive of a response to the challenging cognitive constraints of memorability and communicability which such complexity implies (*Comprehension of Numbers Challenging Global Civilization*, 2014).

Given the inadequacy of the pattern language in which they tend to be articulated, there is the particular challenge of how to recognize the fundamental nature of any limited set of cognitive functions indicated -- as by by Edward de Bono, for example, The specific difficulty being that -- to the extent that they are rendered explicit -- they pose a problem of terminology exacerbated by their expression in English. Ironically the challenge is illustrated by the disparate appreciation of an iconic movie (*Avatar: The Way of Water*, *Metacritic*, 16 December 2022). The challenge is otherwise evident in the dramatic erosion of common terminology with respect to arms control, as described by Scott Ritter (*A Lexicon for Disaster*, *Consortium News*, 19 December 2022).

The array above could be understood as suggestive of the disparate pattern of understandings of either transcendental unity ("deity"), or of the ultimate [theory of everything](#) to which the array of disciplines purportedly claim to aspire. The detailed 180-plus articulations around the circumference are then indicative of the challenging detail (and terminological confusion) to be navigated in achieving any form of transcendental comprehension -- or consensus.

The inner radial lines of the image conveniently avoid the comprehensibility of the "categories" implied -- for which any terms would indeed be inadequate. There is therefore a particular challenge in "generalizing" from the 180-plus articulations at the circumference to the limited number of functions implied by those inner radial lines -- and especially the innermost, a 4-fold set clustering a 20-fold set.

A relevant review is offered with respect to recognition of diversity in ecological terms (Stefano Mammola, et al, *Concepts and applications in functional diversity*, *Functional Ecology*, 35, 2021, 9). From a psychosocial perspective, the pattern above could be understood as indicative of a pantheon, as discussed separately (*Meta-pattern via Engendering and Navigating "Panthons" of Belief?* 2021). This included an exploration of three-dimensional patterns inspired by mathematical experience of interrelationship.

**From hierarchical representation in 2D to polyhedral nesting in 3D:** The earlier exercise with respect to representation of the cognitive bias array in 3D focused on the complete array. Given the comprehensibility constraint, the above argument has emphasized the preliminary need for clarification with respect to a tetrahedral array of functions. It can then be suggested that the subsequent challenge may then be the necessary nesting in 3D of a succession of polyhedra of different degrees of complexity -- understood as an analogue to the hierarchical nesting in 2D which is more conventionally used.

In comprehension terms each polyhedron is then indicative of an integrative cognitive "ceiling". This is more consistent with the implications of a succession of insight barriers typically associated with hierarchies of stages, degrees and initiations. From that perspective, concentric circles of polygons could be used to distinguish layers of the above array in 2D -- then to be represented as

nested polyhedra in 3D. The 4-fold innermost array as a tetrahedron could indeed be understood as embedded within a 20-fold polyhedron (such as a dodecahedron or an icosahedron).

## Achieving credibility of "cognitive bones" via "experiential flesh"?

**Experiential distinctions:** Whilst impressively detailed articulations can be formulated, their credibility in practice is variously challenged to an unfortunate degree -- with implications for the comprehensibility and communicability of strategies, and any possibility of their effective uptake. As a consequence, any assumptions regarding their implementability in practice tend to be fundamentally flawed.

The disciplines used in the articulation of strategies do not offer any sense of what might be missing from the complex frameworks proposed -- nor do they recognize the possibility of such weaknesses. Ironically a natural sense of the missing is evident through other senses -- suggesting the merit of their exploration as disciplines of strategic relevance. Expressed otherwise, are current approaches to strategic articulation cognitively "trapped" in a vision framework in some way (*Metaphoric Entrapment*, 2002; *Aesthetics of Governance in the Year 2490*, 1990).

Most obvious is the sense of taste. This readily evokes recognition of lack of "salt" or "sugar" (for example), or their excess. Beyond such simple contrasts are the distinctions offered and appreciated in cookery of different qualities, as argued separately (*World Governance Cookery Book: food-related insights from home cooking to haute cuisine*, 2002). The distinctions are appreciated to an even greater extent with respect to wine. In both cases, ironically, it is the policy-making leadership which is renowned for cultivating such experiences to excess (*Global dynamics "at the table" inspired by dining and wining in practice*, 2015). Consideration of taste as a metaphor highlights contrasting individual preferences, in addition to those between cultures with regard to styles of cookery.

A similar case can be made for the recognition of tonal distinctions in music and song -- with the advantage that tone patterns and sensitivity have also evoked extensive study regarding the appreciation of harmony, especially with respect to the advantages of **sonification** to pattern recognition. There tends to be immediate sensitivity to what is "missing" in a pattern of tones within any given tuning system -- and recognition of when a pattern is "out of tune". Of particular relevance is detection of the missing in a 4-tone, 5-tone, 6-tone pattern (*Varieties of Tone of Voice and Engagement with Global Strategy*, 2020). In terms of the function of particular instruments, such sensitivity is notably evident with respect to the coherence of **musical ensembles** (Duos, Trios, Quartets, Quintets, Sextets, and the like).

As noted with respect to **musical tuning** by *Wikipedia*:

A tuning system is the system used to define which tones, or pitches, to use when playing music. In other words, it is the choice of number and spacing of frequency values used. Due to the psychoacoustic interaction of tones and timbres, various tone combinations sound more or less "natural" in combination with various timbres... The creation of a tuning system is complicated because musicians want to make music with more than just a few differing tones. As the number of tones is increased, conflicts arise in how each tone combines with every other. Finding a successful combination of tunings has been the cause of debate, and has led to the creation of many different tuning systems across the world. Each tuning system has its own characteristics, strengths and weaknesses.

Curiously, despite its extensive use in product marketing, there has been virtually no effort to explore the role of sonification with respect to strategy uptake -- other than by autocrats (*A Singable Earth Charter, EU Constitution or Global Ethic?* 2006). The contrast with respect to uptake has been noted with respect to sports (*How successful are World Cup music scores and soundtracks?* *Euronews*, 18 December 2022). Given deprecation of regrettable "disconnect" in various contexts, it might be asked whether music offers a key to a remedial response.

Gardening frames the matter otherwise. A gardener may be variously sensitive to what is missing from the garden as an evolving pattern. New varieties may be sought to improve the balance of the whole. There may be particular concern with the quality of the soil, lighting, and the like. Such sensitivity is notably influenced by preferences for different styles of garden and their extremes.

Curiously the selection of varieties for a garden would seem to translate only to a limited degree into recognition of the variety of species necessary (or desirable) for a healthy ecosystem. Particular species engendered by a garden are readily defined as "weeds" -- with animal species considered to be "pests". An analogous situation is evident in the case of any psychosocial system -- ironically with "pest" and "weed" used to frame those who embody characteristic problematic to society (*Eradication as the Strategic Final Solution of the 21st Century?* 2014).

By analogy, what psychosocial functions merit recognition as corresponding to "**keystone species**" (*Role of Keystone Species in an Ecosystem*, *National Geographic*). As suggested by biomimicry, clues from nature could also extend to the forms of functional coherence suggested by the contrast between **bipedal** and **quadrupedal** species and those with a greater number of limbs (**hexapods**, **octopus**, **decapods**, and **millipedes**). There is then a curious parallel to be explored between the contested psychosocial "normalization" and the rapid extinction of species (*Extinction cascades may decimate world biodiversity by 2050*, Flinders University, 19 December 2022; *The New Normal – what needs to be different than before?* *UNESCO Futures of Education*). This occurs in a period when the functional viability of multi-limbed robots is of increasing interest.

Whilst the operability of robots makes readily evident any missing functionality, far less evident is any sense of missing values (*Recognizing missing values and the challenge of configuring value dilemmas*, 2018).

**Hole recognition versus Pattern recognition?** Considerable emphasis is placed on **pattern recognition** in any body of data. Curiously there would seem to be very little focus on the recognition of "holes" in a pattern -- "pattern missing" -- in contrast with their recognition

in garments. In the light of the above examples, what skills are deployed to determine what is "missing" from a pattern?

The question is of relevance to any N-fold strategic articulation. The challenge has even been caricatured by versions of a popular childrens song (*There's a Hole in My Bucket...*). Such skills, to the extent that they are valued in relation to threat or potential disaster, may be framed as "anomaly research" (*Variety of System Failures Engendered by Negligent Distinctions*, 2007). This is now exemplified with respect to UFOs by the newly mandated US All-Domain Anomaly Resolution Office (AARO).

