laetus in praesens

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25 January 2025 | Draft

Eliciting Insight within Complex Polyhedral Configurations of Concepts

AI-assisted clarification of cognitive challenge of fundamentally elusive metaphors

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Introduction

The future may note the remarkable conventional tendency to articulate matters of importance as checklists -- possibly with some clustering of items in the checklist. This is evident in declarations of various kinds, most notably the Universal Declaration of Human Rights and the various regional charters of human rights. The pattern can be recognized in the presentation of the Earth Charter and the articulation of a proposed Global Ethic. The UN's set of **Sustainable Development Goals**, as with it predecessors, also takes list form. Such lists are typically unmemorable. Curiously this evokes little concern in a period in which coherence is desperately sought by many.

The exploration described here follows directly from earlier attempts to configure complex sets in a coherent and memorable manner through mapping them onto polyhedra of appropriate complexity -- polyhedra whose geometric symmetries offer mnemonic keys as well as implying systemic coherence. The previous exercise, which is the primary trigger for this further investigation, focused on the possibility of memorable

organization of the classic 48-fold set of Zen koans of the *Mumonkan* on a truncated cuboctahedron (*Configuring the Paradoxical Insights of 48-fold and 100-fold Sets of Koans*, 2024). That noted in passing the potential value of mapping the traditional set of 147 Delphic Maxims.

Earlier explorations focused on the 64-fold pattern of distinctions shared by the set of genetic codons, the branches of mathematics distinguished by the *Mathematics Subject Classification*, the *Kama Sutra*, and the Chinese classic of the "*Book of Changes*" (*Yi Jing* / *I Ching*) as variously highlighted (*Triadic correspondences between Topology, Kama Sutra and I Ching*, 2011). These were variously mapped onto the relatively unique 64-edged drilled truncated cube (*Proof of concept: use of drilled truncated cube as a mapping framework for 64 elements*, 2015; *Configuring the 64 subjects of mathematics as a 64-edged drilled truncated cube*, 2021). Its relation to the hypercube (or truncated tesseract) -- of concern in studies of oppositional logic -- was also noted (*Implicate order through hypercube and drilled truncated cube?* 2022).

It is one thing to shift from a minimally ordered list format to any such three dimensional mapping configuration. It is another matter to derive new forms of significance from their more integrative order. The concern in what follows is how any such significance might be associated memorably with the geometry of more complex polyhedra. That such a diversity of topics should share a 64-fold pattern recalls the arguments from the perspective of cognitive psychology (George Lakoff and Rafael Núñez, *Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2000).

The focus here is on the patterns of connectivity **"within"** any "superficial" mapping of an array of conceptual distinctions onto a polyhedron -- however that mapping may be organized **"without"** from an "external" perspective as argued in a presentation the World Academy of Art and Science illustrated by a variety of human rights charters (*Topology of Valuing: dynamics of collective engagement with polyhedral value configurations*, 2008 and *Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors* (2008). By contrast here, the geometry of selected polyhedra was used to explore the "underlying" connectivity between "superficial" distinctions. It is assumed that these would be of potential systemic significance -- as well as being a key to the memorability and credibility of the pattern as a whole.

As noted, a point of departure was the array of 48 koans on a truncated cuboctahedron. An effort was made to apply the method to the array of 16 Sustainable Development Goals mapped onto the 16 vertices of the 1-frequency truncated tetrahedral geodesic sphere to clarify their potential internal systemic connectivity. Its applicability to the 30 articles of the Universal Declaration of Human Rights was explored to highlight underlying patterns of connectivity between them. In the light of the relative success of those exercises, the possibility of applying it to the above-mentioned 64-fold arrays was considered.

Of particular interest was the possibility of detecting any "inner" substructures (or patterns of coherence) within each such array -- as implied by the structure of the truncated tesseract, and as explored in some measure with respect to the exploratory mappings in 3D of the UN's SDGs onto the 64-vertex truncated tesseract (*Visualizing the Coherent Configuration of Incommensurable Cognitive Modalities*, 2024). That structure offers the tantalizing image of a form of "inner chamber" of connectivity suggestive of a higher order of coherence and integration. The question here is whether any such "inner chambers" could be identified and visualized in the case of the SDGs or the Universal Declaration of Human Rights -- or indeed with the set of 48 koans.

