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# Identifying Polyhedra Enabling Memorable Strategic Mapping

## Visualization of organization and strategic coherence through 3D modelling

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## Introduction

Strategies, declarations and sets of values and principles typically take the form of lists with a specific number of items. The number selected often varies between 8 and 30. Examples are the 8 Millennium Development Goals of the UN and the 30-fold Universal Declaration of Human Rights. Currently a major focus is given to the 17 Sustainable Development Goals of the UN. There is seemingly a total lack of explanation as to why any given number is appropriate. Nor is there any interest in how such patterns may be more or less appropriate from a systemic perspective. Little consideration is given to the manner in which the items noted in each case are related -- let alone how the many different strategic articulations, based on different choices of numbers, are related to one another.

It is possible to imagine that each such set could be mapped onto a polygon in 2D with a distinctive number of sides -- potentially reflective of seats around a negotiation table. It is also possible to explore how the elements of any such articulation could be mapped in 3D onto a polyhedron -- to be variously rotated for inspection in virtual reality. Possibilities in that respect are discussed separately (*Psychosocial Implication in Polyhedral Animations in 3D*, 2015; *Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors*, 2008).

Especially relevant to this argument is how any such mapping increases memorability and communicability -- and how it enables the set to be comprehended as a whole. These considerations can be considered vital to any sense of coherence of the set as an integrative pattern -- as distinct from a simple checklist or a "to do" list. Are many people able to recall the elements in the patterns identified in the following, or why they include the number of elements in each case:

- 10-foldness: *Habitual use of a 10-fold strategic framework?*
- 12-foldness: *Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources*
- 20-foldness: *Checklist of web resources on 20 strategies, rules, methods and insights*

The question in what follows is **what makes for memorability in the face of a relatively complex set of principles or elements in a strategy**. This question assumes that global governance is faced with a fundamental cognitive challenge, as argued separately (*Comprehension of Numbers Challenging Global Civilization*, 2014). Is the set of 17 Sustainable Development Goals as "comprehensible" or "memorable" as might be assumed to be necessary for their coherent global governance? The challenge of comprehending the risk of civilizational collapse may be in some way related to any tendency to represent it by use of a 2D "mind map" (*Mind Map of Global Civilizational Collapse: why nothing is happening in response to global challenges*, 2011).

Framed otherwise, the question is **at what number does coherence and memorability start to erode** in the case of a 2D pattern of

"constructible polygons". When does the number of representatives around a table characterize fragmentation rather than coherence? Configured in 3D, the question can be framed in terms of the number of "constructible polyhedra", a pattern which is not seemingly explored to the same degree as in the case of polygons. In quest of greater systemic coherence, the exercise which follows is an exploration of the polyhedra which might be suitable for mapping a strategic articulation, depending on the choice of numbers of elements.

In a period in which collective memory is variously challenged, it should be emphasized that **the following exercise is primarily concerned with memorability** (*Societal Learning and the Erosion of Collective Memory*, 1980). There is a very extensive mathematical literature on polyhedra from a variety of perspectives. The literature does not seem to engender or order polyhedra in terms of their suitability for mapping.

Although memorability and mapping are not the focus in such studies, extensive use is made of polyhedral frameworks in computer compiler techniques for analysis and transformation of codes with nested loops (also termed the polytope model). This is indicative of the relevance of the approach to the analysis of patterns of feedback loops which characterize the relation between the many strategic problems (*Feedback Loop Analysis in the Encyclopedia Project*, 2000). A more general review of information mapping is offered by L. John Old (*Information Cartography: using GIS for visualizing non-spatial data Proceedings, ESRI International Users' Conference*, 2002).

Mathematics has indeed developed far more sophisticated tools to explore polygons, polyhedra and polytopes in N-dimensions. As the realm of specialists, these are typically unrelated to any criteria of memorability, comprehensibility or communicability. The quest for comprehension of the symmetry associated with such forms implies could be understood as implying such a preoccupation (*Dynamics of Symmetry Group Theorizing: comprehension of psycho-social implication*, 2008).

As the epitome of preoccupation with patterns of order and relationships, there is some irony to the fact that, in organizing itself -- as in the *Mathematics Subject Classification* -- mathematics is curiously challenged by reliance on the simplest methods of order, namely the nested hierarchy (*Is the House of Mathematics in Order? Are there vital insights from its design*, 2000; *Towards a Periodic Table of Ways of Knowing -- in the light of metaphors of mathematics*, 2009).

The associated thinking could be said to reinforce the nested structures typical of strategic articulations. Thus the UN's 17 Sustainable Development Goals clusters 169 targets, with little consideration of how these are interrelated, even though the 17th Goal ("Partnerships for the Goals") is concerned to a degree with their coordination, namely to: *Strengthen the means of implementation and revitalize the global partnership for sustainable development*. **Is a 17-fold pattern inherently "forgettable"?**

From the perspective of memorability, such challenges could be explored in terms of the skills of a mnemonist, most notably the method of loci, as described by Frances Yates (*The Art of Memory*, 1966). This is a strategy of memory enhancement which uses visualizations of familiar spatial environments in order to enhance the recall of information. It is associated with the term *topoi* (from the Greek for "place") as a metaphor introduced by Aristotle (*Topics*). These characterize the "places" in memory where a speaker or writer may "locate" arguments that are appropriate to a given subject -- as mnemonic aids (Richard Nordquist, *Definition and Examples of the Topoi in Rhetoric*. *ThoughtCo*, 12 February 2020).

Understood otherwise, this is the modern derivation of topics. A topic map in two-dimensions is upheld as a standard for the representation and interchange of knowledge, with an emphasis on the findability of information. The Topic Map paradigm has been adapted to the web by an international consortium (Benedicte Le Grand, *Topic Map Visualization*, January 2003). The possibility of its adaptation to 3D and virtual reality has been envisaged (Alexandre Rocha Oliveira et al, *Supporting Information Visualization through Topic Maps*, Educational Technology, 2002)

The question here is how the vertexes, edges and faces of polyhedra can be used as "*topoi*" or "loci" such as to provide a higher order of coherence to the strategic pattern which connects the information associated with them. The role of music in rendering such patterns memorable is discussed. **The argument concludes by highlighting the developing insights of logic into the geometry of opposition and the manner in which alternative patterns of connectivity are reframed by particular polyhedra**.

This is seen as a specific response to the currently problematic degree of divisiveness, fragmentation and disagreement in governance at all levels of society. In extraordinary times, there is a case for recognizing that extraordinary polyhedral forms may offer insights into unforeen approaches to governance and requisite connectivity. The many examples presented then raise the question as to how global governance might be imagined in that light -- if only by the future.

## Methodology

The following table derives from a procedure using the facilities of *Stella Polyhedron Navigator*. This software application has a very extensive library of polyhedra with a search facility. This enables the number of polyhedra with a given number of faces, edges or vertexes to be identified -- on the assumption that the polyhedron is of sufficient interest to be registered in that library.

The procedure then involved counting the number of polyhedra with N characteristics, whether the number of faces, edges or vertexes. The search started with 4, which resulted in identification of the simplest polyhedron, namely the tetrahedron -- with 4 faces, 6 edges, and 4 vertexes. Given the focus on memorability, no attempt was made in this initial exercise to eliminate:

- **double counting**:. Thus in the case of the tetrahedron, the total polyhedra listed in Column B (below) was 2 (rather than 1, in row 59 of Column A) -- since the number of faces equals the number of vertexes. The total of 2 results from totalling the elements in the corresponding row of Column D.
- **less regular polyhedra**: Notably prisms and pyramids, which some have labelled "second class citizens"
- **polychora**: Namely polytopes as 3D projection of 4D structures, of which there is an extensive collection in the polyhedra library,

in addition to those which may be generated by the software according to user criteria

The assumption here is that, despite such inclusions (perhaps to be removed at a later date), the counts in Column B are an indication of the connectivity associated with a given number of polyhedral elements. As such they are an indication of the potential mapping relevance of the corresponding row in Column C.

At this stage, no special consideration was given to cases giving rise to duplicate totals in Column B. In the case of the tetrahedron, the mappable polyhedral elements are 4 (Column C), namely, the search facility result for the tetrahedron (whether from the number of faces, or vertexes, as indicated in Column D). Meaning that only one polyhedron has 4 faces, 4 edges, or 4 vertexes. Again, note the double counting.

Of particular relevance to memorability is the pattern of symmetry deriving from prime number factors (Column E), assumed to be associated with the total number of mappable elements (Column C). As indicated for the tetrahedron (Column A, row 59), The only prime number of relevance is 2, resulting in an indication of $2^2$ in Column E.

The procedure was initially applied in searches on polyhedra with the total number of mappable elements (Column C) from 1 to 100. The resulting Table 1 was then sorted by the total number of polyhedra resulting from the search, ignoring the consequence of double counting. In descending order, this gave rise to Column B. This shows that -- in terms of memorability at least -- that any polyhedron with 60 mappable elements would constitute a means of configuring 60 elements (Row 1, Column A). Note that in this case the prime number factors are 2x2x3x5 (Column E).

The question is then whether the simpler (and more familiarly memorable) symmetrical polyhedra can themselves be used to render coherent a pattern of 60 strategic elements. For that purpose, Column F is used to distinguish the number of elements (vertexes, edges, faces) onto which 60 (for example) might be mapped. Clearly none of the 5 regular Platonic polyhedra in Column F is suitable for that purpose. In fact, as noted in the table (Column H), a suitable mapping of 60 elements could however be achieved with several of the 13 semi-regular Archimedean polyhedra, necessarily more complex, although their symmetry is comprehensible to a degree when visualized:

- 60 vertexes: truncated dodecahedron, truncated icosahedron, rhombicosidodecahedron, snub dodecahedron
- 60 edges: snub cube, icosidodecahedron
- 60 faces: none

The table has been split with those items less than the possibility of the Platonic polyhedra (namely 62) being presented subsequently in Table 2.

**Table 1: Distinguishing polyhedra of primary value for memorable strategic mapping**
(mauve=Platonic mappabilty; yellow=non-Platonic mappabiliuty; G=dual of cuboctahedron)

| A | B | C | D | | | E | | | | | F (Platonic) | | | | | G | H (Archimedean: semiregular) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Total poly. (desc. order) | Map. poly. elem. | Total polyhedra with indicated number of elements | | | Prime number factors | | | | | Tetra 14 | Cube 26 | Octa 26 | Dod. 62 | Ico. 62 | Rho. dod. 50 | Cubo. 50 | Tun. tetr. 38 | Trun. cube 74 | Trun. oct. 74 | Rho. cub. 98 | Trun. cubo. 146 | Snub. cubo. 122 | Icosi. dod. 122 | Trun. dod. 182 | Trun. icos. 182 | Rho. icod. 242 | Trun. icosi. 362 | Snub dod. 302 |
| | | | Faces | Edges | Vert. | 2 | 3 | 5 | 7 | N | 4v | 8v | 6v | 20v | 12v | 14v | 12v | 12v | 24v | 24v | 24v | 48v | 24v | 30v | 60v | 60v | 60v | 120v | 60v |
| | | | | | | | | | | | 6e | 12e | 12e | 30e | 30e | 24e | 24e | 18e | 36e | 36e | 48e | 72e | 60e | 60e | 90e | 90e | 120e | 180e | 150e |
| | | | | | | | | | | | 4f | 6f | 8f | 12f | 20f | 12f | 14f | 8f | 14f | 14f | 26f | 26f | 38f | 32f | 32f | 32f | 62f | 62f | 92f |
| 1 | 187 | 60 | 65 | 41 | 81 | $2^2$ | 3 | 5 | | | | | | | | | | | | | | | X | X | X | X | X | | X |
| 2 | 108 | 48 | 36 | 28 | 44 | $2^4$ | 3 | | | | | | | | | | | | | | X | X | X | | | | | | |
| 3 | 105 | 24 | 42 | 23 | 40 | $2^3$ | 3 | | | | | | | | | X | X | | | | | | X | | | | | | |
| 4 | 80 | 12 | 35 | 8 | 37 | $2^2$ | 3 | | | | | X | X | X | X | X | X | X | | | | | | | | | | | |
| 5 | 79 | 30 | 24 | 23 | 32 | 2 | 3 | 5 | | | | | | X | X | | | | X | X | X | | | X | | | | | |
| 6 | 78 | 72 | 11 | 49 | 18 | $2^3$ | $3^2$ | | | | | | | | | | | | | | | X | | | | | | | |
| 7 | 70 | 14 | 35 | 5 | 30 | 2 | | | 7 | | X | | | | | | | | X | X | | | | | | | | | |
| 8 | 70 | 32 | 33 | 7 | 30 | $2^5$ | | | | | | | | | | | | | | | | | | X | X | X | | | |
| 9 | 65 | 20 | 20 | 19 | 26 | $2^2$ | | 5 | | | | X | X | X | X | | | | | | | | | | | | | | |
| 10 | 55 | 90 | 18 | 34 | 3 | 2 | $3^2$ | 5 | | | | | | | | | | | | | | | | | X | X | | | |
| 11 | 47 | 26 | 35 | 4 | 8 | 2 | | | | 13 | | X | X | | | X | X | | | | X | | | | | | | | |
| 12 | 47 | 36 | 4 | 38 | 5 | $2^2$ | $3^2$ | | | | | | | | | | | | X | X | | | | | | | | | |
| 13 | 45 | 16 | 20 | 5 | 20 | $2^4$ | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | 42 | 10 | 18 | 3 | 21 | $2^2$ | | 5 | | | X | | | | | | | | | | | | | | | | | | |
| 15 | 40 | 18 | 19 | 8 | 13 | 2 | $3^2$ | | | | | X | X | | | | | X | | | | | | | | | | | |
| 16 | 36 | 40 | 8 | 10 | 18 | $2^3$ | | 5 | | | | | | | | | | | | | | | | | | | | | |
| 17 | 32 | 8 | 13 | 1 | 18 | $2^3$ | | | | | X | X | X | | | | | X | | | | | | | | | | | |
| 18 | 31 | 28 | 5 | 20 | 6 | $2^2$ | | | 7 | | | | | | | | | | | | | | | | | | | | |
| 19 | 27 | 42 | 16 | 5 | 6 | 2 | 3 | | 7 | | | | | X | X | | | | | | | | | | | | | | |
| 20 | 27 | 96 | 4 | 14 | 9 | $2^5$ | 3 | | | | | | | | | | | | | | | | | | | | | | |
| 21 | 25 | 62 | 18 | | 7 | | | | | | | | | | | | | | | | | | | | | | X | X | |
| 22 | 25 | 80 | 11 | 5 | 9 | $2^4$ | | 5 | | | | | | | | | | | | | | | | | | | | | |
| 23 | 22 | 15 | 4 | 9 | 9 | | 3 | 5 | | | | | | | | | | | | | | | | | | | | | |
| 24 | 22 | 44 | 13 | 2 | 7 | $2^2$ | | | | 11 | | | | | | | | | | | | | | | | | | | |
| 25 | 22 | 50 | 9 | 5 | 8 | 2 | | $5^2$ | | | | | | | | X | X | | | | | | | | | | | | |
| 26 | 20 | 22 | 13 | 2 | 5 | 2 | | | | 11 | | | | | | | | | | | | | | | | | | | |
| 27 | 20 | 54 | 15 | 3 | 2 | 2 | $3^3$ | | | | | | | | | | | | | | | | | | | | | | |
| 28 | 20 | 84 | 9 | 6 | 5 | $2^2$ | 3 | | 7 | | | | | | | | | | | | | | | | | | | | |
| 29 | 19 | 9 | 6 | 3 | 10 | | $3^2$ | | | | | | | | | | | | | | | | | | | | | | |
| 30 | 19 | 38 | 16 | 1 | 2 | 2 | | | | 19 | | | | | | | | X | | | | | X | | | | | | |
| 31 | 18 | 52 | 13 | 1 | 4 | $2^2$ | | | | 13 | | | | | | | | | | | | | | | | | | | |
| 32 | 17 | 11 | 9 | 0 | 8 | | | | | 11 | | | | | | | | | | | | | | | | | | | |
| 33 | 17 | 25 | 1 | 6 | 10 | | | $5^2$ | | | | | | | | | | | | | | | | | | | | | |
| 34 | 16 | 21 | 2 | 10 | 4 | | 3 | | 7 | | | | | | | | | | | | | | | | | | | | |
| 35 | 15 | 92 | 9 | | 6 | | | | | | | | | | | | | | | | | | | | | | | | X |
| 36 | 14 | 6 | 8 | 1 | 5 | $2^2$ | 3 | | | | | X | X | | | | | | | | | | | | | | | | |
| 37 | 13 | 7 | 5 | 0 | 8 | | | | 7 | | | | | | | | | | | | | | | | | | | | |
| 38 | 12 | 56 | 6 | 3 | 3 | $2^3$ | | | 7 | | | | | | | | | | | | | | | | | | | | |
| 39 | 12 | 64 | 4 | 3 | 5 | $2^6$ | | | | | | | | | | | | | | | | | | | | | | | |
| 40 | 12 | 100 | 5 | 1 | 6 | $2^2$ | | $5^2$ | | | | | | | | | | | | | | | | | | | | | |
| 41 | 11 | 13 | 5 | 1 | 5 | | | | | 13 | | | | | | | | | | | | | | | | | | | |
| 42 | 11 | 35 | 3 | 5 | 3 | | | 5 | 7 | | | | | | | | | | | | | | | | | | | | |
| 43 | 11 | 70 | 5 | 4 | 2 | | | 5 | 7 | | | | | | | | | | | | | | | | | | | | |
| 44 | 8 | 17 | 5 | 2 | 1 | | | | | 17 | | | | | | | | | | | | | | | | | | | |
| 45 | 8 | 27 | 5 | 3 | | | $3^3$ | | | | | | | | | | | | | | | | | | | | | | |
| 46 | 8 | 55 | 1 | 2 | 5 | | | 5 | | 11 | | | | | | | | | | | | | | | | | | | |
| 47 | 7 | 65 | 2 | 3 | 2 | | | 5 | | 13 | | | | | | | | | | | | | | | | | | | |
| 48 | 6 | 5 | 2 | | 4 | | | 5 | | | | | | | | | | | | | | | | | | | | | |
| 49 | 6 | 37 | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 50 | 6 | 68 | 3 | | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| 51 | 6 | 88 | 5 | 1 | | $2^3$ | | | | 11 | | | | | | | | | | | | | | | | | | | |
| 52 | 5 | 34 | 2 | 1 | 2 | 2 | | | | 17 | | | | | | | | | | | | | | | | | | | |
| 53 | 5 | 45 | 1 | 2 | 2 | | $3^2$ | 5 | | | | | | | | | | | | | | | | | | | | | |
| 54 | 5 | 98 | 1 | 3 | 1 | | | | | | | | | | | | | | | | X | | | | | | | | |
| 55 | 4 | 33 | 1 | 3 | | | 3 | | | 11 | | | | | | | | | | | | | | | | | | | |
| 56 | 4 | 75 | | 2 | 2 | | 3 | $5^2$ | | | | | | | | | | | | | | | | | | | | | |
| 57 | 3 | 31 | 2 | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 58 | 3 | 86 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 59 | 2 | 4 | | 1 | 1 | $2^2$ | | | | | X | | | | | | | | | | | | | | | | | | |
| 60 | 2 | 19 | 1 | 1 | | | | | | 19 | | | | | | | | | | | | | | | | | | | |
| 61 | 2 | 23 | | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 62 | 2 | 39 | | 2 | | | 3 | | | 13 | | | | | | | | | | | | | | | | | | | |