For [Terrence Deacon](#), as previously discussed (*Evolutionary influence of the absent*, 2011):

... have we been looking in the wrong places for clues? ... brain researchers and philosophers of mind have focused on brain processes, neural computations and their correspondences with the material world. But what if we should be focusing on what is not there instead? ... I believe that in order to overcome this stalemate **we need to pay more attention to what is intrinsically not present in everything** -- from life's functions and meanings to mind's experiences and values. (*The importance of what's missing*, *New Scientist*, 26 November 2011) [emphasis added]

A form of quest for the missing, and for anomalies of significance to systemic viability, is the role of bodies charged with "oversight" (*The Role of Parliamentary Oversight Committees; Oversight institutions and advisory bodies monitor implementation*, OECD). Most unfortunately, and most curiously, one interpretation of the term is synonymous with "overlooking" -- which may well be held by critics to be an undeclared purpose of their constitution or membership (David Gillum, Kathleen M. Voge, *After the Fact: another controversial virus study raises questions about US government oversight*, *Bulletin of Atomic Scientists*, 28 October 2022; Maryanne Demasi, *FDA oversight of clinical trials is "grossly inadequate" say experts*, *BMJ*, 16 November 2022; Peter McCullough, *US FDA Willfully Blind on the Safety of COVID-19 Vaccination*, *Global Research*, 18 November 2022).

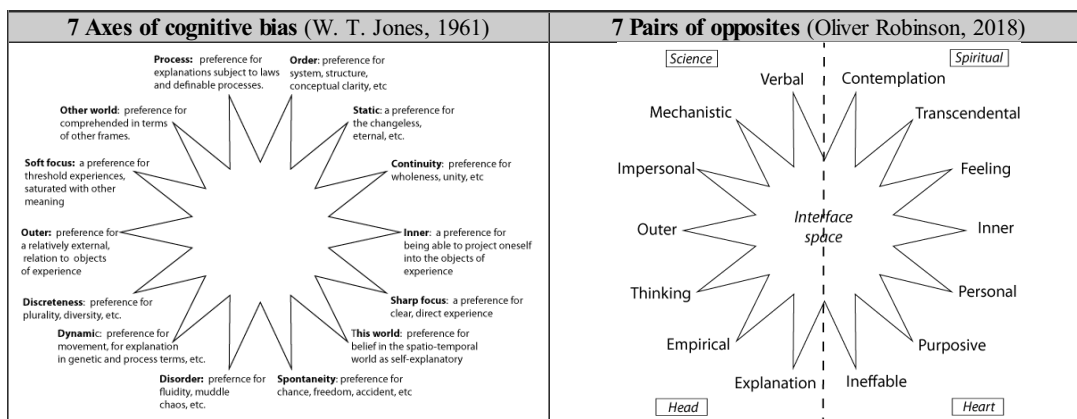
**Constrained dimensionality inhibiting recognition of the missing:** With respect to the functionality of a system, it is intriguing to note a degree of preoccupation with **loopholes** -- recognized as an ambiguity or inadequacy in that system. They can be used to circumvent or otherwise avoid its purpose, as implied or explicitly stated. They are actively searched for and used strategically in a variety of circumstances, including elections, politics, taxes, the **criminal justice system**, or in breaches of security. Bodies responsible for oversight are themselves potentially flawed in lacking capacity to detect them -- *Quis custodiet ipsos custodes?*

Whilst polyhedra could be understood as presenting patterns from which missing features could indeed be recognized as indicative of loopholes, the challenge would appear to be the experiential reality of any such frameworks in indicating systemic inadequacies and the associated vulnerabilities.

**Cognitive functions implied by axes of bias:** A valuable clue to the implication of bias for the potential failure of dialogue is offered by frameworks of bias -- of which that above is then but one example (*Systems of Categories Distinguishing Cultural Biases*, 1991). Of relevance to this argument is then how a framework of experiential biases might be fruitfully articulated as a polyhedron rendering evident any missing emphases.

Frustrated with the predictability of patterns of disagreement, one valuable example is that from a philosophical perspective presented by W. T. Jones (*The Romantic Syndrome: toward a new method in cultural anthropology and the history of ideas*, 1961). Jones distinguishes 7 axes of bias as polarities. These could be contrasted with the 6-fold set of polarities of David Livermore (*The 6 Cultural Systems that Form the Basis for Cultural Intelligence in Leaders, Leadership Essentials*, 10 September 2020). Like the framework of Edward de Bono, that of Livermore suggests the value of a polyhedral framework such as the tetrahedron.

For comparison, presented in 2 dimensions, the image on the right below is a slightly redrawn version of that of [Oliver C. Robinson](#) (*Paths Between Head and Heart: exploring the harmonies of science and spirituality*, 2018), as summarized by the author (*Palintonos Harmonia: the alchemy of opposites*, *Paradigm Explorer*, 2018, 2). That theme is inspired by the insight of Heraclitus and others into "taut harmony" (or "counter-stretched harmony"), as extensively reviewed by [Bernd Seidensticker](#) (*Palintonos Harmonia, Hypomnemata*, 72, 1982)



The extremes in each image frame the need for an appropriate balance. Rather than to be associated "statically" at the median position of each axis however, there is clearly the possibility of fruitful alternation between them -- an elusive "dynamic" whose rhythms are yet to be discovered. Other 7-fold axes are detailed separately in tabular form as a means of clustering the problems and strategies in the *Encyclopedia of World Problems and Human Potential* (*Patterning Problems: patterning the problematique; Patterning Strategies:*

*patterning the resolutique*).

The 2D images above are reproduced from a speculative discussion of the nature of a cognitive "stargate" (*Complementary features of "stargate" de-sign and functionality?* 2018; *Getting to the "stars": understandings of how a "stargate" might work?* 2018). This suggested that any imagined "stargate" is likely to be of higher dimensionality than is seemingly implied by those images (*A "stargate" of higher dimensionality -- the Renaissance as a Gateless Gate?* 2018). The "portal" is somehow to be understood as through a centre of corresponding dimensionality.

## Self-reflexive polyhedralization of dialogue

It is extraordinary to note the focus on the analysis of [passing patterns](#) in professional football and baseball -- as compared with the absence of analysis of the manner in which any point is "passed" in parliamentary discourse. This is rendered even more curious in the light of references to parliamentary dialogue as a game for which other related metaphors are used: keeping the ball in play, dropping the ball, scoring a point, moving the goal posts, etc.

It could be asked whether there is a case for re-imagining [rithmomachia](#), namely the forgotten mathematical board game played in medieval and Renaissance Europe (Ann E. Moyer, *The Philosophers' Game: Rithmomachia in Medieval and Renaissance Europe*, 2001). Known as the "Battle of Numbers", it bore a resemblance to chess and was no doubt an inspiration for [Hermann Hesse](#) in his allusive description of *The Glass Bead Game* (1943).

It could then be asked how such a game might be re-imagined -- shifting from 2D to 3D -- noting the development of [three-dimensional chess](#) since 1907. Three forms of [spherical chess](#) have been recognized. The argument above suggests that, rather than the 2D focus on "passing patterns", such a game might well have a polyhedral emphasis. Just as chess has been a strategic inspiration for territorial warfare, this might well be relevant to space warfare -- however the sense of space and territory could be generalized in psychosocial terms.

Rather than "numbers", the focus then shifts to the geometry and topology implied by the "numbers". There is a case for speculating how mathematicians in dialogue might engage in such a game through making a "point", drawing a "line", and ensuring the intersection of lines at another point. As a reflection of the progress of their discourse, the question is how the game becomes especially self-reflexive -- with point and line inviting metaphorical interpretation. However it is the elaboration of this process to form a "polyhedron" which would take the game to another level -- especially with the possibility of geometrical transformation of one polyhedral form to another.

Tetrahedralization would then be an initial step in polyhedralization. In mathematical terms, polyhedralization is a form of vector space quantization where a vector is assigned to the closest centre point of one polyhedron (*Feature space polyhedralization*, *Living Textbook*, University of Twente). The [map](#) associated with the latter resembles the experimental development of [Tomás J. Fülöpp](#) in endeavouring to elicit patterns of coherence from sets of world problems (*Loop Mining in the Encyclopedia of World Problems*, 2015).

How does a configuration of points and lines invite such a transformation -- when the geometrical transformation "carries" the significance associated with them from one 3D pattern to another? Given references to [Platonic chess](#), it is amusing to reflect on the possibility that the Pythagorians may have (secretly) informed their own discourse in this way -- given their enthusiasm for the Platonic polyhedra. Arguably "Platonic solids" may have been discovered in that period through point-making in dialogue. Of the many transformations possible, especially intriguing is the manner in which one configuration may be elaborated to frame another nested within it.