The detection and visualization of such connectivity, and any formation of an "inner chamber", depended to a large extent on the facilities offered by AI in previous exercises -- in the form of ChatGPT 4o and Claude 3.5 -- given the limited competence of the author (and the relatively inaccessibility of professional mathematical and graphical expertise). In consequence, and somewhat ironically, the interaction with AI did not result in immediate solutions to the question posed and the visualization sought. It became evident that AIs were constrained to a curious degree in responding to a challenge for which they "optimistically" claimed ability. The very lengthy iterative process enabled by artificial intelligence -- with a degree of relative success -- could then be usefully recognized as a metaphor for collective ability to identify higher orders of connectivity -- whether within strategies, human rights, or elusive insights.

The nature of the challenge can then be clarified in terms of the difficulty for AI to transform a series of purportedly connected "points" -- comparable to any conventional checklist of concepts -- into a superficially connected polyhedral configuration, and then to distinguish the patterns of relationship between those points. This can be recognized as the challenge of detecting the connectivity between any array of human rights or of the SDGs. Little effort is made to do so from a systemic perspective -- to the extent that the possibility is even considered to be of any relevance. In detecting and visualizing such connectivity in 3D, the difficulties encountered by AIs could be understood as a metaphor for the human cognitive difficulties in recognizing such patterns.

Despite what is claimed for AI capacity, including imagery, it became very clear that AIs have as yet very limited ability to "think in 3D" -- as might well have been expected. All the more problematic was effective erosion of AI memory in that learnings in an earlier exchange were not retained and applied in a later part of the exchange (for resource management and marketing reasons). Errors previously encountered were simply repeated -- and repeatedly corrected -- undermining any convergence on a viable solution previously achieved successfully. The challenge proved to be all the greater with respect to the detection and visualization of any "inner" patterns of coherence framed by such connectivity -- any "inner chamber".

Metaphorically such a "chamber" might be recognized as a cognitive *sanctum sanctorum* to which only the cognoscenti would currently claim access. The challenge could be comprehended as the quest for a kind of Rosetta Stone enabling a form of "translation" between distinctive cognitive modalities (as previously suggested) -- variously understood to be incommensurable or mutually irrelevant. Such a configuration has been previously explored in the light of that metaphor (*Memorable Packing of Global Strategies in a Polyhedral Rosetta Stone*, 2023).

In cognitive terms the Rosetta Stone might be more appropriately recognized as the archetypal philosopher's stone -- or even more appropriately as the philosophers' stone. Beyond any static emphasis, the question is then the cyclic dynamics through which it "works".

The iterative interactions with either or both AIs proved to be very lengthy and somewhat frustratingly inconclusive. A minimal account of the process is reported here as a means of framing the future challenge in the use of AI to this end -- and the implications for collective human recognition of patterns of connectivity of relevance to governance. Intriguingly the visualizations of partial success included here therefore offer an indication of the partial human capacity to detect complex integrative patterns.

Although the investigation here may be held to be incomprehensible and irrelevant from many perspectives, there is considerable irony to the fact that the highly popular and influential *Star Trek* series featured a polyhedral game Kal-toh -- purportedly played by Vulcans. The cultural impact of Kal-toh among the Vulcans was compared to the human game of chess -- although the comparison was framed in the series by the somewhat insulting remark that "Kal-toh is to chess as chess is to tic-tac-toe".

Although the Vulcan game was only elusively described, it was used as an inspiration in a thesis by Terry David Anderson, in considering a collection of mathematically interesting problems in algorithmic graph theory and game theory (*The Vulcan game of Kal-toh: Finding or making triconnected planar subgraphs*, University of Waterloo, 2011 and *FUN'12: Proceedings of the 6th International Conference on Fun with Algorithms*, 2012). The popular inspiration offered by such speculation can be usefully compared to a widely accessible "polyhedral" game, namely Rubik's Cube -- given its potential relevance to reframng exploration of the UN's Sustainable Development Goals (*Interplay of Sustainable Development Goals through Rubik Cube Variations*, 2017; *Harmonizing Dissonance via a Rubik's Cube Framework*, 2024). As of January 2024, around 500 million cubes had been sold worldwide. As with its variations, it has also been the focus of mathematical studies.