## Commentary on patterns of N-foldness

Is it indeed the Platonic polyhedra which lend themselves most readily to comprehensibility? Is this the reason for the long-standing appeal of the 12-sided dodecahedron, for example (*Associating significance with a dodecahedron*, 2018)? If the challenge of the times can be understood as calling for a radical increase in connectivity (*Time for Provocative Mnemonic Aids to Systemic Connectivity?* 2018), this could be explored in terms of the possibility of *Increasing the dimensionality of the archetypal Round Table* -- from 2D to 3D.

The following remarks could be usefully enhanced and framed otherwise from a mathematical perspective. It should also be stressed that **there is a very wide range of polyhedra, many of which are necessarily omitted from the very extensive range in the Stella library on which this exercise was based**. Again the concern here is however the relevance of the following to memorability and comprehensible mapping possibilities

This exploration originated in a preoccupation with the patterns so obviously preferred in conceptual articulation (*Patterns of N-foldness: comparison of integrated multi-set concept schemes as forms of presentation*, 1980). The preference is rarely explained or explored. The focus on memorability in this argument, especially to the extent it may relate to any preoccupation with so-called sacred geometry, would

be framed as anathema to many specialists in polyhedra studies. An appropriate relationship to numerology or to mathematical theology, as conventionally deprecated, has yet to be fruitfully clarified (*Mathematical Theology: Future Science of Confidence in Belief*, 2011). It is therefore remarkable to note that the seminal thinker on polyhedra, Leonhard Euler had an earlier -- and continuing fascination with the organization of music, as discussed below.

The table above helps to frame the following questions regarding memorable mapping of strategic articulations of N elements (as indicated by Column B):

- **Abundance of polyhedra:** Many more polyhedra are associated with those requiring 60 mappable elements or more -- than tend to be associated with those requiring fewer. It might then be asked to what strategic articulations these might fruitfully correspond -- as may be considered viable and credible by the future.

- **Relevance of more complex polyhedra**: The more complex strategic articulations, requiring 60 mappable elements or more, cannot be associated with the 5 simpler Platonic polyhedra and require the more complex Archimedean set. Their memorability for that purpose is potentially of a lesser order -- given their complexity. Extensive reference has however been made to the "mediating" role and relevance of the cuboctahedron by Buckminster Fuller (*Synergetics: Explorations in the Geometry of Thinking*, 1975/1979). Especially relevant, as discussed below with respect to the logic of opposition, is the role of its dual -- the rhombic dodecahedron (included in the table for tha reason).

- **Relative simplicity of strategic articulations**: As might be expected, many of the conventional articulations of strategic initiatives are potentially associated with more mappable (and memorable) polyhedra, as indicated in the following table

**Table 2: Indicative patterns of coherence and memorability**
(see more complete listing: *Table of strategic structural attributions by number of elements*, 2019)

| N-foldness | Factors | Examples |
|---|---|---|
| 8-foldness | $2^3$ | UN Millennium Development Goals; Noble Eightfold Path; Eightfold Way of particle-physics theory; Eightfold Path of policy analysis |
| 9-foldness | $3^3$ | Planetary boundaries; See checklist of *Indicative symbols* |
| 10-foldness | 2x5 | See checklist: *Habitual use of a 10-fold strategic framework?* |
| 12-foldness | $2^2$x3 | See: *Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources* |
| 14-foldness | 2x7 | *Grand Challenges for Engineering in the 21st Century* (National Academy of Engineering) |
| 15-foldness | 3x5 | Global Challenges (Millennium Project); Principles of transformation (Christopher Alexander) |
| 16-foldness | $2^4$ | UN Sustainable Development Goals (without coordinating 17th); Earth Charter; *The Next Generation of Emerging Global Challenges* (Policy Horizons Canada) |
| 17-foldness | 17 | UN Sustainable Development Goals (with coordinating 17th); *17 Things We Don't Know,,,about Covid-19* (Lisa Rankin); *Top 17 Environmental Problems* (Renewable Resources Coalition) |
| 18-foldness | 2x$3^3$ | *European Convention on Human Rights* |
| 20-foldness | $2^2$x5 | See: *Checklist of web resources on 20 strategies, rules, methods and insights* |
| 30-foldness | 2x3x5 | *Universal Declaration of Human Rights*; note the number of 30-point plans |
| 25-foldness | $5^2$ | *Cairo Declaration on Human Rights in Islam* |
| 53-foldness | 53 | *Arab Charter on Human Rights* |
| 72-foldness | $2^3$x$3^2$ | *"Demonique": a mnemonic aid to comprehension of potential system failure?; "Angelique": evangelisation of the resolutique in the light of angelology?* |
| 82-foldness | 2x41 | *American Convention on Human Rights* |

- **Image memorability**: A polyhedron offers a third dimension in addition to the two-dimensionalithy of an "image" as normally depicted -- whatever the three-dimensionality it may imply. Recent research on image memorability might however be usefully adapted to the memorability of polyhedra (Phillip Isola, et al, *What makes an image memorable? IEEE Conference on Computer Vision and Pattern Recognition*, CVPRW, 2011; Shay Perera, et al, *Is Image Memorability Prediction Solved?* CVPRW, 2019; Nicole C. Rust, et al, *Understanding Image Memorability*, *Trends in Cognitive Sciences*, 24, 2020, 7).

- **Articulations conventionally avoided:** Questions could be raised as to the apparently lower mappability (and potential forgettability) of 7, 9, 11, 15, 16, 21, 22, 28, 40, 52 -- or rather the relative absence of strategic articulations with such a pattern, given the limited number of polyhedra associated with such numbers. There is also the unexplained avoidance of patterns based on numbers considered inauspicious.

- **"Unmappable" articulations**: Some strategic articulations of relative low number do not readily lend themselves to mapping either by the Platonic or Archimedean forms -- most obviously: 16, 48, 15, 28, 9, 21, 22. This frames the question as to the degree to which the memorability of key strategic articulation is associated with exotic polyhedra, rather than those of obvious symmetry.

- **Other memorability factors**: There is of course the possibility that memorability may be associated with the intuitive appeals of other characteristics of polyhedra which have not been highlighted by this study. These include great circles, cells and internal connectivity (golden rectangles, etc).

- **Mapping constraints in using polyhedra**: There is the possibility that memorability may not be strongly associated with the total number of features of a polyhedron (faces, edges, vertexes) rather than with the number of such features separately. This could be a consequence of "cluttering" the map through any attempt to use all three such features in one mapping, rather than one or two

- **Prime numbers**: The incidence of the prime factors and their combination invites further reflection on how they might enable memorability through the patterns of symmetry with which they are associated. An obvious example is the case of 12 ($2^2$x3) and its widespread importance in articulation of strategic initiatives (as noted above). On the one hand they highlight the intuitive appeal of larger numbers of strategic articulations, as with 60 and 72 ($2^3$x$3^2$). On the other hand they raise questions regarding the apparently limited role of 25 ($5^2$) and 100 ($2^2$x$5^2$).

- "**Intuitive appeal**"? Whilst some polyhedra with more associated polyhedra cannot be memorably mapped by simpler polyhedra, it would appear that some articulations of strategic significance are associated with numbers which have some kind of intuitive appeal -- as indicated by those at the head of the list: 60, 48, 72, 32, 90, 36, etc. This appeal may be the focus of elusive symbolic articulations, most obviously 72, as separately discussed (*Variety of System Failures Engendered by Negligent Distinctions: mnemonic clues to 72 modes of viable system failure from a demonic pattern language*, 2016; *Hyperbolic reframing of the Demonique and Angelique of tradition*, 2016). This appeal may be associated with the "simplicity" of the pattern of factors: 72 (as $2^3$x $3^2$), 30 (as 2x3x5). Even more intriguing is the apparently "low mappability" of some complex patterns of fundamental significance to some cultures: 64 hexagrams of the Chinese *I Ching* ($2^6$); the 81 elements of the taoist *Tao Te Ching* ($3^4$), and the 108 of Buddhism ($2^2$x$3^3$). These are discussed below

- **Higher primes**: With respect to higher prime numbers, it is notable that polyhedra are associated with some (probably prisms or pyramids), as in the case of::
  - 17: (1x17), (2x17)
  - 19: (1x19), (2x19)
  - 23: (1x23), (2x23), (3x23)
  - 37: (1x37), (2x37)
  - 41: (2x41)

- **"Islands of strategic stability"**: Nuclear physics now refers to " islands of stability" as being isotopes of superheavy elements that may have considerably longer half-lives than known isotopes of these elements. Their theoretical existence is attributed to stabilizing effects of predicted "magic numbers" of protons and neutrons in the superheavy mass region. The role of prime numbers in this respect is central to explorations of the periodic organization of matter (Jan C.A. Boeyens and Demetrius C. Levendis, *Number Theory and the Periodicity of Matter*, 2008; D. H. Rouvray and R. Bruce King (Eds.), *The Mathematics of the Periodic Table*, 2005). There is also a high correlation between prime-number values of DNA codons and coded amino acids.

  With respect to the exploration here, it might therefore be asked whether such framing is relevant to any quest for "islands of strategic stability" in the organization of governance, as might be indicated by polyhedra. Given the instability of many modes of organization, typically framed in terms of "positive" and "negative", it might also be asked whether any such stability might relate to some analogue to the magic number relation between protons and neutrons. Any such argument might then be especially related to issues of comprehensibility and memorability -- and to "forgettability" (*Towards a Periodic Table of Ways of Knowing -- in the light of metaphors of mathematics*, 2009).

  With respect to governance, such issues relate to the coherence and stability of parliamentary asemblies, given the communication issues between representatives in larger assemblies, as discussed separately notably in the light of potential power law implications (*Dependence of viable global governance on pattern management?* 2020). As noted in the latter, the number of representatives in such assemblies may well exceed 100, with those in the European Parliament numbering up to 750. Clearly there is a challenging compromise between too few representatives (with its undemocratic implications) and too many -- with all that that may imply for effective communication among them. It might even be asked whether some assemblies are designed to be larger in size in order to appear representative, even though they are thereby rendered ineffective. The challenge is obviously of relevance to the design of any World Parliament, as has been variously envisaged. It is in this sense that "pattern management" and its memorability merit attention.

## Polyhedra of secondary value for memorable mapping?

The exercise resulting in the above table extended to polyhedra with 100 mappable elements (whether faces, edges or vertexes). The table above included the first 62 possibilities as an arbitrary cut-off point based on the possible use of the Platonic polyhedra (as indicated by the pattern of colour). The remaining possibilities are presented below for information. They are indicative of configurations which might constitute a challenge to mapping and therefore of less relevant to this quest. Other approaches are considered in the following sections which might however include some of them.

**Table 3: Polyhedra of secondary value for memorable strategic mapping?**
(mauve=Platonic mappability; yellow=non-Platonic mappabiliuty; G=dual of cuboctahedron)

| A | B | C | D | | | E | | | | | F (Platonic) | | | | | G | H (Archimedean: semiregular) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Total poly. (desc. order) | Map. poly. elem. | Total polyhedra with indicated number of elements | | | Prime number factors | | | | | Tetra 14 | Cube 26 | Octa 26 | Dod. 62 | Ico. 62 | Rho. dod. 50 | Cubo. 50 | Tun. tetr. 38 | Trun. cube 74 | Trun. oct. 74 | Rho. cub. 98 | Trun. cubo. 146 | Snub. cubo. 122 | Icosi. dod. 122 | Trun. dod. 182 | Trun. icos. 182 | Rho. icod. 242 | Trun. icosi. 362 | Snub dod. 302 |
| | | | Faces | Edges | Vert. | 2 | 3 | 5 | 7 | N | 4v | 8v | 6v | 20v | 12v | 14v | 12v | 12v | 24v | 24v | 24v | 48v | 24v | 30v | 60v | 60v | 60v | 120v | 60v |
| | | | | | | | | | | | 6e | 12e | 12e | 30e | 30e | 24e | 24e | 18e | 36e | 36e | 48e | 72e | 60e | 60e | 90e | 90e | 120e | 180e | 150e |
| | | | | | | | | | | | 4f | 6f | 8f | 12f | 20f | 12f | 14f | 8f | 14f | 14f | 26f | 26f | 38f | 32f | 32f | 32f | 62f | 62f | 92f |
| 63 | 2 | 41 | 1 | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 64 | 2 | 58 | 1 | | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | | |
| 65 | 2 | 63 | | 2 | | | $3^2$ | | 7 | | | | | | | | | | | | | | | | | | | | |
| 66 | 2 | 66 | | 1 | 1 | 2 | 3 | | | 11 | | | | | | | | | | | | | | | | | | | |
| 67 | 2 | 74 | | | 2 | 2 | | | | 17 | | | | | | | | | X | X | | | | | | | | | |
| 68 | 2 | 76 | 2 | | | $2^2$ | | | 7 | 19 | | | | | | | | | | | | | | | | | | | |
| 69 | 2 | 78 | | 1 | 1 | 2 | 3 | | | 13 | | | | | | | | | | | | | | | | | | | |
| 70 | 2 | 85 | 1 | | 1 | | | 5 | | 17 | | | | | | | | | | | | | | | | | | | |
| 71 | 1 | 43 | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 72 | 1 | 46 | 1 | | | 2 | | | | 23 | | | | | | | | | | | | | | | | | | | |
| 73 | 1 | 47 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 74 | 1 | 49 | 1 | | | | | | $7^2$ | | | | | | | | | | | | | | | | | | | | |
| 75 | 1 | 51 | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 76 | 1 | 59 | | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 77 | 1 | 73 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 78 | 1 | 89 | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 79 | 1 | 91 | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 80 | 1 | 97 | | | 1 | | | | | | | | | | | | | | | | | | | | | | | | |
| 81 | 0 | 29 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 82 | 0 | 53 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 83 | 0 | 57 | | | | | 3 | | | 19 | | | | | | | | | | | | | | | | | | | |
| 84 | 0 | 61 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 85 | 0 | 67 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 86 | 0 | 69 | | | | | 3 | | | 23 | | | | | | | | | | | | | | | | | | | |
| 87 | 0 | 71 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 88 | 0 | 77 | | | | | | | 7 | 11 | | | | | | | | | | | | | | | | | | | |
| 89 | 0 | 79 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 90 | 0 | 81 | | | | | $3^4$ | | | | | | | | | | | | | | | | | | | | | | |
| 91 | 0 | 82 | | | | 2 | | | | 41 | | | | | | | | | | | | | | | | | | | |
| 92 | 0 | 83 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 93 | 0 | 87 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 94 | 0 | 93 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 95 | 0 | 94 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 96 | 0 | 95 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 97 | 0 | 99 | | | | | $3^2$ | | | 11 | | | | | | | | | | | | | | | | | | | |

## Constructible polyhedra in the light of constructible polygons?

If the challenge is one of presenting coherently the elements on a map of some kind -- some form of mind map -- the geometrical constraints in the case of a polygon are one point of departure. As noted above, there is a well-recognized understanding of what constitutes a constructible polygon -- notably because of the constraints on pattern formation by prime numbers.