As argued above, does any such polyhedral discourse depend initially on tetrahedralization, namely through the capacity to make distinctive points as the intersection of lines of distinctive orientation? How elusive is the closure implied by tetrahedral configuration? Is the set of Platonic polyhedra then indicative of distinctive modes or degrees of dialogue? How might dodecahedral dialogue be experienced, especially light for the many appreciative references to 12-fold groups and their symbolic significance, as with the Knights of the Roundtable and the Last Supper (*Increasing the dimensionality of the archetypal Round Table?* 2018). The challenging nature of tetrahedral closure -- as a minimal form of unity -- is exemplified by the contrasting orientations of fundamental disciplines or religions.

Given the insights of [David Bohm](#) into quantum theory (as with [De Broglie-Bohm theory](#)), of relevance is the implication of such insights for the processes of dialogue now known as [Bohmian dialogue](#) (*On Dialogue*, 2013).

Such abstractions (as "cognitive bones") could be explored otherwise (as "experiential flesh") through much-viewed duelling instrumentalists (*Dueling Banjos*, *Celtic Dueling Violins*). Improvisation in song duels is the primary feature of the Basque folk tradition of [bertsolaritza](#) (*Improvisation in Multivocal Poetic Discourse: Basque lauburu and bertsolaritza as catalysts of global significance*, 2016; Alicia Ault, *What Is Bertsolaritza and Who Are the Basque Poets Who Know It?* *Smithsonian Magazine*, 29 June 2016).

These indications frame the question as to what makes for the "interestingness" and coherence of sustainable dialogue -- whether between mathematicians or between instrumentalists (*Guidelines for Sustainable Dialogue*, 2006; *Understanding Sustainable Dialogue: the secret within Bucky's Ball?* 1996; *Imagining Sustainable Dialogue -- and the community it engenders*, 1999).

The increasing complexity of simpler polyhedra is suggestive of the potential of an increasing complexity of dialogues and the distinctive roles they evoke -- as emergent patterns. Beyond the 6-fold pattern proposed by Edward de Bono, is that of the [Belbin Team Role Inventory](#) (or *Belbin Self-Perception Inventory*), whether 8-fold or 9-fold. A 12-fold pattern is implied by the work of Arthur Young on the *Geometry of Meaning* (*Typology of 12 Complementary Dialogue Modes Essential to Sustainable Dialogue*, 1998). How does a viable esports team of 5 or 6 roles set aside any need for other roles from the Belbin array? (*What are the roles in an esports team?* *Esports Transfers*, 23 June 2022; *There are many different roles available within esports*, *British Esports*, 6 December 2016; Gabriel Sciberras, *Understanding the Industry -- Player Roles in Esports Teams*, *SportsGuide*, 8 April 2022)

There is the fascinating possibility that the array of [psychological types](#), as variously understood, is particularly associated with different styles of dialogue. More provocative is the possible association with different expressions of belief as distinctive styles of dialogue (*Meta-pattern via Engendering and Navigating "Pantheons" of Belief?* 2021).

With a pattern of dialogue understood as a particular style of game, it is intriguing to note the potential correspondence between the 36-fold set of recognized ["dramatic situations"](#) and the and 36-fold set of [chess checkmate patterns](#). Any such correspondence would invite reflection on the dialogue processes of governance and their possible representation by more complex polyhedral forms, as discussed separately (*Thirty-six Dramatic Situations faced by Global Governance?* 2022)

## Cognitive challenge of plus/minus 1 or 2?

In exploring the use of polyhedra to map cognitive distinctions, the cases in which the array of polyhedral features differs slightly in number from patterns of distinctions call for further reflection -- as with polyhedra recognized as "near misses" from a more regular array. One interesting example is the set of 16 Boolean logical operations. These are typically reduced to 14 to enable their representation on polyhedra. The accepted justification is that two of the 16 are instances of tautology and contradiction.

Margaret Wertheim offers insight which may be relevant to a difference of one, as between 10-fold string theory and the role of the 11th dimension offered by M-theory (as noted above). Arguing that "mathematics is purely 'the science of symbols', and as such doesn't have to relate to anything other than itself", Wertheim helpfully clarifies the role of perspective developed by artists in shifting the focus of mathematics:

From the 14th to the 16th centuries, artists such as Giotto, Paolo Uccello and Piero della Francesca developed the techniques of what came to be known as *perspective* -- a style originally termed 'geometric figuring'. By consciously exploring geometric principles, these painters gradually learned how to construct images of objects in three-dimensional space. In the process, they reprogrammed European minds to see space in a Euclidean fashion...

The illusionary Euclidean space of perspectival representation that gradually imprinted itself on European consciousness was embraced by Descartes and Galileo as the space of the real world. Worth adding here is that Galileo himself was trained in perspective. His ability to represent depth was a critical feature in his groundbreaking drawings of the Moon, which depicted mountains and valleys and implied that the Moon was as solidly material as the Earth.

Given the tendency of mathematicians to see maths as primarily self-justifying, it could be asked whether some other kind of perspectival shift may prove fundamental to further development of mathematical significance, as a "science of symbols" of relevance to governance. Are the instances of plus/minus 1 indicative of the so-called ["hard problem"](#) and distinctive understanding of the role of the observer?

Given the role of the icosahedron, symbolically and otherwise, it can then be asked whether the [31 great circles of the spherical icosahedron](#) exemplify some form of transcendence of its 30-fold characteristics. Similarly it might be asked how the coherence of a 60-fold patterns is significantly challenged by 59 tetra plus \*\*\*

The implications of plus/minus 2 have been highlighted by the much-cited paper of [George Miller](#) (*The Magical Number Seven, Plus or Minus Two: some limits on our capacity for processing information*, *Psychological Review*, 63, 1956, 2). Arguably this could be seen as characterizing Galileo's memorable assertion -- *E pur si muove!* -- in that recognition of an axis is fundamental to recognition of [rotation and revolution](#) (or their denial). A betrayal of Galileo by science is however evident in the continuing reference to "sunrise" and "sunset" by some disciplines.

Given the 64-fold patterns of the genetic code and of the set of *Yi Jing* hexagrams, further clues may be sought in how they relate to 60-fold and 62-fold patterns. In the case of the hexagrams, the two with a complete set of broken or unbroken lines offer one indication. In the case of the genetic code, it is the sets of [stop codons](#) and [start codons](#) which invite reflection.

## Enabling "experiential flesh" by sonification of polyhedral configurations

The potential of [sonification](#), namely the use of non-speech audio to convey information or perceptualize data, has long been recognized - - as a means of circumventing high orders of information overload to enable pattern recognition. This has been promoted by the [International Community for Auditory Display](#) which prepared a seminal report on the field for the US National Science Foundation (*Sonification Report: Status of the Field and Research Agenda*, 1997). The possibility has been summarized separately (*Technical feasibility of musical sonification*, 2009) with regard to the feasibility of "compressing" information so that the patterns of content can be more easily apprehended

**Agreeableness of music:** As summarized in [Euler's "Degree of Agreeableness" for Musical Chords](#) (*Thats Maths*, 9 August 2018):

The links between music and mathematics stretch back to Pythagoras and many leading mathematicians have studied the theory of music. Music and mathematics were pillars of the *Quadrivium*, the four-fold way that formed the basis of higher education for thousands of years. Music was a central theme for Johannes Kepler in his *Harmonices Mundi* – Harmony of the World, and René Descartes' first work was a compendium of music.

The insights of Leonard Euler into polyhedra have long been recognized, however as noted in the *Thats Maths* summary, it is his understanding of the relation between music and mathematics which has been widely ignored:

He considered music and mathematics as part of a single coordinated system, and his study of music inspired his work on number theory, fluid mechanics and even topology. His book *Tentamen novae theoriae musicae* (1739)... This work did not attract great interest at the time and indeed was described as "too mathematical for musicians and too musical for mathematicians"... In the *Tentamen*, Euler developed an index to indicate the characteristics of different combinations of notes. It was a quantitative measure of the pleasantness of musical intervals and chords. He called it his *Gradus Suavitatis*, or Degree of Agreeableness.