As in the previous experiments, the responses of ChatGPT 4o are distinctively presented below in grayed areas, with those of Claude 3.5. **Given the length of the document to which the exchange gives rise, the form of presentation has itself been treated as an experiment** -- in anticipation of the future implication of

AI into research documents. Web technology now enables the whole document to be held as a single "page" with only the "questions" to AI rendered immediately visible -- a facility (not operational in PDF variants of the page, in contrast with the original). Reservations and commentary on the process of interaction with AI to that end have been discussed separately (*Methodological comment on experimental use of AI*, 2024). Whilst the presentation of responses of two AIs could be readily considered excessive, it offers a "stereoscopic" perspective highlighting the strengths and limitations of each.

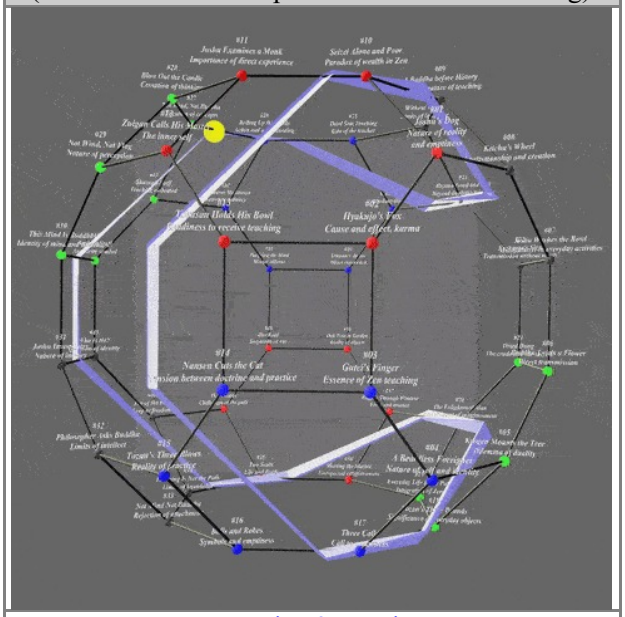
Many of the unusual visualizations presented -- resulting from the interactions with AI -- are indeed remarkable and an imaginative trigger. Potentially of greater interest is what proved impossible to achieve with such assistance and the learnings for future exploration, notably with further development of AI facilities. The "failures", as illustrated visually, can be seen as providing a metaphor for the challenges faced in eliciting insight from coherence. The argument concludes with a speculative exploration of the polyhedral cognitive framing of the widely familiar ball dynamics in goal scoring and targetting.

Show All AI Responses

Exploration of paradoxical cognitive implications of koan mapping

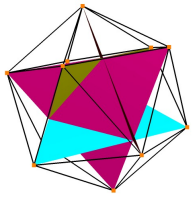
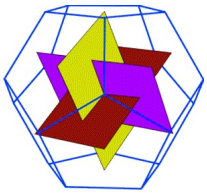
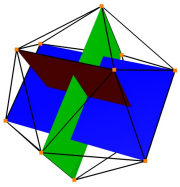
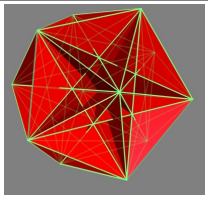
The previous exercise explored a dynamic linkage between koans as illustrated by the reproduction of the following animation (*Configuring the Paradoxical Insights of 48-fold and 100-fold Sets of Koans*, 2024).

Truncated cuboctahedral mapping of 48 Mumonkan koans
with linkages implied by rotation of a tennis-ball seam curve
(note the technical imperfections in the colouring)



Interactive 3D variant

As indicated above, the 48 koans have been speculatively organized on the 48 vertices of the truncated cuboctahedron on the basis of the traditional *BaGua* pattern -- but in 3D. Ths frames the question of the potential significance to be elicited from the 24 pairs of koans on opposite sides of the configuration. That question can be condidered in the light of the geometric significance associated with rectangles of 4 vertices -- a notable feature as golden rectangles integral to the dodecahedron and icosahedron, as shown below, reproduced from (*Coherence, comprehensibility and credibility of a cognitive toolkit*, 2018; *Representation of Creative Processes through Dynamics in Three Dimensions*, 2014).