The table below is of particular interest in that it covers the range of numbers up to 1,000 -- namely the range which typically includes the number of representatives in a legislative assembly. For example, seated in a hemicircle, the European Parliament numbers 705 representatives, the total being restricted to 751 by treaty, according to a system of apportionment.

Table 4: Number of sides of known constructible polygons having up to 1000 sides (bold) or odd side count (red)

| Fermat primes factors | | | | | | Multiples of powers of 2 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_0$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | | $\times 2^0$ | $\times 2^1$ | $\times 2^2$ | $\times 2^3$ | $\times 2^4$ | $\times 2^5$ | $\times 2^6$ | $\times 2^7$ | $\times 2^8$ | $\times 2^9$ |
| | | | | | | **(1)** | **(2)** | **4** | **8** | **16** | **32** | **64** | **128** | **256** | **512** |
| 3 | | | | | = | **3** | **6** | **12** | **24** | **48** | **96** | **192** | **384** | **768** | ... |
| | 5 | | | | = | **5** | **10** | **20** | **40** | **80** | **160** | **320** | **640** | ... | ... |
| 3 | ×5 | | | | = | **15** | **30** | **60** | **120** | **240** | **480** | **960** | ... | ... | ... |
| | | 17 | | | = | **17** | **34** | **68** | **136** | **272** | **544** | ... | ... | ... | ... |
| 3 | | ×17 | | | = | **51** | **102** | **204** | **408** | **816** | ... | ... | ... | ... | ... |
| | 5 | ×17 | | | = | **85** | **170** | **340** | **680** | ... | ... | ... | ... | ... | ... |
| 3 | ×5 | ×17 | | | = | **255** | **510** | ... | ... | ... | ... | ... | ... | ... | ... |
| | | | 257 | | = | **257** | **514** | ... | ... | ... | ... | ... | ... | ... | ... |
| 3 | | | ×257 | | = | **771** | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| | 5 | | ×257 | | = | 1 285 | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 3 | ×5 | | ×257 | | = | 3 855 | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| | | 17 | ×257 | | = | 4 369 | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 3 | | ×17 | ×257 | | = | 13 107 | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| | 5 | ×17 | ×257 | | = | 21 845 | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 3 | ×5 | ×17 | ×257 | | = | 65 535 | ... | ... | ... | ... | ... | ... | ... | ... | ... |

Extracted from table in *Wikipedia* by Cmglee / CC BY-SA

An earlier exercise highlighted the challenge to governance of numbers of elements beyond 100, most evidently the tendency for numbers of parliamentary representatives to be several hundred (*Dependence of viable global governance on pattern management?* 2020). The cases of the European Parliament and any potential World Parliament Assembly were considered. With respect to memorable mappability that exercise noted the mathematical literature on constructible polygons in 2D, usefully summarized by that table.

Of potential interest is whether the numbers in that table are especially indicative of "constructible polyhedra", however that might be understood -- **irrespective of more sophisticated mathematical approaches to the refinement of that question and detection of possible candidates**. To that end a first process was simply to copy into the following table the corresponding elements from the more promising candidates in the range up to 100 (where they matched the numbers in Table 1 above). For the higher numbers, the procedure was then to extend the earlier process with the numbers in the range up to 1000.

Note that in the method for the following table no account is taken of polyhedra generated with prime numbers other than Fermat primes.

**Table 5: Constructible polyhedra in the light of constructible polygons**
(mauve=Platonic mappabilty; yellow=non-Platonic mappabiliuty; G=dual of cuboctahedron)

| A | B | C | D | | | E | | | | | F (Platonic) | | | | | G | H (Archimedean: semiregular) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Total poly. (desc. order) | Map. poly. elem. | Total polyhedra with indicated number of elements | | | Prime number factors | | | | | Tetra 14 | Cube 26 | Octa 26 | Dod. 62 | Ico. 62 | Rho. dod. 50 | Cubo. 50 | Tun. tetr. 38 | Trun. cube 74 | Trun. oct. 74 | Rho. cub. 98 | Trun. cubo. 146 | Snub. cubo. 122 | Icosi. dod. 122 | Trun. dod. 182 | Trun. icos. 182 | Rho. icod. 242 | Trun. icosi. 362 | Snub dod. 302 |
| | | | Faces | Edges | Vert. | 2 | 3 | 5 | 7 | N | 4v | 8v | 6v | 20v | 12v | 14v | 12v | 12v | 24v | 24v | 24v | 48v | 24v | 30v | 60v | 60v | 60v | 120v | 60v |
| | | | | | | | | | | | 6e | 12e | 12e | 30e | 30e | 24e | 24e | 18e | 36e | 36e | 48e | 72e | 60e | 60e | 90e | 90e | 120e | 180e | 150e |
| | | | | | | | | | | | 4f | 6f | 8f | 12f | 20f | 12f | 14f | 8f | 14f | 14f | 26f | 26f | 38f | 32f | 32f | 32f | 62f | 62f | 92f |
| 59 | 2 | 4 | | 1 | 1 | $2^2$ | | | | | X | | | | | | | | | | | | | | | | | | |
| 17 | 32 | 8 | 13 | 1 | 18 | $2^3$ | | | | | X | X | X | | | | | X | | | | | | | | | | | |
| 13 | 45 | 16 | 20 | 5 | 20 | $2^4$ | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 70 | 32 | 33 | 7 | 30 | $2^5$ | | | | | | | | | | | | | | | | | | X | X | X | | | |
| 39 | 12 | 64 | 4 | 3 | 5 | $2^6$ | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 128 | 3 | 4 | 11 | $2^7$ | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 256 | 1 | 0 | 0 | $2^8$ | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 512 | 2 | 8 | 0 | $2^9$ | | | | | | | | | | | | | | | | | | | | | | | | |
| 36 | 14 | 6 | 8 | 1 | 5 | $2^2$ | 3 | | | | | X | X | | | | | | | | | | | | | | | | | |
| 4 | 80 | 12 | 35 | 8 | 37 | $2^2$ | 3 | | | | | X | X | X | X | X | X | X | | | | | | | | | | | | |
| 3 | 105 | 24 | 42 | 23 | 40 | $2^3$ | 3 | | | | | | | | | | X | X | | | | | | X | | | | | | |
| 2 | 108 | 48 | 36 | 28 | 44 | $2^4$ | 3 | | | | | | | | | | | | | | | X | X | X | | | | | | |
| 20 | 27 | 96 | 4 | 14 | 9 | $2^5$ | 3 | | | | | | | | | | | | | | | | | | | | | | | |
| | | 192 | 1 | 3 | 4 | $2^6$ | 3 | | | | | | | | | | | | | | | | | | | | | | | |
| | | 384 | 2 | 2 | 2 | $2^7$ | 3 | | | | | | | | | | | | | | | | | | | | | | | |
| | | 768 | 2 | 4 | 0 | $2^8$ | 3 | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | 42 | 10 | 18 | 3 | 21 | $2^2$ | | 5 | | | X | | | | | | | | | | | | | | | | | | | |
| 9 | 65 | 20 | 20 | 19 | 26 | $2^2$ | | 5 | | | | X | X | X | X | | | | | | | | | | | | | | | |
| 16 | 36 | 40 | 8 | 10 | 18 | $2^3$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |
| 22 | 25 | 80 | 11 | 5 | 9 | $2^4$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | | 160 | 5 | 3 | 5 | $2^5$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | | 320 | 3 | 0 | 0 | $2^6$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | | 640 | 2 | 0 | 1 | $2^7$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 79 | 30 | 24 | 23 | 32 | 2 | 3 | 5 | | | | | | X | X | | | | X | X | X | | | X | | | | | | |
| 1 | 187 | 60 | 65 | 41 | 81 | $2^2$ | 3 | 5 | | | | | | | | | | | | | | | X | X | X | X | X | | X |
| | | 120 | 30 | 71 | 67 | $2^3$ | 3 | 5 | | | | | | | | | | | | | | | | | | | X | X | | |
| | | 240 | 3 | 24 | 8 | $2^4$ | 3 | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | | 480 | 1 | 2 | 1 | $2^5$ | 3 | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | | 960 | 4 | 0 | 2 | $2^7$ | 3 | 5 | | | | | | | | | | | | | | | | | | | | | | |
| 52 | 5 | 34 | 2 | 1 | 2 | 2 | | | | 17 | | | | | | | | | | | | | | | | | | | | |
| 50 | 6 | 68 | 3 | 0 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 136 | 2 | 2 | 0 | $2^3$ | | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 272 | 0 | 0 | 0 | $2^4$ | | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 544 | 1 | 0 | 0 | $2^5$ | | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 102 | 0 | 1 | 1 | 2 | | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 204 | 4 | 0 | 1 | $2^2$ | 3 | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 408 | 1 | 0 | 0 | $2^3$ | 3 | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 916 | 0 | 0 | 0 | $2^4$ | 3 | | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 170 | 0 | 0 | 0 | 2 | | 5 | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 340 | 1 | 0 | 0 | $2^2$ | | 5 | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 680 | 0 | 0 | 0 | $2^3$ | | 5 | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 510 | 0 | 1 | 0 | 2 | 3 | 5 | | 17 | | | | | | | | | | | | | | | | | | | | |
| | | 514 | 0 | 0 | 0 | 2 | | | | 257 | | | | | | | | | | | | | | | | | | | | |
| | | 960 | 4 | 0 | 0 | $2^7$ | | 5 | | | | | | | | | | | | | | | | | | | | | | |

## Memorability of 17 Sustainable Development Goals with 169 tasks

As noted above, currently a major focus of global governance is framed by the 17 Sustainable Development Goals of the UN. There is however, seemingly  at least, a total lack of explanation as to why that number is appropriate -- and seemingly no inquiry into the challenges of the memorability and communicability of that array. Is the set anything more than an essentially arbitrary "to do" list?

Given the importance now attached to the pattern of SDGs in a wide variety of domains, there is a case for taking the 17-fold pattern seriously as a challenge to systemic comprehension and memorability. In this period of chaos, it could even be said that that 17-fold pattern is one of the very few manifestations of coherence engendered in a spirit of global governance.

**Cultural implications?** Given the ever increasing importance of China in global governance, there is a strangely undiscussed possibility that its predecessor, the pattern of 8 Millennium Goals Development, attracted particular support in Asia as a consequence of the considerable popular enthusiasm for 8-foldness in Chinese culture -- an enthusiasm which notably extends into financial investment and architecture. This is accompanied by systematic avoidance of certain other numbers and their combinations, especially evident in addresses, room numbers and skyscaper floor levels,

It might then be asked how the transition to a 17-fold pattern has been promoted and received in that culture. Whereas a 16-fold pattern could be deemed to "work" (as 2x8), the 17 SDGs would be problematic as 16+1. However 17 might be considered to be suitably auspicious as 1+7. Of particular relevance to any such consideration is the Seventeen Point Agreement (1951) affirming Chinese sovereignty over Tibet.

Although such issues may be considered trivial and irrelevant in Western cultures, there is the further peculiarity of the SDGs in their coordination of a peculiar configuration of 169 "tasks". For those in the West who avoid level 13 in skyscrapers, room 13 in hotels, or a meeting at a table of 13, the number 169 is especially questionably as 13x13. As with 13 itself, 169 is especially unmappable as indicated below -- in striking contrast to a degree of enthusiasm associated with 144, namely 12x12. Ironically, the number 4 is especially avoided in Chinese culture.

**Questionable significance**: There is the strange possibility, if only for the Western world, that **the 17-fold pattern of SDGs is eminently "forgettable"**. Or is it inherently interesting because of numeric properties of which most people may only be intuitively aware, if at all, or because of how it features in more exotic polyhedra?. The numeric properties include:

- Since 17 is a Fermat prime, regular heptadecagons can be constructed with a compass and unmarked ruler -- as might be required

in the design of a table at which representatives charged with implementing each SDG could be seated.

- Given the far higher order of memorability of 72 (as noted above), it is curious that the factors of 72 total 17, namely by addition of 8+9 (2+2+2+3+3). This is potentially related to the previous point and to the mathematical curiosity of an emirp (namely a prime number spelled backwards) This is a prime number, like 17, that results in a different prime when its decimal digits are reversed -- in this case 71. The following emirp in the sequence is 73, thereby bracketing 72. Emirps are discussed separately with respect to the case of 37 (*Requirement for emirps and enantiodromia in navigating songlines*, 2015).

- There are exactly 17 two-dimensional space (plane symmetry) groups. These are sometimes called wallpaper groups, as they represent the seventeen possible symmetry types that can be used for wallpaper. This little known property is suggestive of a set of fundamental cognitive patterns implied by the distinctive nature of each SDG -- offering the possibility of an unusually distinctive image by which it could be identified (reminiscent of the role of Scottish tartans). Unfortunately the term "wallpaper" also invites more cynical interpretation as it might relate to SDGs -- in "papering over the cracks" of global governance.

- Relating to the previous point, but potentially of even more fundamental significance in terms of both the string theory of physics and music (as mentioned below) is the role of 17 with respect to the mathematical understanding of orbifolds. In the mathematical disciplines of topology, geometry, and geometric group theory, an orbifold (for "orbit-manifold") is a generalization of a manifold. It is a topological space (called the underlying space) with the structure so named. The compact 2-dimensional connected orbifolds that are not hyperbolic include the 17 parabolic orbifolds which are the quotients of the plane by the 17 wallpaper groups. Reference is also made to an orbihedron..

- The Pythagoreans were alleged to "utterly abominate" 17, which "bars them off from each other and disjoins them". This merits consideration as indicative of the apparent asystemic nature of the set of SDGs.

- Seventeen is the minimum possible number of givens for a sudoku puzzle with a unique solution. Framed in this way, given the conceptual challenge of sudoku, global governance merits recognition as a puzzle -- as continues to be the case in reference to the legendary Gordian Knot. Remarkably, an unusual analysis by CitiBank of the interrelation between SDGs has been presented in matrix form under the heading "Sustainability Sudoku" (*United Nations Sustainable Development Goals: pathways to success -- a systematic framework for aligning investment*, 2018, pp. 20-22).

- The challenging Japanese poetic form of *haiku* is traditionally composed of what have been translated as 17 syllables. Curiously the articulation of the 17 SDGs was celebrated in a *haiku* competition among some of those involved (*Sustainable Development Goals Haiku winner and runners-up announced*! 22 July 2014). The Twitter hashtag of **#SDGhaiku** has since emerged to frame a form of advocacy. In that spirit UNDP has enabled a publication of 82 haiku (*Inspired by Nature: Celebrating Biodiversity with Haikus*, UNDP, 22 May 2017)

To what extent are these to be recognized as trivia, given the serious consideration merited by the pattern of 17 SDGs? Do such curiosities suggest a fundamental recognition of patterning which remains to be explored? Far from trivia, for example, are the facts that:

- the classification of occupations and social status in the USA is inherited from early work on social stratification and mobility using 17 aggregated categories of a larger set of occupations (Peter Blau and Otis Duncan, *The American Occupational Structure*. 1967), strongly critiqued as being without logical justification by Steven Rytina (*Network Persistence and the Axis of Hierarchy: how orderly stratification is implicit in sticky struggles*, 2020, pp. 44-45).
- the traditional rankings of colleges and universities in the USA have been based on a wide range of criteria, most of it with little documented relationship to student engagement, as with the 17-point scale of the report on *Best Colleges* (*U. S. News and World Report*, 2006).
- the Hamilton Rating Scale for Depression is based on 17-points, although variations have been introduced
- the US Intelligence Community is composed of 17 organizations: 2 independent, 8 elements of the Department of Defense, and 7 elements of other departments and agencies. Whether that pattern derives from organizational insights which were a factor in the organization of the SDGs is far from clear, although a case can be made for understanding it in more subtle terms, given the necessity for its coherence (*Envisaging NATO Otherwise -- in 3D and 4D? Potentially hidden faces of global strategy highlighted through polyhedra*, 2017).