The implications have been variously explored:

- Jozsef Sandor: *Euler and Music: a forgotten arithmetic function by Euler* (*Octagon Mathematical Magazine*, 17, 2009, 1)
- Davide Amodio: *Euler and Music Musing Euler's Identity* (*Imagine Math*, 8, 2022)
- Peter Pesic: *Euler's Musical Mathematics* (*Mathematical Intelligencer*, 35, 2013, 2; *Euler: The Mathematics of Musical Sadness* 2014)
- Matthew Hawthorn: *The Unexpected Number Theory and Algebra of Musical Tuning Systems*

The question as to whether music can be meaningfully represented by polyhedra is addressed by Mattia G. Bergomi, et al (*Towards a Topological Fingerprint of Music Lecture Notes in Computer Science*, 02 June 2016):

Can music be represented as a meaningful geometric and topological object? In this paper, we propose a strategy to describe some music features as a polyhedral surface obtained by a simplicial interpretation of the *Tonnetz*. The *Tonnetz* is a graph largely used in computational musicology to describe the harmonic relationships of notes in equal tuning. In particular, we use persistent homology in order to describe the *persistent* properties of music encoded in the aforementioned model.

The question is also explored by Jouko Koskinen<sup>1</sup> and Petteri Mäkiniemi (*Polyhedral Music, Bridges 2019 Conference Proceedings; video*):

When a point moves in 3D space, its path can be described by lists of spatial coordinates. We have tried to hear the music of some regular solids by moving one or more points along their edges and converting the spatial coordinates into auditory ones. Is it possible to hear a 3D shape? If the shape of a regular solid is converted into sound, can we hear similar regularities in the sound as we see in the shape? If we create a musical interpretation of a beautiful polyhedron, is the music produced also beautiful?

Other indications are offered by:

- Debra K. Borkovitz: *Polyhedral Waltz* (4 February 2013)
- Guerino Mazzola: *Geometrie der Töne: Elemente der mathematischen Musiktheorie* (Birkhauser, 1990)
- Jordan Alexander Key: *Euler, Cowell, Polyhedra and the Music Genome* (10 December 2018)
- Tetsuya Maeshiro and Midori Maeshiro: *Polyhedron Network Model to Describe Creative Processes* (*Lecture Notes in Computer Science*, 2014)
- Sonia Cannas: *Geometric representation and algebraic formalization of musical structures* (2018)
- Dmitri Tymoczko: *Geometry of Music* (14 February 2016)

**Icosahedron and music:** Of particular relevance to the argument here are the studies by Yusuke Imai and colleagues (*General Theory of Music by Icosahedron 1: a bridge between "artificial" scales and "natural" scales*, arXiv:2103.10272v4, August 2022; *General Theory of Music by Icosahedron 2: analysis of musical pieces by the exceptional musical icosahedra*, arXiv:2108.10294v3, 3 August 2022). The first focuses on the duality between chromatic scale and Pythagorean chain, and golden major minor self-duality. As summarized there:

Relations among various musical concepts are investigated through a new concept, musical icosahedron that is the regular icosahedron each of whose vertices has one of 12 tones, C, C<sup>♯</sup>, D, E<sup>b</sup>, E, F, F<sup>♯</sup>, G, G<sup>♯</sup>, A, B<sup>b</sup>, B. First, we found that there exist four musical icosahedra that characterize the topology of the chromatic scale and one of the whole tone scales, and have the hexagon-icosahedron symmetry (an operation of raising all the tones of a given scale by two semitones corresponds to a symmetry transformation of the regular icosahedron): chromatic/whole tone musical icosahedra. The major triads or the minor triads are set on the golden triangles of these musical icosahedra. Also, various dualities between musical concepts are shown by these musical icosahedra: the major triads/scales and the minor triads/scales, the major/minor triads and the fundamental triads for the hexatonic major/minor scales, the major/minor scales and the Gregorian modes. Namely, these musical icosahedra connect "natural" scales (Gregorian modes, major/minor scales, hexatonic major/minor scales) and "artificial" scales (chromatic scale and whole tone scales).

Second, we derived duality relations between the chromatic scale and the Pythagorean chain that is a succession of the fifth based on C by using musical icosahedra. We proposed Pythagorean/whole tone musical icosahedra that characterize the topology of the Pythagorean chain and one of the whole tone scales, and have the hexagon-icosahedron symmetry. The Pythagorean chain (chromatic scale) in the chromatic (Pythagorean)/whole tone musical icosahedron is constructed by "middle" lines of the regular icosahedron. While some golden triangles correspond to the major/minor triads in the chromatic/whole tone musical icosahedra, in the Pythagorean/whole tone musical icosahedra, some golden gnomons correspond to the minor/major triads.

Third, we found four types of musical icosahedra other than the chromatic/whole tone musical icosahedra and the

Pythagorean/whole tone musical icosahedra that have the hexagon-icosahedron symmetry. All the major triads and minor triads are represented by the golden triangles or the golden gnomons on each type. Then, these musical icosahedra may be applied to harmonic analysis by combining figures characterized by the golden ratio on the regular icosahedron. All of these musical icosahedra naturally lead to generalizations of major/minor triads and scales through the symmetry of the regular icosahedron.

## Psychosocial implications of a quantum tetrahedron?

As noted by John C. Baez and John W. Barrett, recent work on state sum models of [quantum gravity](#) in 3 and 4 dimensions has led to interest in the "quantum tetrahedron" (*The Quantum Tetrahedron in 3 and 4 Dimensions*, [arxiv.org](#), Mar 1999). This has offered a geometrical explanation of the otherwise puzzling fact that the quantum tetrahedron has more degrees of freedom in 3 dimensions than in 4 dimensions.

[Quantum entanglement](#) has become a resource for fascinating developments in quantum information and quantum communication during the last decades (Walter Thirring, et al, *Entanglement or Separability*, [arxiv.org](#), 18 August 2011). This includes valuable illustrations of the quantum tetrahedron and its double pyramid form, reproduced from Reinhold Bertlmann (*Geometry of Quantum States*, *Annals of Physics*, 324, 2009). Research by J. Belin, et al, presents the *Tetrahedron as a quantum self-imaging system* (*Imaging and Applied Optics*, 2019).

**Quantum tetrahedron:** As argued by John Schliemann, quantum polyhedra constructed from angular momentum operators are the building blocks of space in its quantum description as advocated by [loop quantum gravity](#) (*Classical and Quantum Polyhedra*, [arxiv.org](#), 28 Ausut 2014). For Kai Harris (*Quantum Polyhedra*, [arxiv.org](#), January 2020), the central object of interest is the volume operator of a quanta of space, suggesting that such an object would correspond to some classical polyhedra, and notably to a coupled tetrahedron:

The quantum polyhedra being studied in this paper can be seen as being embedded in Euclidean 3 space. What this means is that we can utilize the language of symmetries in this space to help describe properties of these polyhedrons. Luckily, such a mathematics is widely studied and referred to as the theory of Lie groups and their Algebras

Clarification of the nature of the quantum tetrahedron by Mauro Carfora, et al (*Quantum Tetrahedra*, [arxiv.org](#), 25 Jan 2010) is notably introduced with a comment on a woodcut tetrahedron on a 16th century translation of Plato's *Timaeus*:

This woodcut beautifully illustrates the role of the perfect shape of the tetrahedron in classical culture. The tetrahedron conveys such an impression of strong stability as to be considered as an epitome of virtue, unfailingly capturing us with the depth and elegance of its shape. However, as comfortable as it may seem, this time-honored geometrical shape smuggles energy into some of the more conservative aspects of Mathematics, Physics and Chemistry, since it is perceptive of where the truth hides away from us: the quantum world.

**Geometry of tetrahedron and 6-j symbol:** Seemingly far beyond ordinary comprehension, for Carfora, et al:


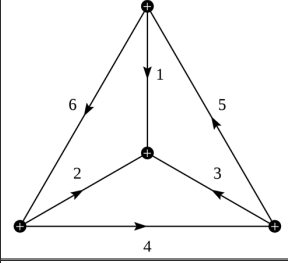
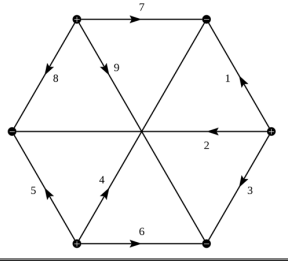
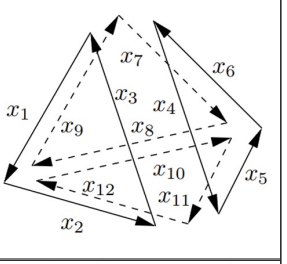
We discuss in details the role of [Wigner 6j symbol](#) as the basic building block unifying such different fields as state sum models for quantum geometry, topological quantum field theory, statistical lattice models and quantum computing. The apparent twofold nature of the 6j symbol displayed in quantum field theory and quantum computing -- a quantum tetrahedron and a computational gate -- is shown to merge together in a unified quantum-computational SU(2)-state sum framework.