Triplcities, quadruplicites and golden rectangles suggestive of internal coherence of a set of operational insights			
Triplicities	Dodecahedron (20 vertices)	Icosahedron (20 faces)	Icosahedron (15 golden rectangles)
			
Generated from the Stella4D software by its developer Robert Webb, from a great icosahedron augmented by an icosahedron, then hiding selected faces			

The assistance provided by AI in what follows was highly dependent on the facilities and polyhedral data provided by the Stella4D application.

Question: In the interpretation of koan mapping onto vertices of a truncated cuboctahedron, particular significance could be attributed to relations between koans which are on: -- opposite vertices (a dilemma?) -- in an equilateral triangular formation, especially when symmetrical with respect to the polyhedron origin (a trilemma?) -- in a coplanar square or rectangular relation (a quadrilemma?) -- with four equilateral triangles forming a tetrahedron (a special quadrilemma?) Could you comment on the sense in which a koan is held to be paradoxical in its own right, and therefore any such configuration is of particular cognitive interest. It would seem that a dilemma -- then passing through the origin (given its significance) -- would be especially problematic as placing the experiencer at the origin. Whereas as a triangular configuration this relates to the arguments of Ron Atkin with respect to Q-analysis -- navigating (safely?) "around" the origin [Jacky Legrand., *How far can Q-analysis go into social systems understanding?* Fifth European Systems Science Congress, 2002; P. Gould, *Q-analysis, or a language of structure: an introduction for social scientists, geographers and planners*, *International Journal of Man-Machine Studies,* 13, 1980, 2]

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Question: If a koan (mapped onto a vertex) suggests a paradox in its own right, could you comment further on the cognitive challenge of reconciling two or more koans. Arguably a single koan can itself be understood as deliberately framing a dilemma. Reconciling two "opposing" koans is therefore even more intriguing

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Preliminary identification of triangular and square substructures

Prior to considering whether such 4-fold features are present in the truncated cuboctahedron, it may then be asked whether there are regular (equilateral) triangles between koan vertices within the configuration. These might have particular cognitive significance, given the fundamental importance of triangulation in surveyng and global navigation (*Triangulation of Incommensurable Concepts for Global Configuration*, 2011). Rather than "opposing koan" pairs, their significance might be less challenging and elusive -- in contrast to any rectangular configurations of koans.

The question presented to the AIs was framed as follows:

> You have access to the truncated cuboctahedron OFF data -- as analyzed. The vertices have been used in previous exchanges to map 48 koans. The question is how to elicit cognitive significance from obscure text (labels). In the past you have distributed the koans on that polyhedron in the light of the 8-fold *Bagua* pattern -- already a cognitive challenge. Assumptions can be made that there is significance to be elicited from koans in opposing positions on the configuration -- 24 pairs. Of interest is whether triplets with their centres at the origin exist -- or possibly offset from the centre across the structure. Clearly rectangles of vertices exist through the centre -- whether they are golden

> rectangles is of interest. The argument is that such structures are cognitive templates -- as with the triangulation of surveying and navigation. For a start, could you assist in detecting regular triangles of vertices

Whilst the question appeared to be correctly understood, the capacity to detect and generate results in the form of an X3D file for visalization in 3D proved to be problematic. Purportedly 64 regular triangles were detected of which 5 were closest to the origin and at the same distance from it. The set of 64 relates to a related interest explored separately with respect to the 64 hexagrams.

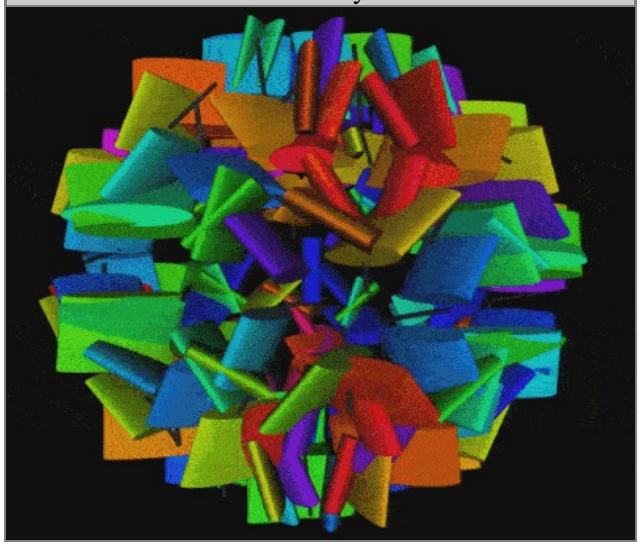
The focus was switched to the detection of square and rectangular forms framed by the following question:

> My specific interest (initially) is the projection of square patterns of a truncated cuboctahedron through the structure. With 6 (of the 12) orthogonally positioned squares projected in this way an internal cube is formed by the intersection of such projections (as with a tesseract). This is relatively simple to comprehend in contrast with the intersection resulting from the projection of the other 6. Does that form an internal form analogous to the truncated cuboctahedron itself

Unfortunately there was no sense of convergence on a solution --- after 5 or more hours of assiduous iteration -- especially with the reversion to testing aspects which have already worked. It would have been useful to offer some indication of probability of convergence in the light of an inference regarding the competence of the questioner. With respect to responses, considerable time is wasted -- introducing appropriate uncertainties -- regarding placement of corrective snippets in the light of such incompetence. The results are disappointing in the light of your insightful response to related X3D possibilities which must clearly be set aide

The following images are suggestive visual metaphors of any chaotic jumble of sets of concepts and insights held to be attractive

"Unsuccessful" detection and 3D representation of triangles within the truncated cuboctahedron
Animation of aesthetically remarkable result



Displacement of surface configurations towards a common centre

Question: In your response with regard to "substructures" framed by projections of polygonal patterns on the surface of a polyhedron) through it to an identical pattern on the opposite side, there is a sense in which this framing by intersecting triangular or rectangular prisms (etc) effectively subtend internal structures which you appropriately name as tesseract-like. Do you have any trace of studies of this process and the manner in which

such subtended geometric forms are named

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Question: Before reverting to the unresolved Python script issues, I would like to take further your commentary on the variety of polyhedral "internal" substructures. With respect to possible X3D dynamics, I now distinguish the following possibilities: -- indicate internal links between all vertices as cylinders, with the longer edges then being distinguished with relatively larger radius compared to the shorter (and therefore closer to the circumsphere surface. These would enable detection of internal structures by inspection -- move progressively selected surface patterns of edges (eg squares, hexagons, etc) with ROUTES towards the centre (and towards corresponding opposite features).

The movement towards each other of opposing features would result in their intersection around the centre (eg squares forming a cube) which you have highlighted in terms of hemipolyheda (when the faces tilt). although the result is more obvious in their dual forms (Tetrahemihexacron vs tetrahemihexahedron) -- detection of triangles of parallel or polyhedral configurations of internal edges (or rectangles, etc) -- whether the consequence of internal edges based on surface vertices or based on intersection of internal edges as substructures -- a form of inversion of surface configurations of edges as they pass through the centre to their corresponding opposing configuration, namely each collapsing edge vertex passing through the centre to be expanded thereafter -- possible consideration of spherical polyhedral substructure forms Could you comment further

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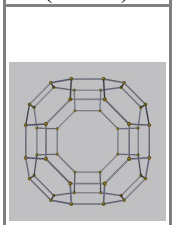
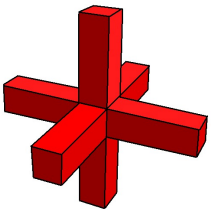
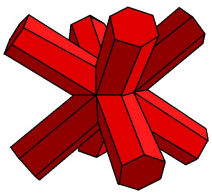
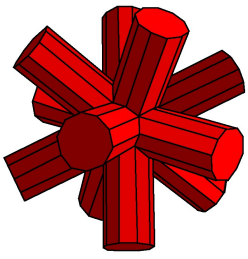
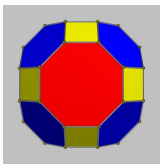
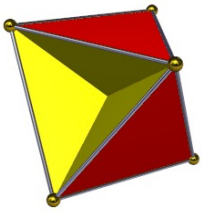
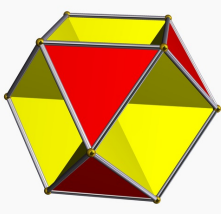
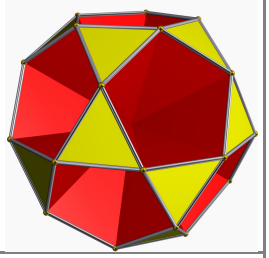
The hemipolyhedra offer useful clues to the manner in which the polygonal faces of the truncated cuboctahedron "collapse" towards (and even pass through) the centre. Their square, hexagonal and octagonal cross-sections (as shown below) suggest trajectories through the centre of the truncated cuboctahedron.