What insight is potentially associated with Ian Dunmur's performances of the *17-step Lakeland Clog Routine* at traditional step dance festivals (*Lakeland steps Ian Dunmur 2002 Performance*; *Norman Robinson: 17 Step Routine*, *Instep*, 11 January 1984)? **Given the long-established attraction of both *sudoku* and *haiku* for many (if not all), there is a case for exploring whether and how such attraction might be "translated", so as to render the 17 SDGs attractive globally**.

**Memorable mapping possibilities**: The challenge of rendering a pattern of 16(+1) strategic goals coherent, memorable and communicable can be variously explored:

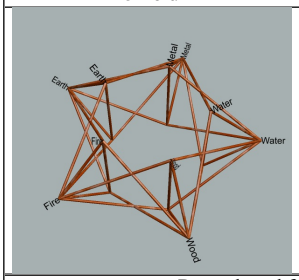
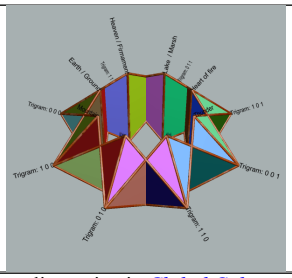
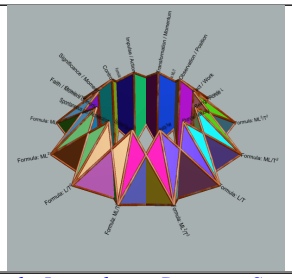
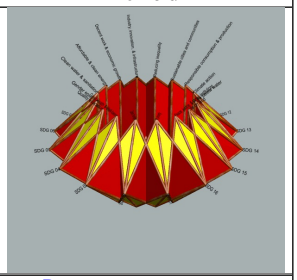
- Using Rubik's cube: *Interplay of Sustainable Development Goals through Rubik Cube Variations: engaging otherwise with what people find meaningful* (2017). That argument explored the integrative role of the 3x3 magic square with respect to the 8-fold pattern of Millennium Development Goals of the UN (*Elusive requisite "magic" for sustainability?*). The approach was then applied to the SDGs by exploring a 4x4 magic square (*Magic square of sustainability: from 3x3 to 4x4?*). There is of course the possibility of applying this approach to a mapping of the array of 16 (+1) bodies forming the US Intelligence Community. Especially in the case of the USA, this would be consistent with the importance of magic squares to coherent organization. This was a major preoccupation of one of its Founding Fathers, namely Benjamin Franklin. His particular interest was in 16x16 magic

squares (Harvey Heinz, *Franklin Squares*, 2013). There is clearly a case for exploring whether such understanding influenced the pattern of SDGs, consciously or unconsciously.

- Using a star torus: An earlier exploration focused on the pattern of 16 articulated by the star torus, since the 17th is specifically intended as coordinating the others, then to be understood as somehow centrally located as a point of focus (*Current relevance of the "simplest torus"?* 2019; *Modifying the cyclic symmetry of the star torus*, 2019).

As a mnemonic mapping aid, the star torus enables a variety of relevant considerations of a 16-fold pattern, as indicated by other sections of that argument (*Framing an operating context of 16 "dimensions"*, *Functional dynamics of a 16-fold configuration of strategic goals*, 2019). Of particular interest, is the manner in which the cyclicity of such a torus can be varied to interrelate otherwise disparate mapping configurations -- including the 8-fold and the 12-fold.

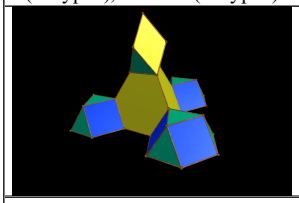
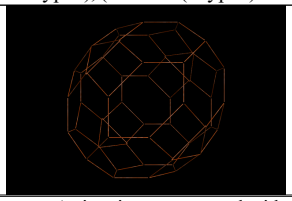
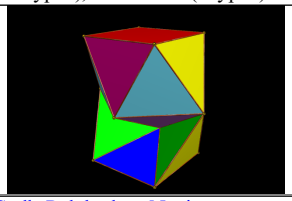
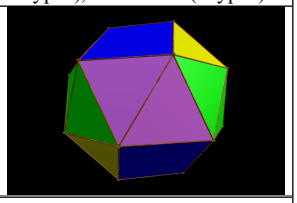
**Illustrative use of geometry of star torus for mapping purposes**
(use browser facilities to enlarge animations and labelling)

| 5-fold | 8-fold | 12-fold | 16-fold |
|---|---|---|---|
|  |  |  |  |

Reproduced from discussion in *Global Coherence by Interrelating Disparate Strategic Patterns*
*Dynamically Topological interweaving of 4-fold, 8-fold, 12-fold, 16-fold and 20-fold in 3D* (2019)
Animations generated with Stella Polyhedron Navigat

Rather than focusing on 16 as a means of circumventing the difficulty of mapping 17, an alternative approach that can be explored is through mapping 34, namely 2x17. In such cases, 17 can then be associated with the axes linking 34 vertexes, or the parallelism of 34 edges or 34 sides. Examples of unusual possibilities -- and therefore potentially memorable -- are indicated below. In that respect it is intriguing to note that the 4x4 magic square mentioned above has a so-called magic constant of 34, namely the sum of its rows, columns or diagonals.

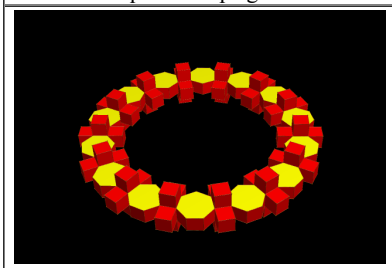
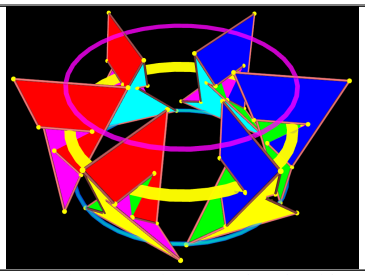
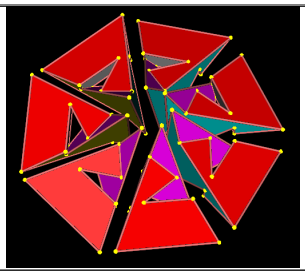
**Contrasting possibilities of mapping 17 SDGs onto polyhedra** (as 2x34 features)

| "Moon base" polyhedron | 4-frequency tetrahedral geodesic sphere (dual) | Stewart toroid Z4 | Gyroelongated square bicupola |
|---|---|---|---|
| 34 vertexes (12 types), 63 edges (21 types), 31 faces (11 types) | 34 faces (4 types), 96 edges (8 types), (64 faces (6 types) | 22 faces (11 types), 34 edges (17 types), 14 vertexes (8 types) | 34 faces (5 types), 56 edges (8 types), 24 vertexes (3 types) |
|  |  |  |  |

Animations generated with Stella Polyhedron Navigator

**Mapping the unimaginable**: The associated challenge of providing a degree of coherence to the "unmappable" pattern of 169 SDG "tasks" could be explored by a similar process, namely a focus on 378 (2x169), as indicated in the reasonably memorable pattern on the left below.

**Polyhedra suggestive of mappings of potential relevance**

| 169 SDG tasks onto 378 features | Configuration of Szilassi polyhedra | |
|---|---|---|
| Squares+Heptagons | 6 | 7 |
|  |  |  |
| 378 faces (15 types), 798 edges (29 types) 420 faces (15 types) | Interactive 3D; variants (vrml; x3d); videos (solid mp4; wireframe mp4). | Interactive vrml; Video mp4 |

In this light, it might be usefully asked how 7-fold articulations can be usefully mapped, given the significance attached to that number as being an appropriate ("comfortable") size for an expert meeting, or a strategic articulation, especially in the light of the classic study by George Miller (*The Magical Number Seven, Plus or Minus Two: some limits on our capacity for processing information*, *Psychological*

*Review*, 63, 1956, 2).

One notable candidate in that case is the highly unusual Szilassi polyhedron of 7 sides -- all touching one another, exemplifying an ideal in any dialogue process. This possibility is discussed separately (*Mapping of WH-questions with question-pairs onto the Szilassi polyhedron*, 2014; *Dynamics of discord anticipating the dynamics of concord*, 2018).

## Relevance of polyhedral mapping of patterns of complexity with symbolic appeal

As noted above, of particular interest is the possibility of mapping complex patterns with long-standing appeal, whether intuitively appreciated or embodied in traditional symbolism. Part of the interest lies in whether or not the larger numbers involved are associated with strategic preoccupations, or have been usefully explained.

Their "extraordinary" nature is consistent with the relatively unique polyhedra (even exotic) which may render them especially memorable for that reason. Examples include the otherwise incomprehensible, if not "meaningless", patterns of:

- **64-foldness** ($2^6$): The number 64 is of considerable signifiance in a variety of domains, most obviously in computer technology. As shown below, the pattern can be usefully mapped by the drilled truncated cube (*Proof of concept: use of drilled truncated cube as a mapping framework for 64 elements*, 2015). This is a feature of:
  - the 8x8 board pattern of the (Western) strategic game of chess
  - the 64-hexagrams of the Chinese *I Ching*
  - the 64 codons of the genetic code

- **72-foldness** ($2^3x3^2$): The number 72 is of particular significance in a range of religions. It is notably characteristic of traditional (Western) articulations of angels and demons, explored separately as a potential pattern of protein spikes of the COVID-19 virus (*Cognitive Engagement with Spike Dynamics of a Polyhedral Coronavirus*, 2020). The Rule of 72 is a feature of analysis of exponential growth and is used for calculations of compound interest as opposed to simple interest. The relationship of that pattern to 144, of similar significance in some domains (as discussed below), merits consideration -- ironically so, given the challenge to governance of reconciling any articulation of whatever are to be recognized as 72 "angels" faced with 72 "demons" (*Engaging with Hyperreality through Demonique and Angelique? Mnemonic clues to global governance from mathematical theology and hyperbolic tessellation*, 2016). As discussed there, the challenge is all the greater given the tendency by world leaders to label as "evil" those opposing them, thereby effectively framing themselves as "angelic".

- **81-foldness** ($3^4$): The number 81 is a feature of the Chinese *Tao Te Ching* (*9-fold Magic Square Pattern of Tao Te Ching Insights -- experimentally associated with the 81 insights of the T'ai Hsüan Ching*, 2006). It is the number of squares on a *shogi* playing board -- the Japanese form of chess as a strategy game. For a Christian, saying a complete rosary involves 13 prayers, five of which vary from time to time, and in total, these 13 prayers are repeated 81 times (with the Hail Mary repeated 53 times). Whilst the 81-fold pattern does not lend itself to a polyhedral mapping, it is of potential interest that this could be rendered memorable with the 162 vertices (namely $2x3^4$) of the 4-frequency icosahedral geodesic sphere, complemented by its dual with 162 faces. In the animations below, 81 one-word mnemonics from the above exercise are accordingly attributed randomly to faces or vertexes (duplicated as positive or negative). Those labelled "+" or "-" could of course be carefully repositioned to be on opposite sides of each figure.

- **108-foldness** ($2^2x3^3$): The number 108 is considered sacred by the Dharmic Religions, such as Hinduism, Buddhism, and Jainism. Malas or rosaries are typical made up of that number of prayer beads as a guide to meditation. The number is characteristic of sets of questions and statements by Buddha. For the East Asian martial arts tracing their roots back to Buddhism, 108 has become an important symbolic number in a number of martial arts styles, moves, techniques and pressure points. Curiously it is the number of double-stitches on the common baseball seam.

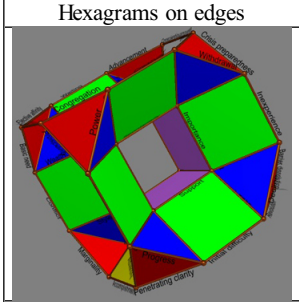
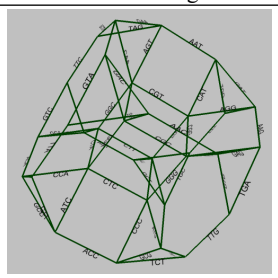
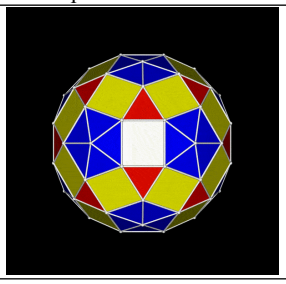
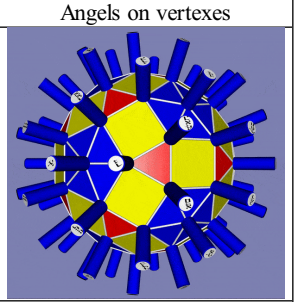
- **144-foldness** ($2^4x3^2$): The number 144 is of mathematical significance as a "highly totient number". The Chinese game of Mahjong commonly has 144 tiles -- potentially unique in distinguishing the 144 by visual means, with many variants presented in 3D configurations. As 2x72, this invites more speculative mapping exploration (*Challenge of 144 Distinctions -- Mahjong as "Angels" versus "Demons"*, 2020).

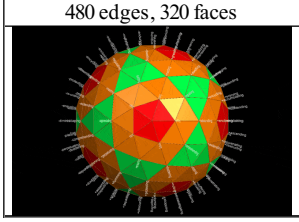
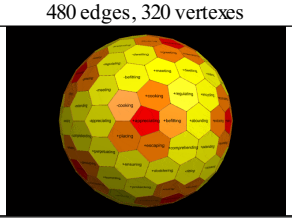
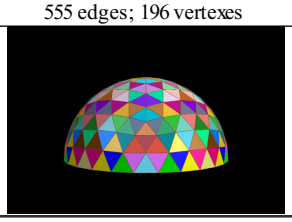
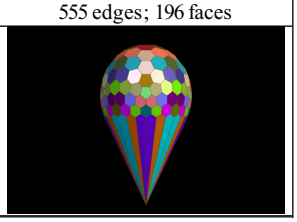
  As the square of the strategically and religiously significant number 12, the number 144 has acquired further significance (by extension) to the number 144,000 as a focus of religious prophecy. In contrast with the absence of polyhedral preoccupation with respect to the other numbers cited here, a specific relationship is inferred from a New Age perspective with respect to what is termed a double penta-dodecahedron defining an Earth grid pattern, as described by Joseph Jochmans: *Beyond the Platonic series of Solids is another form being geometrically generated out of the old Icosa-Dodeca crystal. If you take an Icosahedron and join together with lines every other point inside the form, you create twelve pentacles or five-pointed stars. If you extend the outer edges of the Icosahedron and join these node points together, you create a second group of twelve pentacles or stars. This becomes the seed crystal that gives birth to a new crystalline form called a double penta-dodecahedron, composed of twelve double-pentacles equally spaced across the surface of the globe* (*Earth: A Crystal Planet,* 1996). The polyhedral relevance of 144 is also highlighted by *The Mereon Matrix: everything connected through (k)nothing* (2018) as noted below.

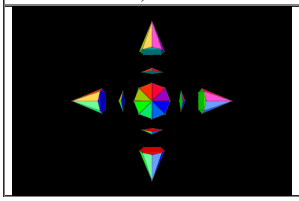
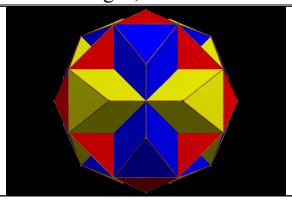
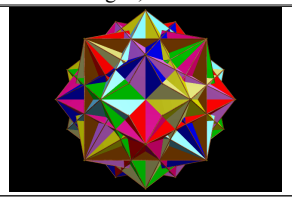
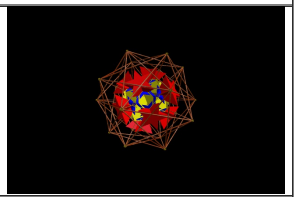
- **361-foldness** ($19^2$): The number 19 is of particular significance in Islamic religions. It is a feature of the 19x19 grid of the (Asian) abstract strategy board game of go -- now a focus of artificial intelligence programming through Alphago. A possible mapping of relevance to such strategy is the 6-frequency icosahedral geodesic hemisphere and its dual, as shown below in terms of 361.