Wigner's [6-j symbols](#) were introduced by [Eugene Paul Wigner](#) in 1940 and published in 1965. They are defined as a sum over products of four [Wigner 3-j symbols](#). A remarkable formula for the asymptotic behaviour of the 6-j symbol was first conjectured by Ponzano and Regge, and later proven by Justin Roberts (*Classical 6j-symbols and the Tetrahedron*, *Geometry and Topology*, 3, 1999). The asymptotic formula applies when all six quantum numbers  $j_1, \dots, j_6$  are taken to be large then associating the geometry of a tetrahedron to the 6-j symbol. For Roberts:

A classical 6j-symbol is a real number which can be associated to a labelling of the six edges of a tetrahedron by irreducible representations of SU(2). This abstract association is traditionally used simply to express the symmetry of the 6-j symbol, which is a purely algebraic object; however, it has a deeper geometric significance. Ponzano and Regge, expanding on work of Wigner, gave a striking (but unproved) asymptotic formula relating the value of the 6-j symbol, when the dimensions of the representations are large, to the volume of an honest Euclidean tetrahedron whose edge lengths are these dimensions. The goal of this paper is to prove and explain this formula by using geometric quantization. A surprising spin-off is that a generic Euclidean tetrahedron gives rise to a family of twelve scissors-congruent but non-congruent tetrahedra.

[Jucys diagram](#) for the Wigner 6-j symbol The plus sign on the nodes indicate an anticlockwise reading of its surrounding lines. Due to its symmetries, there are many ways in which the diagram can be drawn. An equivalent configuration can be created by taking its mirror image and thus changing the pluses to minuses. \*\*\*\*

[Jucys diagram](#) for the Wigner 9-j symbol. The diagram describes a summation over six [3-jm symbols](#). Plus signs on each nodes indicate an anticlockwise reading of the lines for the 3-jm symbol, whereas minus signs indicate clockwise. Due to its symmetries, there are many ways in which the diagram can be drawn.

Woodcut tetrahedron on a 16th century translation of Plato's <i>Timaeus</i>	Wigner 6- <i>j</i> symbol (Jucys diagram)	Wigner 9- <i>j</i> symbol (Jucys diagram)	Tetrahedral configuration of 12 vectors (with mirror variants)
			
Reproduced from Mauro Carfora, et al ( <i>Quantum Tetrahedra</i> , <i>arxiv.org</i> , 25 Jan 2010)	Fylwind, CC BY 4.0, via Wikimedia Commons	Fylwind, CC BY 4.0, via Wikimedia Commons	Reproduced from Justin Roberts ( <i>Classical 6j-symbols and the tetrahedron</i> , 1999)

With respect to any psychosocial interpretation of the 9-fold pattern of the Wigner 9-*j* symbol, there is a case for exploring the significance invested in the [enneagram](#) and its associated controversies (Kenneth Ireland, *The Jesuit Transmission of the Enneagram*, 14 November 2020; *Muddied Roots, Psychobabble, Inoculation*, 3 April 2021; A. G. E. Blake, *The Intelligent Enneagram*, 1997). Curiously, in his discussion of management cybernetics, Stafford Beer has noted the manner in which the enneagram is embedded within an icosahedron, as illustrated separately (*Use of virtual reality for 3-dimensional articulation of connectivity*, 2014). More generally there is the question of the significance attributed to 9-fold patterns (*Ninefold configuration in practice and its comprehension constraints*, 2016).

**Comprehending angular momentum?** In quantum mechanics, the [angular momentum operator](#) is one of several related operators analogous to classical [angular momentum](#) -- as the rotational analogue of linear momentum. The angular momentum operator plays a central role in the theory of atomic and molecular physics and other quantum problems involving [rotational symmetry](#). Such an operator is applied to a mathematical representation of the physical state of a system and yields an angular momentum value if the state has a definite value for it. There are several angular momentum operators, but it is the total angular momentum which is usually denoted **J** -- and with its energy conserved. The six 6-*j* symbols arise in the coupling of [three angular momenta](#).

To the extent that such insights are reflective of a fundamental understanding of reality, the challenge is how the angular momentum in question is to be comprehended and explained to the uninitiated. In the light of classical mechanics, angular momentum is defined as the property of any rotating object given by moment of inertia times angular velocity. This can be explained as:

- If you try to get on a bicycle and balance without a kickstand, you will probably fall off. But once you start pedalling, these wheels pick up angular momentum. They are going to resist change, thereby making balancing easier (*Angular Momentum*).
- Picture a small child on a spinning playground ride, such as a merry-go-round, and she's yelling that she wants to get off. You have to stop the spinning ride, but it's going to take some effort. Why? Because it has angular momentum (Steven Holzner, *How to Calculate Angular Momentum*. *Dummies*, 5 January 2017)
- In a sport such as ice skating, a person in a tucked position decreases the moment of inertia and spins faster -- in contrast with someone in an extended position when the moment of inertia is increased, decreasing the rate of spin (Michael Walden, *Angular Momentum in Sport*, *TeachPE*, 24 March 2021)

The matter is clarified otherwise by Brian Dorney (*Angular Momentum in Quantum Mechanics*, *Quantum Diaries*, 17 September 2011):

In Classical Mechanics (CM), angular momentum is associated with rotational motion. As an example, let's look at the spinning tea cup ride available at most amusement parks/carnivals (i.e. something similar to the one seen in this [YouTube Video](#)). Here the tea cups have what's called an *orbital* angular momentum associated with their motion around the center of the ride (similar to the Earth revolving around the Sun). Now the tea cups also have *spin* angular momentum due to the of the cup spinning on its own axis (similar to the masses that make up the Earth rotating about the planet's axis)....

Now in Quantum Mechanics (QM) it should not shock you to learn particles also have orbital and spin angular momentum (the sum of these two is the again total angular momentum for the particle).... Now another key difference is that *elementary particles are true point particles, and thus have no internal substructure*. This causes a profound difference in how angular momentum is handled in QM versus how it is handled in CM.

Take for example spin angular momentum. The notion of a piece of an electron rotating about an electron's axis is nonsense, there simply isn't "a piece of an electron!" Thus, spin angular momentum in QM is an *intrinsic* property of a particle and is not associated with some spatial variables. You cannot describe spin angular momentum in QM via a function of position variables or a vector in 3D space as we know it. Spin angular momentum in QM exists in the abstract world of linear algebra (aka matrix algebra), for which I will try not to delve to far into here.

The quest for an appropriate metaphor in the case of quantum mechanics has been addressed to physicists (*Metaphor for orbital angular momentum?* *Physics Forums*, 7 August 2015):

... someone asked me to clarify the difference of spin angular momentum and orbital angular momentum without math. I was trying to think of a metaphor, but I wanted to make sure it's a fair one -- the spin angular momentum is like Earth rotating on its own axis and orbital angular momentum of an electron in an orbital is analogous to the rotation of the Earth around the Sun.

*Response:* So it's a relatively fair analogy and won't be that misleading for a mere metaphor. Unfortunately the spin angular momentum has no classical analog, the spin operators... cannot be represented by any other operators which have classical analogue as is the case for orbital angular momentum... It may be safer to just "dictate" to your student that spin angular momentum is just there for every particle, i.e. there happened to be another angular momentum aside from orbital angular momentum in the quantum world, this being one of the distinguishing trademarks of quantum mechanics as opposed to classical mechanics.