Truncated cuboctahedron -- with non-transparent faces (below)	Hemipolyhedra (and their duals)		
	Tetrahemihexacron -- dual of Tetrahemihexahedron (below)	Octahemioctacron -- dual of Octahemioctahedron (below)	Small icosihemidodecacron -- dual of Small icosihemidodecahedron (below)
			
			
	Tomruen, CC BY-SA 4.0, via Wikimedia Commons		

Although efforts were made with the AIs to elicit 3D models of movement of the faces towards the centre, it became curiously apparent that considerable difficulty was encountered in calculating face normals from the OFF file data where the faces were however defined with great precision. It was agreed that the Newell

method offered a generic solution for the future (*Calculating surface normal in Python using Newell's Method, StackOverflow*).

Rather than seeking the ease and speed of AI assistance, it would otherwise have been relatively straightforward to produce an X3D model in which clones of selected faces moved to the centre. The movement could have been variously timed such as to ensure that faces connected periodically at the centre -- substructures thereby defined dynamically. Thus the square faces of the truncated cuboctahedron could meet to form a cube. Less evident is the inernal structure formed (or implied) as the 8 hexagonal faces do so -- or the 6 octagonal faces

Variety of approaches to identification of potentially insightful substructures

In the light of the problematic results to which reference is made above, the approach was reframed through clarifying the variety of possible approaches to substructure detection.

Question: In a configuration of insights, as the mapping of koans on the truncated cuboctahedron can be held to be, could you classify the substructures of coherence it might embody -- if every vertex (indicative of cognitive significance) is potentially linked directly to every other vertex. For example, the patterns of koans on the "surface", the binary "opposites through the origin, the 3D substructure configurations at the intersection of projections of surface patterns between opposite sides (square patterns potentially forming a cube around the origin, for example), coplanar triangles (squares, or rectangles) between surface vertices, other coplanar (or 3D) configurations resulting from intersections of links between vertices (rather than intersecting at the vertices). All these offer distinctive cognitive implications between the koans so linked. Is the set complete

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Quest for internal substructures of significance in the 48-fold Mumonkan

Question: With respect to your question of priorities, we have paused detection of parallel triangles in favour of "collapsing" square face structures. This was held to be simplest with respect to processing the OFF file data. That approach has itself been paused. The internal connection between all vertices of a polyhedron appears straightforward except for the clutter problem. Setting aside the squares collapse (for which I now conclude that there is a fundamental difficulty in calculating face normals prior to configuring the faces to be moved), could we consider the feasibility of one of the other options of interest. This would be indicating with cylinders all internal vertex-to-vertex links. As you have indicated this would result in a lot of clutter -- to be remedied by reducing the radius of the cylinders according to the height -- namely those close to the surface to be very thin, those closer to the origin to be thicker. The internal links to be highlighted would necessarily exclude the external links between vertices (by which the polyhedron is defined) as well as those through the origin between opposite vertices.

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Question: Radius scaling possibly linear in 5 to 10 steps. Remove only the longest provisionally. Uniform initially. I can experimentally adjust, especially if the Materials are defined by step Group