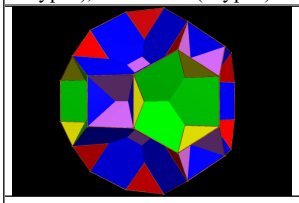
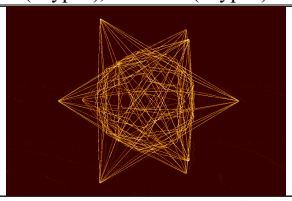
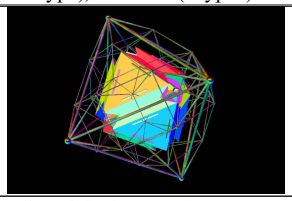
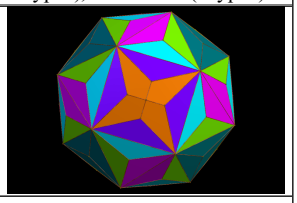
From a strategic perspective, it is appropriate to recall the argument that Mao acquired strategic advantage in the Vietnam war by playing go -- poorly matched in retrospect by efforts of the opposing Western coalition using a strategy framed by chess (Scott Boorman, *The Protracted Game: a wei-ch'i interpretation of Maoist revolutionary strategy*, 1969).

The more complex polyhedra of potential interest in memorable mapping of such configurations are indicated by the following. Given the numbers in each case, the polyhedra identified are relatively unique.

| Indication of possible 64-fold and 72-fold ma[ppings | | | |
|---|---|---|---|
| Drilled truncated cube (64 edges) | | Pentakis rhombicosidodecahedron (72 vertexes) | |
| Hexagrams on edges | Codons on edges | Spikes on vertexes | Angels on vertexes |
|  |  |  |  |

| **Tao Te Ching principles** | | **19x19 grid of the Game of go** | |
|---|---|---|---|
| 4-frequency icosahedral geodesic sphere<br>(162 mnemonic terms randomly mapped +/-) | | 6-frequency icosahedral geodesic hemisphere<br>(dual on right) | |
| 162 vertices<br>480 edges, 320 faces | dual: 162 faces<br>480 edges, 320 vertexes | 361 faces (73 types)<br>555 edges; 196 vertexes | 361 vertexes (73 types)<br>555 edges; 196 faces |
|  |  |  |  |

| **Alternative mappings of 108-fold principles of Buddhism, Hinduism, Jainism and martial arts** | | | |
|---|---|---|---|
| 12 part compound | Tuncated cube 3 | 8+1 cube compound | 921-Ditdiddip |
| 108-faced; 84 vertexes | 108-edges; 72 vertexes | 108-edged; 72 vertexes | 108-faced 4D rotation |
|  |  |  |  |

| **Alternative mappings of 144 elements, notably cultivated in relation to the 144,000 of the *Book of Revelations*** | | | |
|---|---|---|---|
| Faceted truncated cuboctahedron 2 | 12 Pentagonal antiprisms (dual) | 516-Offadac 4D rotation | Dual of Hendeca-faced Poly. |
| 72 faces (5 types), 144 edges (7 types), 48 vertexes (2 types) | 144 vertexes (4 types), 240 edges (4 types), 120 faces (2 types) | 144 vertexes (1 type), 576 edges (1 type), 960 faces (3 types) | 144 faces (8 types), 228 edges (10 types), 86 vertexes (6 types) |
|  |  |  |  |
| Animations generated with Stella Polyhedron Navigator | | | |

## Strategic viability of global governance enabled by mappings on exotic polyhedra

**Possibilities from 100 to 1000**: Given the complexity of the challenges of global governance -- obviously suggested by the 169 tasks of the SDG articulation -- there is clearly a case for exploring the mapping of numbers above 100. This is especially the case since that range tends to correspond to the number of representatives in a parliamentary assembly -- and any concern for the coherence of the pattern of their interrelationship and the communicability of that pattern.

Clearly some of these articulations are considered worthwhile and memorable for reasons which merit clarification in a wider context -- as indicated in the previous section. What other patterns might be significance to viable global governance in the fact of complexity? Which types of polyhedra can be usefully excluded from such consideration as essentially unmemorable or merely confusing in some manner? **Are there polyhedra which could catalyze a greater degree of unforeseen coherence?**

The categories of polyhedra in the library of *Stella Polyhedron Navigator* could frame this response. Again it should be stressed that many other polyhedra exist or may be generated by other methods.

Also noteworthy is the possibility that the requisite mappability required for the requisite strategic coherence might be better enabled in 4D -- of which the Stella application includes a selection of "3D aspects" and is able to generate many more. Known as polychora, such polytopes of 4D and more constitute a very extensive category whose strategic significance remains to be explored -- especially if they imply a time dimension beyond the static "timeless" implications of a 3D comnfiguration (*Comprehending the shapes of time through four-dimensional uniform polychora*, 2015).

The capacity of the application to generate many more polyhedra than are listed in the table below is a further limitation of the figures presented in the tables here. For example, in the case of cupolae, prisms and geodesic spheres, it is for the user to specify what should be generated -- beyond what is indicated in the library of models.

| **Table 6: Models directly accessible from Stella Polyhedron application** (excluding those which can be generated as user options; and with possibility of double counting) | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Uniform** | | | | **Stella library** | | **Miscellaneous** | |
| Regular | | Degenerates | 7 | Augmented uniforms | 114 | 4D library (3D Aspects) | 26 |
| -- Platonic | 5 | Johnson solids | 78 | Bruckner | 130 | Geomag library | 25 |
| -- Kepler-Poinsot | 4 | Near misses | 15 | Compounds | 128 | | |
| Semi-regular | | Stewart toroids | 51 | Facetings | 58 | | |
| -- Archimedean | 13 | Pyramids / Cupolae | 23 | Geodesic hemispheres | 12 | | |
| -- Tetrahedral symmetry | 2 | | | Geodesic spheres | 22 | | |
| -- Octahedral symmetry | 10 | | | Leonardo-style | 23 | | |
| -- Icosahedral symmetry | 58 | | | More Stewart toroids | 60 | | |
| -- Nonconvex snubs | 11 | | | Parts | 15 | | |
| -- Prisms / Antiprisms | 22 | | | Rectangular isohedra | 4 | | |
| | | | | Stellations | 78 | | |
| | | | | Sub-symmetric | 7 | | |
| | | | | Topological | 11 | | |
| **Totals** | **125** | | **174** | | **559** | | **51** |
| | | | | | | **Cumulative total** | 909 |

**Memorability of "knowledge architecture" of relevance to governance**: In considering the variety of polyhedra, the question of memorabilty could be associated with:

- **degree of symmetry**: This is clearly a preoccupation of mathematics in their quest for ever higher orders of symmetry groups. The challenge for memorabiltty is that excessive symmetry may be experienced as "**boring**" and essentially unmemorable (potentially as with a "pretty face")
- **nature of any asymmetry**: Curiously memorability may depend on forms of asymmetry which render the polyhedra especially "**interesting**" -- suggesting the need for their investigation in terms of what those might imply for governance.

It is also appropriate to note that patterns considered "interesting" may be anticipated in the arts and in the architecture of buildings -- especially given the competitive innovation between modern architects. Some of the patterns chosen may be echoed in polyhedral forms. Examples include:

- stellated polyhedra
- polyhedra with concave features
- variously "drilled" hollow structures

With respect to the "architecture" of buildings, it is appropriate to note the extensive use of this metaphor to refer to the architecture of knowledge and of computer memory (Michel Foucault, *The Archaeology of Knowledge*, 1969; Serdar Erisen, *The Architecture of Knowledge from the Knowledge of Architecture*, *Athens Journal of Architecture*, 2020).

It is however appropriate to note that the relatively recent focus on information design and knowledge graphs is predominantly focused on the adequacy of their 2D representation, despite explicit recognition of multidimensionality and the challenge of semantic integration (Yucong Duan, et al, *Specifying architecture of knowledge graph with data graph, information graph, knowledge graph and wisdom graph*, *2017 IEEE 15th International Conference on Software Engineering Research, Management and Applications*; David Meza, *How NASA Finds Critical Data through a Knowledge Graph, NASA*, 17 May 2017).

There is a degree of irony to the conventional choice of the metaphor implied by data mining, namely as the process of discovering patterns in large data sets. The irony derives from those forms of mining especially focused on the quest for precious metals and precious stones. The implication is that the patterns discovered by data mining are in some way especially precious -- in the extreme case to be considered comparable with diamonds. As with other precious stones, particular value is associated with the beauty whereby they focus light -- in turn offering a variety of insightful metaphors (*Patterning Archetypal Templates of Emergent Order: implications of diamond faceting for enlightening dialogue*, 2002; *Summary of Gemstone Faceting and Crystals*, 2002).

Use of "facet" has long been borrowed as a metaphor in the information sciences. A faceted classification system uses a set of semantically cohesive categories that are combined as needed to create an expression of a concept. In this way, the faceted classification is not limited to already defined concepts.With respect to geometry, faceting is the process of removing parts of a polygon, polyhedron or polytope, without creating any new vertices.

The question would then be how exotic polyhedra are suggestive of possibilities for "strategic architecture" initiatives and organizational architecture in the sense of organization design (Missoum Mohamed Rafik, *What is "Strategic architecture"?* Ecole Nationale supérieure de Management, 2017).

**Identifying extraordinary patterns of potential organizational relevance**: The procedure used for the following table was to isolate

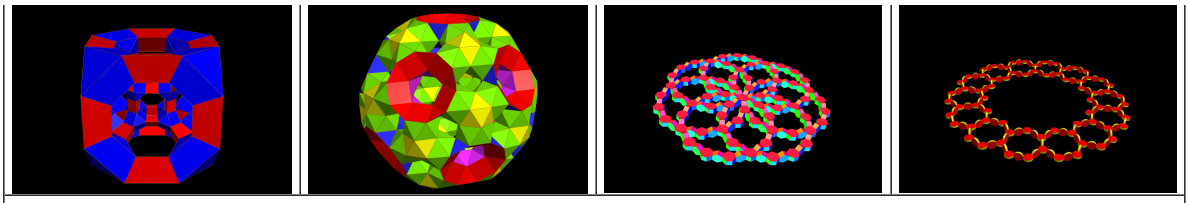
polyhedra which were especially singular (Column B), and therefore potentially "interesting" in memorable terms (despite their complexity) in the range from 100 to 1000 (excluding those below 100 already identified above as having symbolic value). Column Y is added given some suggestion that memorability may be related to the total number of prime factors. Column Z indicates those polyhedra selected as examples of those potentially interesting as images or animations -- as presented in this document. Included in the table (for information) are some items with potentially fruitful prime number factors -- but with no detected polyhedra.

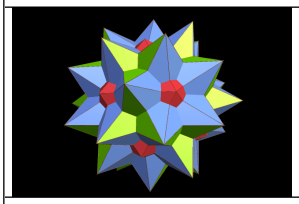
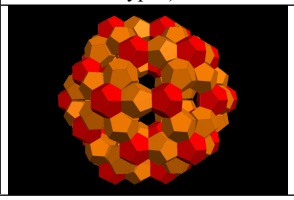
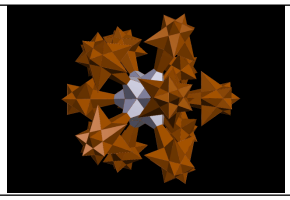
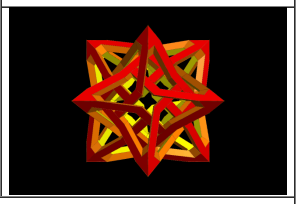
**Table 7: Exotic polyhedra of potentially vital significance for memorable strategic mapping?**

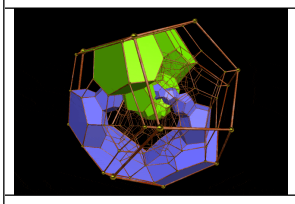
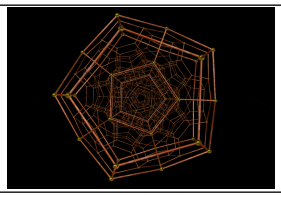
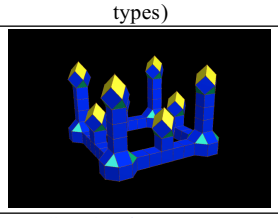
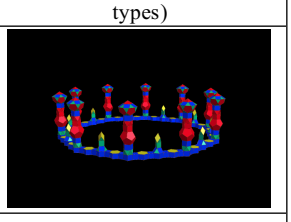
| A | B | C | D | | | E | | | | | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Total poly. | Map. poly. elem. | Total polyhedra with indicated number of elements | | | Prime number factors | | | | | Total factors | Selected for presentation as images or animations |
| | | | Faces | Edges | Vert. | 2 | 3 | 5 | 7 | N | | |
| | 2 | 102 | 0 | 1 | 1 | 2 | | | | 17 | 2 | |
| | 6 | 108 | 3 | 3 | 0 | $2^2$ | $3^3$ | | | | 5 | 12 part compound; Tuncated cube 3; 8+1 cube compound; 921-Ditdiddip |
| X | 8 | 120 | 30 | 71 | 67 | $2^3$ | 3 | 5 | | | 5 | |
| | 18 | 128 | 3 | 4 | 11 | $2^7$ | | | | | 7 | |
| | 4 | 136 | 2 | 2 | 0 | $2^3$ | | | | 17 | 3 | |
| | 34 | 144 | 3 | 9 | 22 | $2^4$ | $3^2$ | | | | 6 | Dual of Hendeca-faced polyhedron; 12 Pentagonal antiprisms (dual); Faceted truncated cuboctahedron 2 |
| X | 29 | 150 | 2 | 22 | 5 | 2 | 3 | $5^2$ | | | 4 | |
| | 13 | 160 | 5 | 3 | 5 | $2^5$ | | 5 | | | 6 | |
| | 1 | 162 | 0 | 0 | 1 | 2 | $3^4$ | | | | 5 | 4-frequency icosahedral geodesic sphere |
| | 0 | 170 | 0 | 0 | 0 | 2 | | 5 | | 17 | 3 | |
| X | 83 | 180 | 2 | 77 | 4 | $2^2$ | $3^2$ | 5 | | | 5 | |
| | 8 | 192 | 1 | 3 | 4 | $2^6$ | 3 | | | | 7 | 358-Proh Proj |
| | 1 | 196 | 0 | 0 | 1 | $2^2$ | | | $7^2$ | | 4 | 6-frequency icosahedral geodesic hemisphere |
| | 5 | 204 | 4 | 0 | 1 | $2^2$ | 3 | | | 17 | 4 | |
| | 5 | 216 | 0 | 4 | 1 | $2^3$ | $3^3$ | | | | 6 | 12-Gishi Slice_5 |
| | 3 | 228 | 1 | 2 | 0 | $2^2$ | 3 | | | 19 | 4 | Dual of Hendeca-faced Polyhedron |
| | 35 | 240 | 3 | 24 | 8 | $2^4$ | 3 | 5 | | | 6 | 12 Pentagonal antiprisms (dual) |
| | 1 | 256 | 1 | 0 | 0 | $2^8$ | | | | | 8 | 358-Proh Proj |
| | 0 | 272 | 0 | 0 | 0 | $2^4$ | | | | 17 | 5 | |
| | 7 | 280 | 1 | 1 | 5 | $2^3$ | | 5 | 7 | | 5 | Castle |
| | 1 | 296 | 1 | 0 | 0 | $2^3$ | | | | 37 | 4 | Castle |
| | 3 | 320 | 3 | 0 | 0 | $2^6$ | | 5 | | | 7 | 4-frequency icosahedral geodesic sphere |
| | 1 | 340 | 1 | 0 | 0 | $2^2$ | | 5 | | 17 | 4 | |
| | 1 | 350 | 1 | 0 | 0 | 2 | | $5^2$ | 7 | | 5 | 12-Gishi Slice_5 |
| | 1 | 361 | 1 | 0 | 0 | 2 | | | | $19^2$ | 3 | 6-frequency icosahedral geodesic hemisphere |
| | 1 | 378 | 1 | 0 | 0 | 2 | $3^3$ | | 7 | | 3 | Squares+Heptagons |
| | 6 | 384 | 2 | 2 | 2 | $2^7$ | 3 | | | | 8 | Topo Stellated Rhombic Dodecahedron |
| | 1 | 408 | 1 | 0 | 0 | $2^3$ | 3 | | | 17 | 5 | |
| | 1 | 452 | 0 | 0 | 1 | $2^2$ | | | | 113 | 3 | 14-Gofix Slice |
| | 4 | 480 | 1 | 2 | 1 | $2^5$ | 3 | 5 | | | 7 | 4-frequency icosahedral geodesic sphere; 358-Proh Proj; 12(J6-Q5S5) + 20J63 + 60S5 |
| | 1 | 510 | 0 | 1 | 0 | 2 | 3 | 5 | | 17 | 4 | |
| | 10 | 512 | 2 | 8 | 0 | $2^9$ | | | | | 9 | |
| | 0 | 514 | 0 | 0 | 0 | 2 | | | | 257 | 2 | |
| | 1 | 544 | 1 | 0 | 0 | $2^5$ | | | | 17 | 6 | |
| | 1 | 555 | 0 | 1 | 0 | | 3 | 5 | | 37 | 3 | 6-frequency icosahedral geodesic hemisphere |
| | 3 | 560 | 3 | 0 | 0 | $2^4$ | | 5 | 7 | | 6 | |
| | 1 | 575 | 0 | 1 | 0 | | | $5^2$ | | 23 | 3 | Castle |
| | 1 | 583 | 1 | 0 | 0 | | | | | 11.53 | 2 | |
| | 7 | 600 | 1 | 3 | 3 | $2^3$ | 3 | $5^2$ | | | 6 | Topo Stellated Rhombic Dodecahedron; 120 Cell-Linked rings-Spinning |
| | 3 | 640 | 2 | 0 | 1 | $2^7$ | | 5 | | | 8 | |
| | 0 | 680 | 0 | 0 | 0 | $2^3$ | | 5 | | 17 | 5 | |
| | 1 | 702 | 1 | 0 | 0 | 2 | $3^3$ | | | 13 | 5 | |
| | 9 | 720 | 4 | 4 | 1 | $2^4$ | $3^2$ | 5 | | | 7 | 120 Cell-Linked rings-Spinning |
| | 1 | 750 | 0 | 1 | 0 | 2 | 3 | $5^3$ | | | 5 | 12-Gishi Slice_5 |
| | 6 | 768 | 2 | 4 | 0 | $2^8$ | 3 | | | | 9 | |
| | 1 | 780 | 0 | 0 | 1 | $2^2$ | 3 | 5 | | 13 | 5 | 14-Gofix Slice |
| | 1 | 798 | 0 | 0 | 1 | 2 | 3 | 7 | | 19 | 4 | |
| | 2 | 864 | 2 | 0 | 0 | $2^5$ | $3^3$ | | | | 8 | Dodecahedra 92 (RTC) |
| | 0 | 916 | 0 | 0 | 0 | $2^4$ | 3 | | | 17 | 6 | |
| | 1 | 952 | 0 | 0 | 1 | $2^3$ | | | 7 | 17 | 5 | |
| | 4 | 960 | 4 | 0 | 0 | $2^7$ | | 5 | | | 8 | 516-Offadac |
| | 0 | 980 | 0 | 0 | 0 | $2^2$ | | 5 | $7^2$ | | 5 | |
| | 9 | 990 | 0 | 0 | 0 | 2 | $3^2$ | 5 | | 11 | 5 | |
| | 0 | 999 | 0 | 0 | 0 | | $3^3$ | | | 37 | 4 | |
| | 0 | 1000 | 0 | 0 | 0 | $2^3$ | | $5^3$ | | | 6 | |
| | 0 | 1001 | 0 | 0 | 0 | | | | 7 | 11.13 | 3 | |
| | 1 | 1003 | 1 | 0 | 0 | | | | | 17.59 | 2 | Heptagonal rings |
| | 0 | 1050 | 0 | 0 | 0 | 2 | 3 | $5^2$ | 7 | | 5 | |
| | 5 | 1056 | 4 | 1 | 0 | $2^5$ | 3 | | | 11 | 7 | Topo Stellated Rhombic Dodecah |
| | 1 | 1060 | 1 | 0 | 0 | $2^2$ | | 5 | | 53 | 4 | 12(J6-Q5S5) + 20J63 + 60S5 |
| | 0 | 1080 | 0 | 0 | 0 | $2^3$ | $3^3$ | 5 | | | 7 | |
| | 1 | 1270 | 1 | 0 | 0 | 2 | | 5 | | 127 | 3 | Fortress |