**Quantum consciousness?** Conventional considerations of the meaning to be associated with geometrical configurations can now be fruitfully challenged by speculative reflection on the implications of quantum theory for human consciousness. This has been notably articulated from an international relations perspective by [Alexander Wendt](#) (*Quantum Mind and Social Science: unifying physical and social ontology*, 2015; [video](#); [interview](#)). Those implications are discussed separately (*Quantum consciousness implications of fundamental symbol patterns*, 2017; *On being "walking wave functions" in terms of quantum consciousness?*, 2017). A range of considerations have been variously presented:

- Harald Atmanspacher: *Quantum Approaches to Consciousness* (*Stanford Encyclopedia of Philosophy*, 2020 with extensive bibliography)
- Thomas R. Verny: *Consciousness and the Quantum Mind* (*Psychology Today*, 18 July 2022)
- Diana Darie: *Quantum Consciousness: The Emperor's new Mind and the Implications of the Incompleteness Theorems* (*Medium*, 20 October 2019)
- Robert Oates Jr.: *Consciousness and the Quantum: The Next Paradigm* (Maharishi International University Press, 2022)
- Shelli Renée Joye: *Sub-Quantum Consciousness: a geometry of consciousness based upon the work of Karl Pribram, David Bohm, and Pierre Teilhard De Chardin* (Viola Institute, 2019)
- Deepak Chopra, et al: *Cosmology of Consciousness: Quantum Physics and Neuroscience of Mind* (Science Publishers, 2017)

**Tetrahedral entanglement as a metaphor:** References are increasingly made to entanglement, inspired by the insights of quantum mechanics, as helpfully explained separately (*What Is Entanglement and Why Is It Important?* *CalTech*).

Recent research on the possibility that human consciousness may rely on entanglement is discussed by Elizabeth Fernandez (*Brain experiment suggests that consciousness relies on quantum entanglement* *Big Think*, 22 November 2022), citing the work of Christian Matthias Kerskens and David López Pérez who argue:

Recent proposals in quantum gravity have suggested that unknown systems can mediate entanglement between two known quantum systems, if the mediator itself is non-classical. This approach may be applicable to the brain, where speculations about quantum operations in consciousness and cognition have a long history....Our findings suggest that we may have witnessed entanglement mediated by consciousness-related brain functions. Those brain functions must then operate non-classically, which would mean that consciousness is non-classical. (*Experimental indications of non-classical brain functions*, *Journal of Physics Communications*, 6, 2022, 10)

There are some references to "tetrahedral entanglement", presumably in the light of implications of the tetrahedral  $6-j$  pattern, as noted above. There is the intriguing possibility that the inherently obscure articulations of quantum mechanics by physicists may distract from recognition of widespread intuitive understanding of the distinctions framed by the  $6-j$  pattern or spin angular momentum (existing only in "the abstract world of linear algebra").

Arguably, if the reality inferred by quantum mechanics is one in which people are necessarily intimately engaged -- if not "entangled" in ways the future may clarify -- then the patterns articulated by quantum physicists may be comprehensible otherwise (if only intuitively). The question is then what form this intuitive engagement may take in relation to the tetrahedral  $6-j$  symbol pattern -- as the sum over products of four  $3-j$  symbols.

Provocatively it may then be asked whether the 6-fold distinctions noted above by Edward de Bono could not be usefully explored as instances of metaphorical insights into the distinctions implied by the  $6-j$  tetrahedral pattern of quantum mechanics (*Six Thinking Hats*, 1985; *Six Action Shoes*, 1991; *Six Value Medals*, 2005). The 6-fold generalizations of such a pattern by de Bono as *Six Frames For Thinking About Information* (2008), or by [Raymond Abellio](#) as *La Structure absolue* (1965), could then be understood as efforts to articulate the nature of the  $6-j$  tetrahedron.

The same could be said of the 6 distinctive polarities in the 12-fold schema of Arthur Young (*Geometry of Meaning*, 1976). It is however the qualitative significance of "distinction" which is elusive -- seemingly with little insight from a quantum perspective. The challenge to comprehension can be explored otherwise in terms of any viable system of "disparate" features (*Dynamics of N-fold Integration of Disparate Cognitive Modalities*, 2021; *Global Coherence by Interrelating Disparate Strategic Patterns Dynamically*, 2019). One instance is the 8-fold patterns characteristic of computer memory (*Torus interconnect -- as used in supercomputers*, 2019). This is suggestive of the organization of the various "8-fold ways".

Of further relevance from a quantum perspective is the manner in which the  $6-j$  symbols and  $3-j$  symbols are complemented by  $9-j$  symbols. These are related to [recoupling coefficients](#) involving four angular momenta. It might be asked how their significance relates to 9-fold patterns recognized otherwise (*Ninefold configuration in practice and its comprehension constraints*, 2016).

**Experiencing distinctions in a tetrahedral framework:** The clarification required is how any such "distinction" may be experienced in a pattern of requisite variety -- as "experiential flesh" in contrast with the "cognitive bones" offered by the abstractions of more complex algebras. Ironically, from a metaphorical perspective, the distinctions derive from placing a distinctive "spin" on a cognitive modality

enabling a topic to be explored from a different "angle" -- even obsessively so, as a feature of cognitive "momentum" and "inertia".

There is potentially great irony in the extent to which esports now constitute an instance of "doing [quantum dynamics](#)" (however "badly") through the manner in which two tetrahedra of 6 roles (plus or minus 1) engage with one another -- with the tetrahedral coupling potentially understood as a form of entanglement. The experiential requirement for viable teams engenders a need for 6 distinctive roles, irrespective of the number of players associated with each distinct role. The same might be said of the distinct [rugby union positions](#) and the [association football positions](#). Participants have a strongly developed "re-cognition" of the experiential relationship between roles. The distinctive abstractions of "angular momentum" may indeed be intimately related to the experiential coherence of those roles. Team-play may well be "tetrahedralization" of roles.

To the extent that ball games are indeed an instance of "doing quantum dynamics", there is then a peculiar degree of role reversal to any comparison with the iconic satire of Moliere's *Le Bourgeois gentilhomme* (1670) in which the protagonist discovers: *For more than forty years I have been speaking prose while knowing nothing of it, and I am the most obliged person in the world to you for telling me so.* To the extent the "spirit of the game" is celebrated, is it through such understanding of quantum dynamics that the disconnect highlighted above might be remedied?

Provocatively again, it may also be asked whether comprehension of the distinctions of the 3-*j* pattern is helpfully enabled and illustrated in the extensive analysis of how [Dante Alighieri](#) describes the three rings (*tre giri*) of the Holy Trinity in *Paradiso 33* of the *Divine Comedy* (Arielle Saiber and Aba Mbirika, *The Three Giri of Paradiso XXXIII*, *Dante Studies*, 131, 2013, pp. 237-272). The remarkable interdisciplinary exploration combines insights from speculative theology, geometry and knot theory.

## Closest packing of concepts and compactification of insight

An understanding of closest packing has been a concern of physics since the time of Kepler -- and hence the recognition of [Kepler's conjecture](#) in relation to spheres. It is a continuing concern with respect to the [closest packing of atoms in molecules](#). Of particular relevance to this argument is the [closest packing of Archimedean polyhedra](#), as articulated by [Keith Critchlow](#) (*Order in Space: a design source book*, 1969). The question raised in that regard is the nature of closest packing of concepts and insights as they might relate to memorability.

One intriguing approach to any understanding of "closest packing of insight" is the nature of aphorism in relation to ignorance (*Ignorance and What We Do Not Know*, 1998). This is especially the case in the light of the comment by Karl Kraus: A great deal of learning can be packed into an empty head (*Aphorisms and More Aphorisms*, 1909). The nature of aphorisms is explored by [Andrew Hui](#) (*A Theory of the Aphorism: from Confucius to Twitter*, 2019; *In praise of aphorisms*, *Aeon*, 1 June 2020). Given the traditional importance associated with the 48 koan of the so-called [Gateless Gate](#), this frames the question of how an array of aphorisms might be configured to elicit insight of a higher order (*Configuring a Set of Zen Koan as a Wisdom Container: formatting the Gateless Gate for Twitter*, 2012). In relation to the wisdom of the United Nations, it is appropriate to recall the compilation by V. S. M. de Guinzbourg (*Wit and Wisdom of the United Nations: proverbs and apothegms on diplomacy*, 1961).

In the language of Buckminster Fuller:

Humans may be quite unconscious of their unavoidable employment of isotropic vector matrix fields of thought or of physical articulations....However, their physical brains...are always and only most economically interassociative, interactive, and intertransforming only in respect to the **closest-packed isotropic vector matrix fields** which altogether subconsciously accommodate the conceptual geometry picturing and memory storing of each individual's evolutionary accumulation of special-case experience happenings, which human inventories are accumulatively stored isotropic-vector-matrix-wise in the brain... (*Synergetics: explorations in the geometry of thinking*, 1975/1979I, 426.473) [emphasis added]

In the discussion of dimensionality in relation to fundamental physics, [Margaret Wertheim](#) notes:

So this gets us to the 10 dimensions of string theory. Here there are the four *large-scale* dimensions of spacetime (described by general relativity), plus an extra six 'compact' dimensions (one for electromagnetism and five for the nuclear forces), all curled up in some fiendishly complex, scrunched-up, geometric structure.