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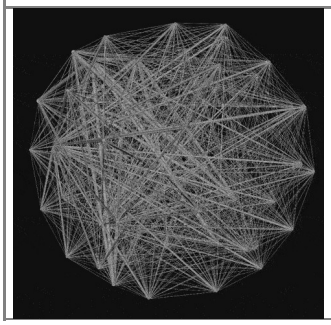
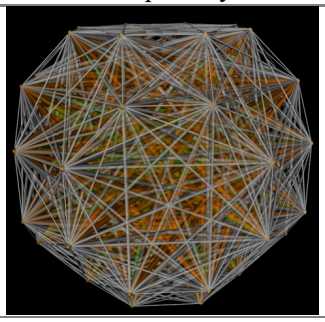
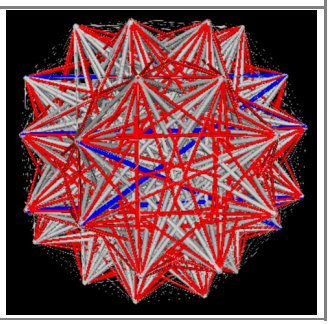
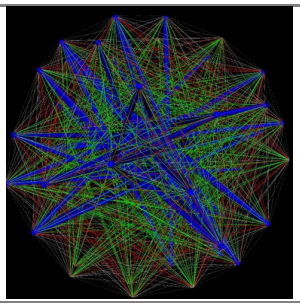
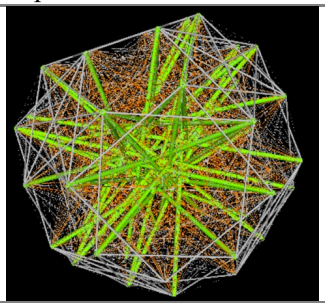
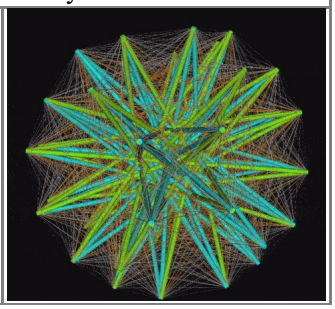
Following this stage of the exchange, ChatGPT successively produced an extensive set of X3D scripts in response to feeback in quest of visually helpful displays. Some doubts were raised and addressed regarding potential sources of error which call for review and clarification in future work. These notably related to the lengths of links across the polyhedron and whether the detected differences justified clustering such lengths

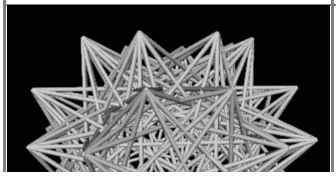
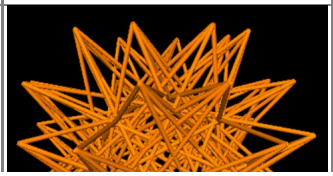
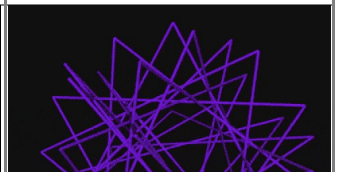
together or separately (given the precision of the coordinates in terms of decimal places and potential consequences of rounding).

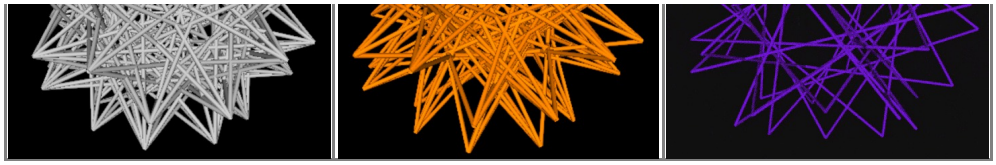
The process then switched to visual inspection of the displays (without AI assistance) and successive manual modification of the X3D models to highlight contrasting patterns. A particular assumption made in this process was that patterns of shorter links between vertices could be presented either with very fine lines or omitted in favour of the patterns of longer internal inks across the polyhedron. This was provisionally justified by the assumption that the shorter links approximated most closely to the external edges of the polyhedron, whereas the longer links could be potentially associated with deeper or more challenging insights -- however this might be subequently understood.

Initially the focus was on the aesthetics of decluttering the complex of internal links between vertices of the truncated cuboctahedron, as shown below (**without** the koan labels which had featured in earlier models). Techniques explored included reducing the radius of shorter links (near the surface), even rendering them increasingly transparent. Colouring the longer (deeper) links was also tried.