**Images suggestive of extraordinary forms of coherence in governance:** The following are examples of polyhedra which frame the question as to how global governance might be understood if organized according to such patterns. Of particular interest is the manner in which otherwise incomprehensible numbers of elements are rendered memorable to a degree by their incorporation into structures which are relatively memorable. Of some relevance is the possibility that competitive advantage may be derived in the future through organization in terms of unusual mappings (eching to a degree the history of use of global maps for navigation purposes).

| **Polyhedra of potential interest in mapping organization of global governance** | | | |
|---|---|---|---|
| 358-Proh Proj | 12(J6-Q5S5) + 20J63 + 60S5 | Heptagonal rings | 14-gon Toroid |
| 256 faces (13 types), 480 edges (23 types), 192 vertexes (8 types) | 1060 faces (19 types), 1620 edges (27 types), 480 vertexes (8 types) | 1003 faces (74 types), 2401 edges (174 types), 1358 vertices (97 types) | 2128 faces (78 types), 5796 edges (208 types), 3640 vertexes (130 types) |
| | | | |



| 12-Gishi Slice 5 | Dodecahedra 92 (RTC) | 14-Gofix Slice | Topo Stellated Rhombic Dodecahedron |
|---|---|---|---|
| 350 faces (14 types), 750 edges (13 types), 216 vertexes (8 types) | 864 faces (16 types), 2160 edges (36 types), 1240 vertexes (22 types) | 780 faces (13 types), 1230 edges (21 types), 452 vertexes (9 types) | 384 faces (16 types), 1056 edges (44 types), 600 vertexes (26 types) |
|  |  |  |  |

| 120 Cell-Linked rings-Spinning | | Castle | Fortress |
|---|---|---|---|
| 720 faces (1 type), 1200 edges (1 type), 600 vertexes (1 type) | | 296 faces (74 types), 575 edges (144 types), 280 vertexes (70 types) | 1270 faces (127 types), 2440 edges (244 types), 1170 vertexes (117 types) |
|  |  |  |  |
| Images and animations generated by *Stella Polyhedron Navigator* | | | |

## Polyhedral "memory palaces": an ordering pattern for sustainable self-governance?

**Geometrical metaphors**: As noted above, the work of Frances Yates (*The Art of Memory*, 1966) on the method of loci -- as used by orators of the past and by mnemonists of the present day -- focused on memory palaces and memory theatres. The role of virtual reality in this respect has been recently explored ( Jan-Paul Huttner, et al, *Immersive Ars Memoria: Evaluating the Usefulness of a Virtual Memory Palace*. *Scholar Space*, 8 January 2019). Understood as forms of knowledge architecture, there is potential in extending that approach to polyhedra -- whether as palaces or theatres. In this case it is the mapping of topics onto features of a polyhedron which are then to be recognized as "loci".

There are significant traces of this approach in the use of geometrical metaphors in common discourse, most notably in politics, as for example in use of: "making a point", "pursuing a line of argument" or "drawing a line", "taking sides, and the like -- as separately discussed (*Engaging with Globality through cognitive lines, circlets, crowns or holes*, 2009). More explicitly, proposals have been made to understand the organization of Europe in terms of "variable geometry" (*Alternation between Variable Geometries: a brokership style for the United Nations as a guarantee of its requisite variety*, 1983).

Of particular interest is how the distinctive features of polyhedra might serve to carry a memory -- effectively extending the simpler notion of mapping information and knowledge onto a more conventional 2D map, as with mind maps. As noted separately (*Projective geometry of discourse: points, lines, frames and "hidden" perspectives*, 2018), distinctions which then merit investigation include:

- **vertexes**, specifically in the light of the various types of vertex of a polyhedron. These are notably distinguished by the number of edges which intersect at such a point, otherwise to be understood in terms of the valency of the vertex. Especially obvious are the vertexes at the confluence of lines in any polyhedron forming a star, notably as a result of stellation.

- **lines**, most obviously recognized as external edges of which various types may be distinguished. Edges may be recognized as parallel, whether in relation to a given side or across the polyhedron. Also of relevance are the implicit axes of symmetry -- especially since that notion from geometry is widely implied in global strategic consideration (as with Axis of Evil). Additionally there are the implicit lines across the polyhedron -- internally, and distinct from axes -- possibly parallel, possibly forming triangles or rectangles (notably golden rectangles in the case of the icosahedron, for example).

- **faces**, namely the polygonal sides of the polyhedron, defined in geometrical terms as N-gons, according to the number of sides. Most obvious are triangles (3-gonal), squares (4-gonal), and hexagons (6-gonal). Less evident are those of 4-sided rhombic form and octagonal form. Rarer still are heptagonal (7-gonal), octagonal (8-gonal), enneagonal (9-gonal), decagonal (10-gonal), hendecagonal (11-gonal), dodecagonal (12-gonal) faces. These can be understood as framing spaces, possibly of discourse or agreement. As indicated with respect to lines, such polygons may be evident internally rather than as characterizing the surface of the polyhedron. The number of sides of constituent polygons is necessarily distinct from the number of faces, irrespective of the

number of their sides (*Polyhedra with N-faces*).

- **circles**: especially when these take the form of great circles, namely defining reflection planes splitting the polyhedron into two halves. These may be especially significance to navigation the space defined by the polyhedron, as discussed with respect to the spherical geometry of the so-called *Pentagramma Myrificum* (*Global Psychosocial Implication in the Pentagramma Mirificum: clues from spherical geometry to "getting around" and circumnavigating imaginatively*, 2015). The circle metaphor is variously borrowed in social discourse to indicate a particular pattern of linkage (as with a social circle,)

- **spheres**: whether recognized as a circumsphere or an insphere. An obvious metaphorical use of "sphere" is with respect to the problematic reference to spheres of influence, whether or not these can be readily associated with polyhedral geometry

**Perspective in relation to a polyhedral memory palace:** Clearly a polyhedron can be viewed externally from a variety of perspectives -- some of which may highlight memorable symmetry effects. It might then be asked how many such distinct perspectives are associated with a given polyhedron. This could be understood in terms of the diversity of insights effectively integrated by the polyhedron as a whole.

In such terms, a 3D polyhedron clearly offers a more complex challenge than the 2D form of a conventional map -- about which it could be asked how many contrasting perspectives it offers. The question is highlighted by provocative maps presented "upside down" -- "south-up map orientation" -- thereby drawing attention to seldom-recognized biases. Of greater relevance is the challenge of projecting the 3D form of the Earth onto a 2D surface, selectively optimizing distorting constraints. There are many such projections (*List of map projections*, *Wikipedia*).

There have been relatively few attempts to use a polyhedron as a mapping surface to interrelate a diversity of topics. Most significant in that respect is the Dymaxion Map designed by Buckminster Fuller as a consequence of his magnum opus, as mentioned above (*Synergetics: Explorations in the Geometry of Thinking*, 1975/1979). The World Game, a collaborative simulation game in which players attempt to solve world problems, is played on a 70-by-35-foot version.

The preference has been for a sphere, notably widely deployed as Science On a Sphere, namely as a spherical projection system created by the US National Oceanic and Atmospheric Administration. Another approach is virtual globe, namely a three-dimensional (3D) software model or representation of the Earth or another world. This provides the user with the ability to freely move around in the virtual environment by changing the viewing angle and position.

**Embodying a perspective cognitively?** A polyhedron with a unique central point offers the possibility, potentially highly relevant, of a 360-degree perspective ("wrap-around") from that position (for which software patents have been accorded, notably as a screen display technology for video-gaming). The features of the polyhedron, onto which memorable knowledge and information might be projected, are then configured around the "observer", as is otherwise characteristic of a planetarium.

The question is then the nature of the experience in which memorable knowledge is configured in this way -- according to the particularities of the selected polyhedron. Whereas a 2D mind map offers a highly distorted "flat" image of an integrative experience, a 3D configuration (experienced from within a polyhedron) reinforces a unique sense of embodied knowledge. It then constitutes both the sense of a memory palace and of a memory theatre.

An approximation to such architecture in the processes of governance is to be recognized in a "situation room" -- recognized as an intelligence management centre (Michael Bohn, *Nerve Center: Inside the White House Situation Room*, 2004). This can be understood as a nexus of collective cognitive fusion, whose configuration the future may imagine otherwise (*Enactivating a Cognitive Fusion Reactor Imaginal Transformation of Energy Resourcing (ITER-8)*, 2006).

The experience is even more intimate if personal attributes, interests (topics) and values are "deployed" in this way -- possibly extended provocatively to an array of personal roles (or "multiple personalities"). As a theatre, the inner surface could be understood as offering a form of screen on which the dynamics between roles and/or topics could then be observed. So framed, it raises the questions of how any set of roles or frameworks might be configured by a polyhedron inviting embodiment framing the process of "donning" and "doffing" each "bias" (*Systems of Categories Distinguishing Cultural Biases*, 1993). How might requisite variety for sustainability then be understood as configured -- 8-fold, 12-fold, 20-fold, or more?

A potentially valuable metaphor, to explore further the process of requisite "cognitive fusion", may be through the functioning of a polyhedral array of optical lenses through which significance associated with the vertexes is brought to a focus at its centre. To the extent that effective governance involves juggling the relationship between "sides" of various orientation, each side of the polyhedron might be understood as a lens (*Governance as "juggling" -- Juggling as "governance": dynamics of braiding incommensurable insights for sustainable governance*, 2018). A potentially relevant articulation in electromagnetic terms is offered in a patent (*Polyhedral antenna and associated methods*, EP08828124A).

Provocatively it could be argued that the mysterious 17th SDG goal is potentially at the centre of an array of 16 polyhedral features -- an integrative nexus of enactive decision-making. Given metaphorical reference to the global brain, this nexus could be understood as a collective *corpus callosum*, as yet to be appropriately framed (*Corpus Callosum of the Global Brain? Locating the integrative function within the world wide web*, 2014).

More imaginatively, as anticipated by the iconic science fiction movie *Contact* (1997), the experience of being centered in this way might even be compared to a "stargate" (*Topology of a Renaissance "Stargate" of Higher Dimensionality: complementary ways of imagining engagement with otherness*, 2018). The "memorial" role of iconic monuments such as Stonehenge can be imaginatively associated with this function, as with legendary accounts of ancient races "withdrawing into the stones".

# Memorability implied by Euler's isomorphism of musical and polyhedral order?

**Degrees of agreeableness**: In the quest for greater understanding of the memorability of order through polyhedral patterns, it is somewhat extraordinary to note the role of music in the thinking which framed the articulation of insight into the patterns highlighted above -- effectively prefiguring it. That key insight was formulated by Leonhard Euler (1707-1783) in what is now universally recognized as the Euler characteristic. This was the discovery relating the number of vertexes, edges and faces of a convex polyhedron, namely V-E+F=2.

Euler's early interest in music, which persisted throughout his life, culminated in a focus on what has been translated as "degree of agreeableness' -- a *gradus-suavitalis* function, as variously explained by the following:

- József Śandor: *Euler and Music: a forgotten arithmetic function by Euler* (*Octogon Mathematical Magazine*, 17, 1, 2009)
- Roger Mathew Grant: *Leonhard Euler's Unfinished Theory of Rhythm* (*Journal of Music Theory,* 57, 2013, 2).
- Jordan Alexander Key: *Euler, Cowell, Polyhedra and the Music Genome: Leonard Euler's theory of music and its anticipation of modern musical discourse* (10 December 2018)
- *Euler's "Degree of Agreeableness" for Musical Chords* (*Thats Maths*, 9 August 2018)

Although the relation between music and order has been a preoccupation since the Pythagoreans, it could be said that, despite Euler, mathematicians and musicians have essentially gone their separate ways with respect to the implications of the relationship between polyhedra and music. This disassociation would seem to extend to any understanding of memorability, despite the fascination of mathematicians with symmetry -- diffidently framed in terms of the quintessentially non-mathematical reference to "mathematical beauty".

Of particular interest is the manner in which Euler's preoccupation with music can be understood as "prefiguring" his approach to polyhedra. A similar phenomenon has been documented with respect to the philosopher Ludwig Wittgenstein by Susan Sterrett (*Wittgenstein Flies a Kite: a story of models of wings and models of the world*, 2005; *Pictures, Models and Measures*, *Belgrade Philosophical Annual*, 30, 2017). As the author shows in that case, the glimpse of a solution to the problem of language in 1914 had to do with experimental models which had been so crucial to the Wright brothers' solution to the problem of flight. An analogous prefiguration can be speculatively explored in relation to Albert Einstein's fundamental insight (*Einstein's Implicit Theory of Relativity -- of Cognitive Property? Unexamined influence of patenting procedures*, 2007).