... here are many versions of string-theory equations describing 10-dimensional space, but in the 1990s the mathematician Edward Witten, at the Institute for Advanced Study in Princeton (Einstein's old haunt), showed that things could be somewhat simplified if we took an 11-dimensional perspective. (*Radical Dimensions*, *Aeon*, 10 January 2018)

The argument can be taken further by considering how much more insight might be appropriately and succinctly "packed" into any conceptual container for mnemonic purposes -- or as a "pregnant" symbol of their implications. It is necessarily the convenience of any particular pattern of attributions for a given observer which then reflects what is valued as "experiential flesh". The tetrahedron is then effectively a cognitive "coat rack" or "hat rack" on which "thinking hats" may be variously hung. Possibilities include:

- 6-fold pattern of edges, as argued above
- 10-fold pattern (combining edges with sides or vertices), offering a device for representation of the Ten Commandments, for example
- 12-fold pattern (considering each of the 4 vertices as a 3-fold nexus of edges), suggesting a mnemonic device through which the elements of any of the many 12-fold articulations might be presented
- 14-fold pattern (totalling edges, sides and vertices), of potential relevance to the 14-fold clustering of significance to such

extremes as oppositional logic and Shakespearean poetry, as argued separately (*Pattern of 14-foldness as an Implicit Organizing Principle for Governance?* 2021; *Variety of Rhyming Patterns in Standard 14-line Sonnets*, 2021)

Less obvious, except through intuitive appreciation of symmetry, are other features including the *tetrahedral great circles* of three types (Nicholas Shea, *Great Circles of the Tetrahedron*, 2018). Also of relevance are considerations of *tetrahedral symmetry*: As noted above, a regular tetrahedron has 12 rotational (or orientation-preserving) symmetries, and a symmetry order of 24 including transformations that combine a reflection and a rotation. Difficulties in recognizing all 12 are usefully addressed separately (*Symmetries of Tetrahedron*, *Mathematics Stack Exchange*, 2022).

Given the apparent simplicity of a tetrahedron as commonly viewed, the question is how complex patterns of insight might be held by it in some way -- if its potential in that respect is as significant as the above argument. Physics has offered an important insight in that respect, at least in terms of the perceptibility of the so-called *extra dimensions*. In string theory, *compactification* is a generalization of *Kaluza–Klein theory* which endeavours to reconcile the gap between the conception of the universe based on its four observable dimensions with the 10, 11, or 26 dimensions by which theoretical equations suggest the universe is composed. For this purpose it is assumed the *extra dimensions* are "wrapped" up on themselves, or "curled" up on *Calabi–Yau spaces*, or on *orbifolds*.

With respect to the above argument, the question here is whether there is a form of intuitive cognitive engagement with what is otherwise readily held to be beyond human ken. Given the reference to sonification above, it is intriguing to note the role of an orbifold in ordering music (Dmitri Tymoczko, *The Geometry of Musical Chords*, *Science*, 313. 5783, 7 July 2007). In the case of a tetrahedron, for example, is it the case that additional significance may be effectively experienced as "curled up" within its seemingly simple form? Is that how it may be held to be of symbolic significance -- as with other Platonic forms valued from a perspective of sacred geometry?

Given the association of David Bohm, as a theoretical physicist, with his valued insights into dialogue (as noted above), it could then be asked how "curled up" significance relates to his understanding of implicate order. Bohm contrasts this with the explicate order and suggests a *holomovement* process by which they are related (*Wholeness and the Implicate Order*, 1980).

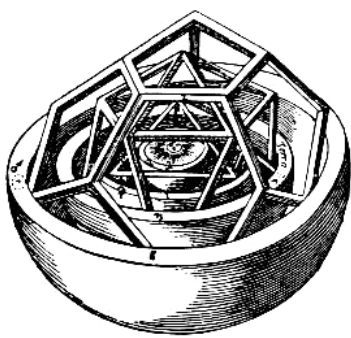

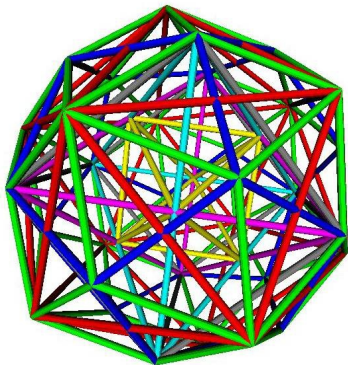
For Bohm, *implicate and explicate order* are ontological concepts for quantum theory used to describe two different frameworks for understanding the same phenomenon or aspect of reality. Various interpretations have been given to holomovement (Renée Weber, *Meaning as being in the implicate order philosophy of David Bohm: a conversation*, *Quantum Implications*, 1987; David Storoy, *David Bohm, Implicate Order and Holomovement*, *Science and Nonduality*; Emanuel Kuntzelman and Jill Robinson, *The Holomovement*, *Kosmos*, Autumn 2021; Kavita Byrd, *Creating a Quantum "Holomovement" with the Spheres of Co-Creation*, *Ubiverse*, 24 July 2020).

## Polyhedral cosmograms as Rosetta Stones -- "cosmohedra"?

As indicated in introducing this argument, there is a case for imagining how a complex of "antitheses" might be envisaged, namely through what images it might be variously framed, as considered separately (*Interrelating Multiple Ways of Looking at a Crisis: beyond the pandemic discipline of the one right way*, 2021). With the current rumours of nuclear war in the imminent future, something radically distinctive would appear to be required if any solution is to avoid being a feature of the problem (*Time for Provocative Mnemonic Aids to Systemic Connectivity?* 2018).

One inspiration in that regard is the role that has been played by the *bombing of Guernica* as a tragedy of the not-so-distant past -- commemorated through the symbolic significance attributed to it through the much-cited painting by Pablo Picasso (*Guernica*, 1937). This suggests the possibility of envisaging coherence in the surreal as a kaleidoscope of imagery forming a hyperobject, as argued separately (*Reimagining Guernica to Engage the Antitheses of a Cancel Culture*, 2022).

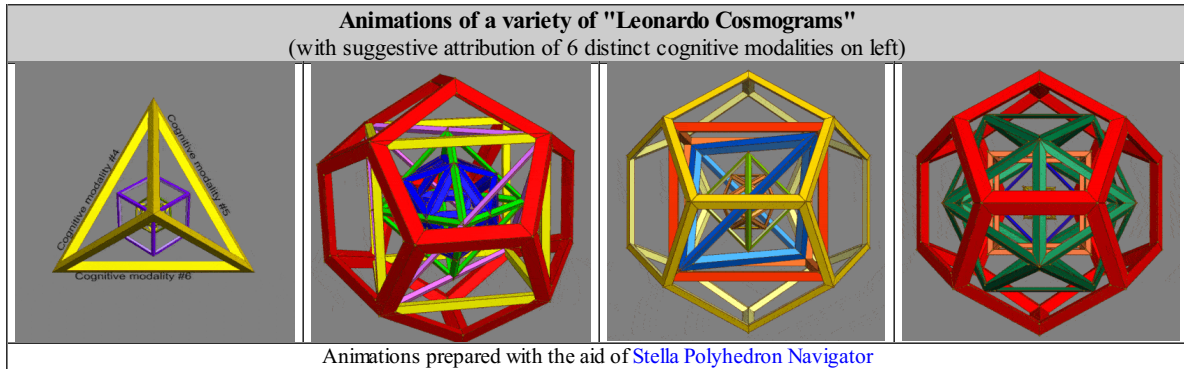
Understood as a challenge to visualization in 3D (at least), possibilities with respect to dialogue are discussed separately (*Visualization Enabling Integrative Conference Comprehension*, 2018). That argument reproduced the contrasting images below.

Contrasting images of structural coherence		
Nested polyhedral model of solar system of Johannes Kepler (1596)	Chinese ivory puzzle ball	Rhombic Triacontahedron (green) as a nesting framework
		
Reproduced from <i>Wikipedia</i> entry on <i>Mysterium Cosmographicum</i> (1596)	British Museum [CC BY 2.0 ], via <i>Wikimedia Commons</i>	Virtual reality variants: <i>static</i> : <i>vrml</i> or <i>x3d</i> ; <i>mutual rotation</i> : <i>vrml</i> or <i>x3d</i> ; "pumping": <i>vrml</i> or <i>x3d</i> ; videos: "pumping" <i>mp4</i> ; "rotation" <i>mp4</i>

To the extent that such configurations are indeed enhanced by music in some way, they offer an allusion to the philosophical reference to the "music of the spheres" -- possibly to be understood as a musical Rosetta Stone. It is then useful to appreciate the aesthetic potential

of "cosmograms", of which the animations below are examples in 3D. Given the argument above, with its quantum implications for cognition and psychosocial order, such representations could be termed "cosmohedra".