The relation between Euler's insight into musical order and polyhedral order has been described in terms of isomorphism by Peter Pesic (*Music and the Making of Modern Science,* 2014, *Euler's Musical Mathematics. The Mathematical Intelligencer* 35, 2013, 2). As Pesic describes Euler's exploration of a "degree of agreeableness" in music:

> The structure of this relation between vertices, edges, and faces is strikingly similar to the structure of the degree of agreeableness of musical intervals, s - n +1. Without intending any direct connection between polyhedra and Euler's hierarchy of musical intervals, as such, both these relations (V + F - E = 2 and s - n + 1) give the kind of general categorization we now think of as *topological* and which Euler thought of in terms of *geometria situs*. To be sure, these relations are very different, and not just in the objects they describe. Euler's formula is an equation describing a necessary and sufficient condition for closed, convex polyhedra; his formula for musical degree defines a hierarchy between different intervals. They both pose a general schematization that categorizes a vast domain, of polyhedra or of musical intervals, respectively, subsuming many different individuals under a larger genus.
>
> ... his criterion for setting up his degrees is freely chosen according to his notions of what would be more "intelligible" and hence more "agreeable" (*suavis*). 1n his musical work, Euler first devised the general classificatory strategy that he then applied to the bridge problem and later to polyhedra. To use a later mathematical term, his approaches in these cases were *isomorphic*, that is, they had the same essential structure. Because the musical example came first, it arguably was the arena in which he first found and applied the kind of approach that he later (and perhaps without realizing it) then found appropriate to bridges and polyhedra (2014, p. 148)

The reference to the "bridge problem" relates to Euler's fundamental contribution to graph theory -- with respect to the problem of the so-called Seven Bridges of Königsberg. .

***Spectrum musicum* and *Gradus suavitatis*:** As further clarified in the *Wikipedia* entry with respect to Euler's music theory:

> A first point of Euler's musical theory is the definition of "genres", i.e. of possible divisions of the octave using the prime numbers 3 and 5. Euler describes 18 such genres, with the general definition $2^mA$, where A is the "exponent" of the genre (i.e. the sum of the exponents of 3 and 5) and $2^m$ (where "m is an indefinite number, small or large, so long as the sounds are perceptible" (Leonhard Euler, *Tentamen novae theoriae musicae*, St Petersburg, 1739, p. 115), expresses that the relation holds independently of the number of octaves concerned... Genre 18 ($2^m.3^3.5^2$) is the "diatonico-chromatic", "used generally in all compositions"...Euler later envisaged the possibility of describing genres including the prime number 7. Euler devised a specific graph, the *Speculum musicum*, to illustrate the diatonico-chromatic genre, and discussed paths in this graph for specific intervals, recalling his interest in the Seven Bridges of Königsberg (Euler, 1739, p. 147; Euler, *De harmoniae veris principiis*, 1774, p. 350)... Euler further used the principle of the "exponent" to propose a derivation of the *gradus suavitatis* (degree of suavity, of agreeableness) of intervals and chords from their prime factors -- one must keep in mind that he considered just intonation, i.e. 1 and the prime numbers 3 and 5 only.

As articulated by Jordan Alexander Key:

> ...Euler used the principle of the "exponent" when comparing two similar musical structures (pitches in harmonies, rhythmic values in rhythmic successions, etc.) to propose a derivation of what he termed the "gradus suavitatis" or "degree of agreeableness" between these structures. This degree could then be used to compare seemingly similar or dissimilar structures in music (different harmonies, rhythms, melodies, meters, forms, etc.); through this mathematically derived comparison, deep similarities could be uncovered that might not lie at the surface of the music or which might be obscured due to historical aesthetic bias. (*Euler, Cowell, Polyhedra and the Music Genome: Leonard Euler's theory of music and its anticipation of modern musical discourse*, 10 December 2018)

Of particular relevance to the role of music in relation to polyhedra framed in terms of factoral exponents (as above), is the use of exponential analysis by Ernest McClain as a musicologist with an historical preoccupation of significance to governance (*Myth of Invariance: the origins of the gods, mathematics and music from the Rg Veda to Plato,* 1976; *The Pythagorean Plato: prelude to the song itself,* 1978; *Meditations Through the Quran: tonal images in an oral culture,* 1981; *Music and Deep Memory: speculations in ancient mathematics, tuning, and tradition*, 2018).

With respect to any insight into memorability enabled by either music or polyhedra, **it is the relation between Euler's "agreeableness" and the "beauty" -- which mathematicians struggle to relate to symmetry -- that calls for further clarification, notably in the light of comprehensibility and communicability**. Euler's effort to recognize a "degree of agreeableness" suggests the possibility of an insight into a "degree of memorability", as this might contrast with "degrees of beauty" -- especially given the controversy which such recognition would imply from a cross-cultural perspective (Leonhard Euler, *On the true principles of harmony as presented through the Speculum Musicum*, St. Petersburg Academy, 1773; translated by Larry Blaine and Douglas Kendall).

The potential for necessary (?) "controversy" with respect to any relevance to memorable strategic articulations is usefully implied in a thesis on consonance in music by Julián Villegas (*Local Consonance Maximization in Realtime*, University of Aizu, 2006). Specifically discussing the *gradus suavitatis,* Villegas introduces his argument in the following terms:

> There seems to be agreement that **music is more interesting when passages of 'tension' and 'relaxation' alternate** while it is performed. Interchangeable expressions for **the same patterns include 'pleasant' and 'unpleasant,' 'rest' and 'motion,' 'euphony' and 'cacophony,'** etc. Different explanations for the subjective perception of these patterns exist and depend on the context in which they are studied: physiology, music, psychology, genetics, etc. The psychoacoustical explanation is one of the most accepted. According to this theory, the separation in frequency of tones sounding concurrently determines directly the consonance; the more separated they are, the more 'relaxed' their interaction is perceived. In general, it's desirable that in the alternation of tension and relaxation sections, the sounds corresponding to the relaxation state have a maximum consonance, **so the problem of achieving this can equivalently be considered the problem of maximizing the consonance of the interaction of simultaneous tones at a given time**. These issues have been addressed for about six centuries, since musicians started to formalize the use of multiple sounds at the same time coherently. (p. 3, emphasis added)

Given the degree of controversy with regard to any strategy in practice, the insight into "harmony" offered by music -- in the light of the recognized role of dissonance -- suggests that such necessary "dissonance" could be usefully used to reframe simplistic understandings of "positive versus negative", "agreement versus disagreement" and "consensus versus dissent" (*The Consensus Delusion: mysterious attractor undermining global civilization as currently imagined*, 2011). Arguably some new form of alternation merits consideration to engage more fruitfully with the prevailing degree of divisive fragmentation (*Development through Alternation*, 1983). Such fragmentation is of course curiously echoed in the world of music.

Of particular interest, given the reference above to 17 parabolic orbifolds, it is appropriate to note the role that orbifold structure has been held to play in the organization of music as articulated by Dmitri Tymoczko, who models musical chords as points in orbifold space (*The Geometry of Music*, 2011). As discussed separately (*Musical implications of orbifolds for comprehension of questioning dynamics*, 2014), there is the possibility that the distinctive cognitive feel for logical distinctions and connectivity might be associated with chords.

**Dissonance and memorability**: This emphasis on the requisite interrelationship between "pleasant" and "unpleasant" merits consideration in imagining how any global strategy might be rendered memorable through song or music, as can be otherwise considered (*A Singable Earth Charter, EU Constitution or Global Ethic?* 2006; *Aesthetics of Governance in the Year 2490*, 1990; *Reversing the Anthem of Europe to Signal Distress*, 2016).

To whatever degree **dissonance is to be be considered essential to memorability**, in his review of dissonance Villegas notes (p. 16):

> A successful single theory to explain consonance and dissonance remains elusive, and some theoreticians and researchers argue that there's no single theory that could explain it but a set of them....:
>
> - **frequency ratio**: the auditory 'preference' for small integer ratios, because of the resulting periodicity of the stimuli,
> - **harmonic relationship**: the expected dissonance when the harmonic relationships of a composition doesn't follow the classical canons of western harmony,
> - **temporal dissonance**: related to the beating of a pair of sounds when the difference of their frequencies is small enough to partially cancel the effect of each other (amplitude modulation),
> - **tonal fusion**: the perceived euphony of simultaneous sounds that can be perceived as a single tone,

- **tonotopic dissonance**: the perceived dissonance of a pair of sinusoidal waves when their frequency difference is less than one critical bandwidth,
- **virtual pitch**: the component of dissonance that arises from competing (unclear) virtual pitches,
- **expectation dissonance**: alterations on learned harmonic patterns, as in 'cadences' where a leading tone resolves to a different note than the tonic or its equivalent,
- **interval category**: or the difficulty to classify the formed interval of a pair of sounds into a learned category of intervals,
- **absolute pitch category**: the perceived dissonance of a tone by a person with absolute pitch when it's impossible to classify it into one of the learned pitches due to the ambiguity of its frequency
- **stream incoherence theory**: the component of dissonance that arises due to confusion regarding streaming, and
- **relative dissonance**: the contextual relative consonance of a sonority when it is preceded by other sonorities of contrasting dissonance. This effect is related to the sensation of rest and peace experienced when the most dense and dark dissonant composition, listened as loud can be stood, finishes

The metaphor of a "memory theatre", as mentioned above, could be usefully extended to that of a "concert hall", even an open-air concert hall -- given the manner in which any performance might be rendered "memorable". The concert metaphor has however already been borrowed in reference to a "concert of democracies" as an alternative form of international organization -- but without any consideration of memorability (Ivo H. Daalder, *Who and Why: The Concert of Democracies*, *Brookings*, 15 December 2006; *Beyond a "Concert of Democracies"?* 2011).

**Poetry-making and policy-making:** The question of what makes music memorable, in contrast to the kind of strategic laundry list exemplified by the UN's Sustainable Development Goals, is presumably a matter of great concern to the music industry in competitively marketing its products (Eleanor Crane, et al, *Musical Hit Detection*, 12 December 2008).

With respect to governance, a potentially useful approach is through recognition of the extent to which iconic leaders -- of contrasting political persuasion -- have claimed a particular interest in poetry (*Poetic Engagement with Afghanistan, Caucasus and Iran: an unexplored strategic opportunity?* 2009). From that perspective it may be asked whether the relations between the elements of a poem, most obviously through "rhyme", complement and enhance the strategic thinking otherwise associated with "reason". How the articulation of strategic elements "rhyme" then invites investigation -- as a complement to any understanding of cybernetic feedback loops between them through "reason" (*Poetry-making and Policy-making: arranging a Marriage between Beauty and the Beast*, 1993).

The argument can be reinforced by the much-publicized engagement with *haiku* of Dag Hammarsköld, an early Secretary-General of the UN, and of Herman Van Rompuy, as a recent President of the European Council. As noted above, curiously the 17-fold pattern of the UN's SDGs has been celebrated in the 17-fold organization of *haiku* poetry, itself supported by UNDP (*Inspired by Nature: Celebrating Biodiversity with Haikus*, UNDP, 22 May 2017). The notable subtlety of *haiku* merits reflection on the insights it offers into strategic resilience, as argued separately (*Ensuring Strategic Resilience through Haiku Patterns: reframing the scope of the "martial arts" in response to strategic threats*, 2006).

Poetry can be especially valuable in interrelating into a coherent pattern themes evoked in successive parts of a poem -- effectively feeding back and forward in a manner reminiscent of the positive and negative feedback valued from a cybernetic perspective. This ability may be reflected in the thematic content, perhaps most succinctly indicated by the poet John Keats as;

> ... Negative Capability, that is, when a man is capable of being in uncertainties, mysteries, doubts, without any irritable reaching after fact and reason... [rather than] being incapable of remaining content with half-knowledge (1899)

**Nonlinearity and multidimensionality**: Negative capability is otherwise comparable with the Zen Buddhist understanding of *shoshin* (Christian Jarrett, *How to foster "shoshin"*, *Aeon-Psyche)*. This contrasts with the assumption of perfection, or the righteous "unidirectional" quest for a form of unity, characteristic of many belief systems having problematic dynamics with whatever calls them into question. Paradoxically that process of designing out any sense of imperfection, from what they assume they embody, engenders a deniable process of enantiodromia whereby they eventually embody imperfection of a kind. This is especially evident in religions (most obviously Catholicism at the present time), political ideologies, and science itself (*Knowledge Processes Neglected by Science: insights from the crisis of science and belief*, 2012).

The absence of any poetic aspiration is especially evident in the essentially linear articulations of conventional strategies and principles -- dependent on forgettable "reason" in the absence of memorable "rhyme". In their assumption of righteousness, they could indeed be understood as characterized by "negative incapability" -- otherwise evident in avoidance of any form of negative feedback or acknowledgement of ignorance. This is currently a documented characteristic of world leaders.

In contrast with the asystemic approach of strategic linearity, this can be understood as an essentially nonlinear narrative, whether poetic or note (*Difference Between Linear and Nonlinear Text*, *Difference Between*, 18 June 2018). It is in this sense that any linear understanding of degrees of agreeability, unity, perfection or memorability can be fruitfully called into question. As with polyhedral forms, **strategies may be differently memorable and variously exemplifying unity**.

## Reframing forms of connectivity through the logic of oppositional geometry

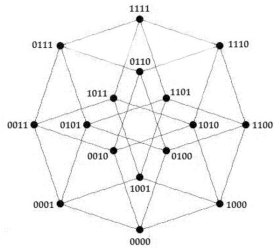
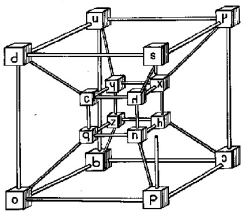
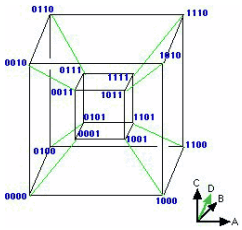
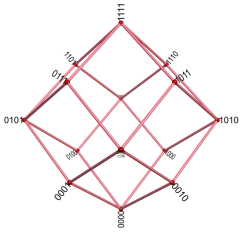
**"Oppositional logic"?** Given the inherently vexatious incapacity to address strategic disagreement, other than by seeking (violently) to repress it, the relevance of polyhedra can be fruitfully illustrated by reference to oppositional geometry fundamental to logical geometry in discourse (*Oppositional Logic as Comprehensible Key to Sustainable Democracy: configuring patterns of anti-otherness*, 2018;

*Oppositional logic and its requisite polyhedral geometry*, 2018). The matter is framed academically in terms of Aristotelian diagrams, and the square of opposition (Lorenz Demey, *Aristotelian Diagrams in the Debate on Future Contingents*, *Sophia*, 58, 2019; Alessio Moretti, *The Geometry of Logical Opposition*, University of Neuchâtel, 2009).

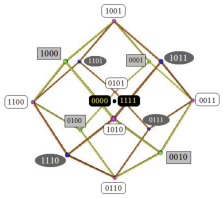
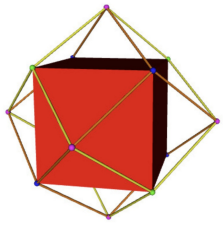
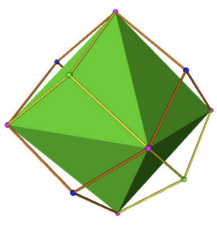
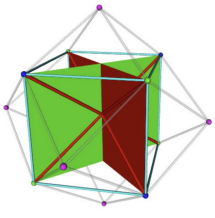
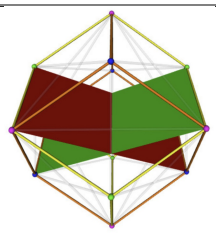
Arguably if there is one characteristic of psychosocial reality which is a fundamental challenge to governance it is that of "opposition" -- and the framework within which it can be appropriately comprehended and integrated. The argument for doing so is that literature is particularly focused on the **geometrical representation of opposition** as articulated in truth tables through the set of 16 Boolean connectives (logical operations on two variables) of basic logic.

As explained by Steven H. Cullinane, and illustrated below left (*The Geometry of Logic: finite geometry and the 16 Boolean connectives*, *Finite Geometry Notes*, 2007), a Hasse diagram of a Boolean lattice, may also be viewed as a tesseract (4-dimensional hypercube). There the vertices represent the 16 traditional "binary connectives". The tesseract's 16 vertices may also be regarded as representing either the 16 subsets of a 4-set or the 16 elements of the affine 4-space A over the two-element Galois field. The pattern was originally depicted by Shea Zellweger, as a "logic alphabet", as shown below.

A key polyhedron used to map the 16 Boolean logical connectives in that approach is the rhombic dodecahedron of 14 vertices (namely 16-2) with its 12 faces. The distorted mapping from 16 to 14 can be discussed as a "fudge" to avoid the challenges of 4D comprehension, as discussed separately (*Governance beyond the logical focus on true vs false?* 2019; *Questionable confusion in configuring strategic frameworks: "fudging" self-reflexivity?* 2019).

| Haase diagram<br>or Tesseract | **The Logic Alphabet Tesseract**<br>- a four-dimensional cube (see coding).<br>by Shea Zellweger | **Topologically faithful 4-statement Venn diagram**<br>is the graph of edges of a 4-dimensional cube<br>as described by Tony Phillips | Organization of contingent bitstrings<br>on a rhombic dodecahedron |
|---|---|---|---|
|  |  |  |  |
| | Diagram by Warren Tschantz (reproduced from the Institute of Figuring) . | A vertex is labeled by its coordinates (0 or 1) in the A, B, C and D directions; the 4-cube is drawn as projected into 3-space; **edges going off in the 4th dimension are shown in green.** | Adapted from Lorenz Demey and Hans Smessaert (2017) |

In terms of logical geometry, the relation between the rhombic dodecahedron and more conventional Platonic polyhedra is illustrated in images from the Logical Geometry website below. Of further interest is the connectivity within that structure as illustrated by the images on the right.

| **Aristotelian logic diagrams related to the rhombic dodecahedron** | | | | |
|---|---|---|---|---|
| Standard bitstring mapping | Embedding of a cube | Embedding of an octahedron | Embedding of classical balanced Aristotelian squares | Embedding of unbalanced Aristotelian squares |
|  |  |  |  |  |
| Reproduced from *3D Aristotelian diagrams* (*Diagram database of logical geometry*, June 2020) | | | | |

Whether undertstood as a pattern of 16 (or questionably reduced to 14), is this pattern strangely related to that of the 16 SDGs discussed above. In ystemic terms, does each such "sSustainable Development Goal" imply a distinctive forms of connectvity essential (if not vital) to sustainmability? This possibility might also be explored through an appropriately diverse pattern of feedback loops characteristic of a viable system in cybernetic terms.

**Comprehension?** Given this subtle complexity, there is great irony to the fact the governance of society depends formally on the binary distinction between "true-or-false", or "guilty-or-not guilty". A rare exception with respect to the latter is the "not proven" of Scottish law, and the cultivated evanescent nature of fake news and its deniability (*Varieties of Fake News and Misrepresentation: when are deception, pretence and cover-up acceptable?* 2019; *Deniable responsibility for any ultimate crime against humanity?* 2019). The emergence of a "post-truth" society, characterized by post-truth politics, even suggests the need for a post-truth adaptation of truth tables (*Towards articulation of a "post-truth table"?* 2016)

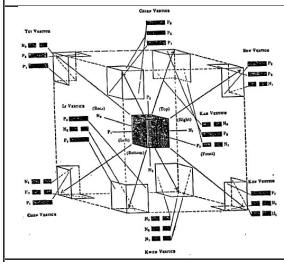
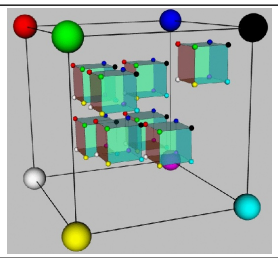
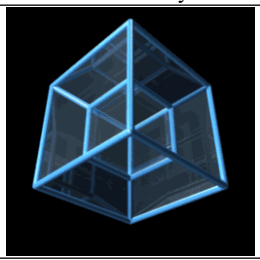
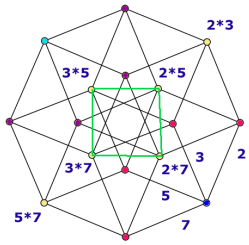
As discussed separately, curiously missing from any discussion of an "eightfold way", or of the subtle intricacies of "truth tables", is the

challenge they may imply to comprehension (*Memorability: "comprehension tables" as complement to "truth tables"*, 2019). It is as though the simple presentation of such patterns is naively assumed to trigger comprehension of the knowledge implied -- as with declarations regarding the threat of global warming and other crises. Whereas the focus of truth tables is on the "shades of grey" in the relation between "true" and "false", their presentation is seemingly to be recognized as constituting a simple binary distinction between "knowledge" and "ignorance". The reality that any "eightfold way" (as encoded by such tables) may be meaningless (or incomprehensible) is not a consideration.

**"Oppositional comprehension"?** It is profoundly curious, if not tragic, that there is seemingly no "translation" between the logical articulation above and the evident reality of divisive strategic disagreement. One suggestive indication to that end is the early framing offered relating a form suspiciously similar to the tesseract above to the 8-fold Chinese *BaGua* articulation, as shown below left.

Given the experiential dynamics traditionally associated with that articulation, this suggests a complementary mode of exploring opposition -- in which the complementarity is emphasized within a framework, arguably more general than the limitations of logic alone. The dynamics implied are suggested by the virtual reality animation, reproduced from a separate discussion with related imagery (*Neglected recognition of logical patterns -- especially of opposition*, 2017). This offers a reminder that the "edges" of a polytope, presented sttically, may be more fruitfully understood dynamically as indications of feedback loops and trajectories, or portions thereof. As originally emphasized by Buckminster Fuller: *All systems are polyhedra: All polyhedra are systems.* (1979, 400.56)

The limitations of the rigid polyhedral structure, especially when "reduced" to the rhombic dodecahedron -- understood statically -- are further highlighted by the frutiful challenge to comprehension indicated by the dynamics of the tesseract animation below.

| **Cubical representation** of *BaGua* pattern of *I Ching* | **Interactive virtual reality variant in 3D** | **Tesseract animation** simulating requisite 4-dimensionality? | **Hexany** as a mapping of musical tuning systems |
|---|---|---|---|
|  |  |  |  |
| Reproduced from Z. D. Sung, *The Symbols of Yi King or the Symbols of the Chinese Logic of Changes* (1934, p. 12) | Virtual reality variants: vrml/wrl; x3d. Reproduced from *Neglected recognition of logical patterns -- especially of opposition*, (2017) | by Jason Hise [CC0], via Wikimedia Commons | Modified by Robert Walker from Tilman Piesk's *Hypercubestar* on *Wikipedia* |

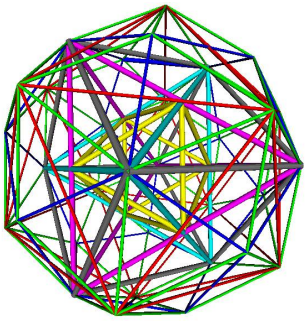
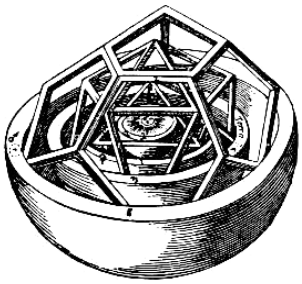
Of potential relevance to the reference above to a musical articulation, is the argument with regard to musical tuning systems framed as the hexany by Erv Wilson (above right). As described by Robert Walker (*Hexany*), this can be thought of as analogous to the octahedron (geometric dual of the cube). The notes are arranged so that each point represents a pitch and every edge and interval with each face represents a triad. It thus has eight just intonation triads where each triad has two notes in common with three of the other chords. Each triad occurs just once with its inversion represented by the opposing 3 tones. The edges of the octahedron show musical intervals between the vertices, usually chosen to be consonant intervals from the harmonic series. The points represent musical notes, and the three notes that make each of the triangular faces represent musical triads. Wilson*** also pointed out and explored the idea of melodic hexanies.

**Dynamic embodiment of perspective?** Comprehended from within, as an encompassing pattern of dynamically shifting connectives in discourse, the rhombic dodecahedron (as with simpler or more complex polyhedra), might be better imagined metaphorically as a "frozen poem" or a "concert hall" (rather than a "memory theatre", as mentioned above).

A perspective "from within" then evokes consideration of any mapping as a cognitive projection, as may be variously explored (*Cognitive Embodiment of Nature "Re-cognized" Systemically: radical engagement with an increasingly surreal reality*, 2018; Interface challenge of inside-outside, insight-outsight, information-outformation, 2017; *Embodying Global Hegemony through a Sustaining Pattern of Discourse Cognitive challenge of dominion over all one surveys*, 2015; *The Territory Construed as the Map: in search of radical design innovations in the representation of human activities and their relationships*, 1979)

The challenge to comprehension and memorability, as it relates to governance could then be explored in terms of shifting cognitively between such patterns -- somewhat reminiscent of shifting gears for different terrain in an automobile or truck. (*Psychosocial Implication in Polyhedral Animations in 3D Patterns of change suggested by nesting, packing, and transforming symmetrical polyhedra*, 2015).

| **Nesting 5 Platonic polyhedra: octahedron, icosahedron, dodecahedron, tetrahedron, cube** | |
|---|---|
| Rhombic Triacontahedron (green) as a nesting framework: Interactive display. | Polyhedral model of solar system of Johannes Kepler |
| | |

| | |
|---|---|
|  |  |
| Virtual reality variants *static*: vrml or x3d; *mutual rotation*: vrml or x3d; *"pumping"*: vrml or x3d; videos: "*pumping*" mp4; "*rotation*" mp4) Developed with X3D Edit and Stella Polyhedron Navigator | Reproduced from *Wikipedia* entry on *Mysterium Cosmographicum*(1596) |

***

**A tragic strategic postscript ?**

Curiously, even coincidentally, the dynamic nesting of polyhedra is a notable feature of the Mereon Matrix -- the focus of study and commentary by a range of specialists in the last two decades. It gave rise to a book in 2013 (Lynnclaire Dennis, Jytte Brender McNair and Louis H. Kauffman (Eds.). *Mereon Matrix: Unity, Perspective and Paradox*, 2013). The book was retracted when the publisher, Elsevier, allegedly decided to terminate its series on "Applied Mathematics" (Peter McNairm, Lynnclaire Dennis and Jytte Brender Mcnair, *Meron Matrix*, *ResearchGate*, 1 January 2017). A second edition was later produced by a different publisher (Louis H Kauffman, Jytte Brender Mcnair and Lynnclaire Dennis (Eds.). *The Mereon Matrix: everything connected through (k)nothing*, 2018). The editors are given different precedence in different sources for the different editions.

The book, as claimed by its authors, variously describes the ultimate model:

- The dynamics of the geometry unites all Platonic and Kepler Solids into one united structure and creates 11 unique trefoil knots. Its topology is directly related to the dynamics of the polyhedra.
- The Mereon Matrix is an approach to the unification of knowledge that relies on whole systems modelling. it is a framework charting the emergence of the Platonic and Kepler solids in a sequential, emergent growth process that describes a non-linear whole system, and includes a process of 'breathing' as well as multiplying ('birthing');
- This dynamic/kinematic structure provides insight and a new approach to General Systems Theory and non-linear science, evolving through a new approach to polyhedral geometry. A set of 11 First Principles is derived from the structure, topology and dynamics of the Mereon Matrix, which serve well as a template information model.
- The Mereon Matrix is related to a large number of systems, physical, mathematical, and philosophical, and in linking these systems, provides access to new relationships among them by combining geometry with process thinking. The new perspective on systems is hypothesized as universal -- this is, applicable in all areas of science, natural and social. Such applicability has been demonstrated for applications as diverse as pre-life evolution, biological evolution and human molecular genetics, as well as a classroom management system for the educational system.
- Care has been taken to use images and languaging that are understandable across domains, connecting diverse disciplines, while making this complex system easily accessible. (McNairn, et al, 2017)

Otherwise described:

> One specific investigation has been into the Platonic polyhedra emerging and defined through dynamics. As a result of this research a matrix of intersecting polyhedra has been discovered which unifies all the fundamental Platonic polyhedra. This matrix, known as Mereon, shows a way that all the Platonic polyhedra (and other polyhedra) relate to one another in an amazing coordination. This is similar to Fuller's investigations into a rational matrix of the polyhedra. Specifically, Dennis' matrix includes at least 10 Tetrahedra, 5 Cubes, 5 Octahedra, 5 Cube-Octahedra (Fuller's "Vector Equilibrium"), 5 Rhombic Dodecahedra, 1 regular Dodecahedron, 5 Icosahedron and 1 Rhombic Triacontahedron. Fuller's matrix of polyhedra does not accommodate all these polyhedra into a single matrix. (*The Mereon Matrix: everything connected through (k)nothing*, 2018)

The dynamics have been defined in the fiollowing terms:

> 'Breathing' is the dynamics of the two polyhedra that together constitute the Mereon Matrix. In the dynamics, the 120 polybedron expands to a 180 polyhedron and back to the 120 polyhedron. Similarly, the 144 polyhedron expands to a 300 polyhedron and then back to the 144 polyhedron. Breathing is paralleled to the mechanism that drives the progression from one function to the next until tbe end of a traversal, through the succession of functions, and then continuing with the first function again. (J. Brender McNair. *A Unifying Theory of Evolution Generated by Means of Information Modelling*, IOS Press, 2016 , p. 184)

Or otherwise as:

> In specific terms, it is a dual polyhedra: one polyhedron inside another. The inner most polyhedron is diamond-like; has 144 triangular faces and grows to 300. In its dynamics, light emerges from 48 vertices. The outer polyhedron is watery, akin to a bubble; it has 120 triangular faces and grows to 180 faces. In the space between the two polyhedra, the energetic interconnection between the two, ties into multiple pattern knots. (Kevin Williams, *The Science of Life*

*Discovered from Lynnclaire Dennis' Near-Death Experience*, 2014)

The clearest visualizations are however offered by Robert W. Gray (*Lynnclaire Dennis' Geometry: The Pattern*).

Most unfortunately, given the claim to provide a unique modelling insight into the crisis dynamics of humanity (which global governance seeks to address), the manner of that claim undermines its purported intention to a regrettable degree. Despite claims for its universal applicability, readily available visualizations of the model are either superficial, subject to commercial and/or copyright constraints, or a feature of exorbitant image marketing. Is that to be expected of future creative insights into the strategic challenges of governance in this period -- "buy my book"?

It is noteworthy that the global crisis of COVID-19 has triggered a widespread relaxation of such barriers by major publishers. Should a key insight of global strategic relevance only be available in a publication costing several hundred dollars? Is the world to be held to ransom in the face of global catastrophe by those with "killer apps" -- as is evident with respect to the marketing of vaccines in response to COVID-19?

The book has also invited the assiduous attention of anonymous sceptical critics readily dismissing some of its aspects with the label "woo" -- as pseudoscientific -- and with little ability to appreciate its other dimensions (*Lynnclaire Dennis NDE: a skeptical look*, *Skeptic*, 12 June 2014). Typically such critiques have absolutely nothing of consequence to offer in response to the challenges of the times. Ironically the systematic use of "woo" by sceptics has become an equivalent to alkahest -- the hypothetical universal solvent of alchemists.

The argument above with respect to a polyhedral articulation of strategic alternatives (conventionally perceived as competing unfruitfully) sought ways of reframing their opposition to one another in terms of complementarity. *The Mereon Matrix*, in positioning itself as unique and of universal relevance, unfortunately fails to engage with those who challenge its claims from other perspectives -- as with the lengthy critique in the *Skeptic*.

How is the nature of that dynamic to be understood, if it is not fruitfully reframed by *The Mereon Matrix* itself? The failure in this respect exemplifies the failure of global strategy-making at this time -- each proposed strategy, however creative, dismissing or suppressing the relevance of others -- and indulging in questionable copyright and commercial strategies on the side, as with science itself (*Knowledge Processes Neglected by Science: insights from the crisis of science and belief*, 2012; *Future Coping Strategies: beyond the constraints of proprietary metaphors*, 1992).

The underlying interpersonal and institutional dynamics, as evident in the tragic intellectual copyright disputes regarding many vital insights, merit systematic exploration as case studies to clarify the challenge of communication of innovation. The dynamics are especially complex because of the psychosocial dimension -- deprecated by the natural sciences as "woo", when associated with "belief" of any kind. A striking example of such dynamics is offered by the assertion of a renowned specialist in crystallography (twice awarded a Nobel Prize) with respect to quasicrystals: *There is no such thing as quasicrystals, only quasi-scientists*. Their discoverer was subsequently awarded the Nobel Prize in Chemistry (*Quasicrystals Scoop Prize*, *Chemistry World*, November 2011). Is the deprecated role of "pesudoscience" to be compared with the dynamics giving rise to recognition of quasicrystals? Is such deprecation to be understood as an indication of "quasi-science"?

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