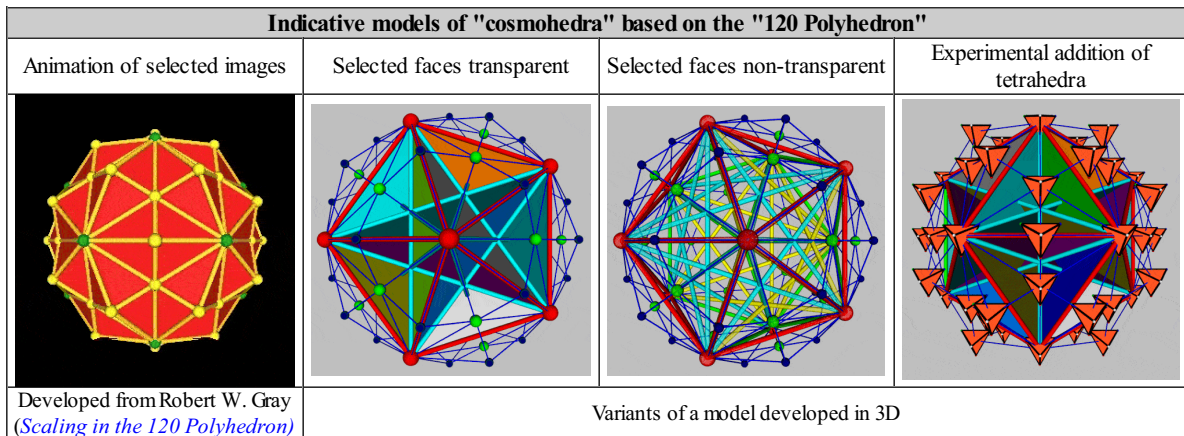
In contrast to metaphorical references to a "Rosetta Stone", with the correspondences between distinct languages implied by their juxtaposition on sides of the stone, use of a polyhedron offers contrasting implications between use of the sides, the edges and the vertices -- and the possibility of its topological transformation to other polyhedra (*Pathway "route maps" of potential psychosocial transformation?* 2015).



The above animations are suggestive of the manner in which richer patterns of significance could be variously packed and carried by polyhedral configurations. Of particular interest is the focus given to the so-called "120 polyhedron" of 62 vertices and 120 faces (Robert W. Gray, *What's In This Polyhedron?* 2000). One animation is provided by Roman Chijner (*Polyhedron, whose coordinates are golden numbers, GeoGebra*). It is extensively discussed by Jytte Brender Mcnair, Lynnclaire Dennis and Louis Kauffman (*The Mereon Matrix: everything connected through (k)nothing*, 2018).

The polyhedron "holds" an icosahedron of 12 vertices, 20 vertices of a regular dodecahedron, together with the 30 vertices of 5 octahedra ( $12+20+30=62$ ). However in total it is a container for: 10 tetrahedra; 5 cubes; 5 octahedra; 5 rhombic dodecahedra; 1 icosahedron; 1 regular dodecahedron; 1 rhombic triacontahedron; and many jitterbugs. The coordinates of each are most helpfully provided by Robert Gray (*Polyhedra Coordinates*, 2001).

An extensive presentation of a variety of ways in which the 120 Polyhedron may be explored is also provided by Robert Gray (*Scaling in the 120 Polyhedron - Part 2 - Part 3 - Part 4*). As an indication, a selection of images from the first section have been combined in the animation on the left below. The coordinates of the embedded icosahedron and the 10 embedded tetrahedra were used to produce a 3D model (as shown in the central animations below). As an exercise, 62 tetrahedra were presented (and rotated) in the animation on the right.



## References

Raymond Abellio. *La Structure absolue, essai*. Gallimard, 1965.

Christopher Alexander:

- The Nature of Order: an essay on the art of building and the nature of the universe. Center for Environmental Structure, 2002-5 [summary]
- A Foreshadowing of 21st Century Art: the color and geometry of very early Turkish carpets. Oxford University Press, 1993 [review]
- A Pattern Language: towns, buildings, construction. Oxford University Press, 1977
- Notes on the Synthesis of Form. Harvard University Press, 1965 [summary]

Stafford Beer:

- Beyond Dispute: the invention of team synteegrity. Wiley, 1994 [summary]

- Decision and Control: the meaning of operational research and management cybernetics. Wiley, 1966
- Brain of the Firm. Wiley, 1981
- Diagnosing the System for Organizations. Wiley, 1985
- The Viable System Model: its provenance, development, methodology and pathology. *Journal of the Operational Research Society*, 35, 1984
- Platform for Change. Wiley, 1995 [[summary](#)]

A. G. E. Blake. The Intelligent Enneagram. Shamballa, 1997

David Bohm:

- On Dialogue. Routledge, 2013
- Wholeness and the Implicate Order. Routledge, 1980 [[summary](#)]

R. Buckminster Fuller. Synergetics: explorations in the geometry of thinking. Macmillan, 1975/1979

Roberto Casati and Achille C. Varzi, Holes and Other Superficialities. MIT Press, 1994

Edward de Bono:

- I Am Right, You Are Wrong. Penguin, 1968 [[text](#)]
- Six Thinking Hats. Little Brown, 1985
- Six Frames For Thinking About Information. Vermilion, 2008
- Six Value Medals. Vermilion, 2005
- Six Action Shoes. Longman Higher Education, 1991
- Water Logic: an alternative to I am Right, You are Wrong. Penguin, 1987

Terrence W. Deacon:

- Incomplete Nature: how mind emerged from matter. W. W. Norton, 2011
- What's Missing from Theories of Information? In: Paul Davies and Niels Henrik Gregersen (Eds.), Information and the Nature of Reality: From Physics to Metaphysics. Cambridge University Press, 2010, pp. 146-169 [[text](#)]

Marcus du Sautoy. Symmetry. HarperCollins, 2008

Underwood Dudley. Numerology: Or, What Pythagoras Wrought. Mathematical Association of America, 1998

Anagarika Govinda. The Inner Structure of the I Ching; the Book of Transformations. Weatherill, 1981

Edward Haskell. Full Circle: The Moral Force of Unified Science. New York, Gordon and Breach, 1972 [[text](#)]

Andrew Hui. A Theory of the Aphorism: from Confucius to Twitter. Princeton University Press, 2019 [[summary](#)]

W. T. Jones. The Romantic Syndrome: toward a new method in cultural anthropology and the history of ideas. Martinus Nijhoff, 1961) [[summary](#)]

R. Maher, S. Mann, and C. A. McAlpine. MetaMAP: a graphical tool for designing initiatives to support multiple sustainability goals. *Sustainability Science*, 17, 2022 [[text](#)]

Jytte Brender McNair, Lynnclaire Dennis and Louis Kauffman (Eds.). The Mereon Matrix: everything connected through (k)nothing. World Scientific, 2018

Arthur I. Miller. 137: Jung, Pauli, and the Pursuit of a Scientific Obsession. W. W. Norton, 2010

Ann E. Moyer. The Philosophers' Game: Rithmomachia in Medieval and Renaissance Europe. University of Michigan Press, 2001 [[contents](#)]

Roger Penrose. The Emperor's New Mind: concerning computers, minds and the Laws of Physics. Oxford University Press, 1989 [[summary](#)]

Jean Piaget. Studies in Reflecting Abstraction. Taylor and Francis, 2014 [[summary](#)]

Nicholas Rescher. The Strife of Systems: an essay on the grounds and implications of philosophical diversity. University of Pittsburg Press, 1985

Alexander Wendt. Quantum Mind and Social Science: unifying physical and social ontology. Cambridge University Press, 2015

Margaret Wertheim:

- Pythagoras' Trousers: God, Physics, and the gender wars. W. W. Norton, 1995
- The Pearly Gates of Cyberspace: a history of space from Dante to the Internet. Virago Press, 2000
- Physics on the Fringe: smoke rings, circlons and Alternative Theories of Everything. Walker Books, 2012

Arthur M. Young:

- Geometry of Meaning. Delacorte Press, 1976
- The Reflexive Universe: evolution of consciousness. Delacorte Press, 1976
- The Bell Notes: a journey from physics to metaphysics. Delacorte Press, 1979



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