Initial results of efforts to display the array of potential connections between 48 koans		
Black and white	Selective use of transparency	Selective colouring
		
More complex use of colour and selectivity		
		

ChatGPT enabled an animation which cycled through patterns of links of different lengths across the truncated cuboctahedron (internally), distinctively coloured, to get a sense of the contrasting internal structures. This started with the shortest length (necessarily part of the surface structure), through to the longest through the origin to an opposite vertex. Some of the patterns render visually detectable (to a degree) a form of "inner chamber" at the centre of the structure. Any such "chamber" could be understood as indicative of a higher order of insightful coherence or integration.

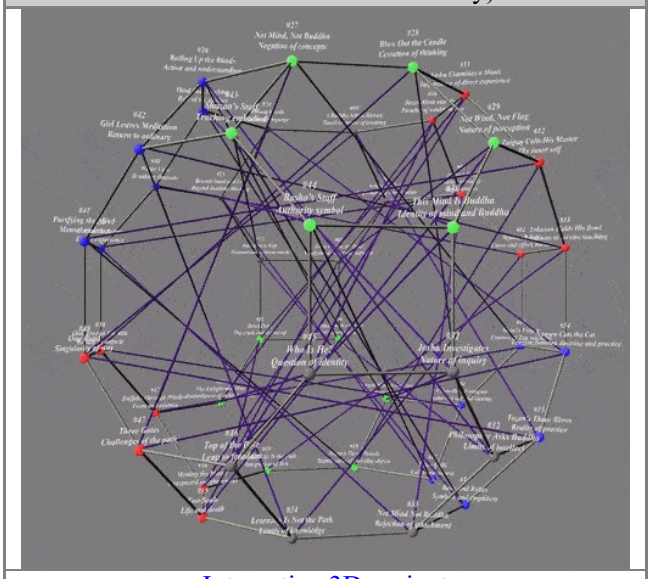
Indication of use of cycling between 48 koans through patterns of links of different length		
Black and whote screenshot	Thicker link colour screenshot	Cycling animation
		



Provisionally setting aside the quest for specific substructures, both AIs responded proactively to the possibility of representing the different levels of connectivity between vertices -- to which concept labels might be variously attached. In order to avoid the visual clutter apparent above, the choice was made to cycle through patterns of connectivity from the most "superficial" to the antipodal, as illustrated below.

In a third stage the internal patterns of links were embedded as an animation **within** the truncated cuboctahedron (with addition of koan-labelled vertices), as shown below

Illustrative animation of changing patterns of links between koans
(coloured by distinctive levels of separation within the truncated cuboctahedral array)



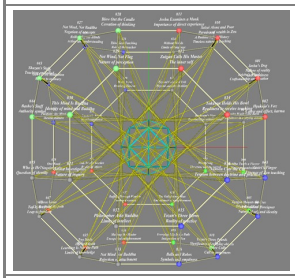
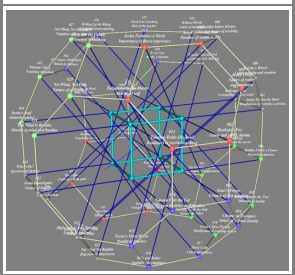
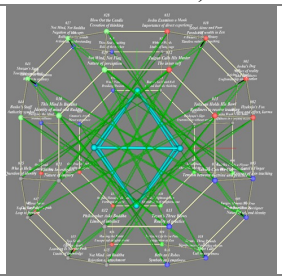
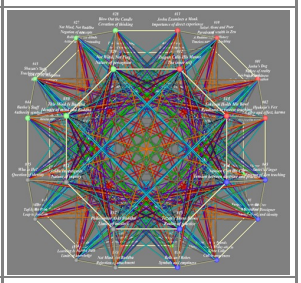
Interactive 3D variant

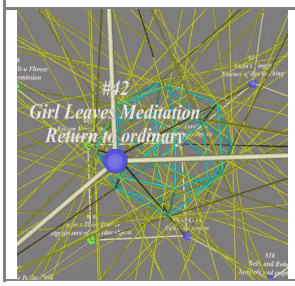
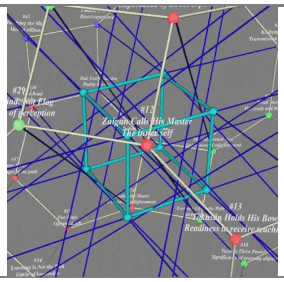
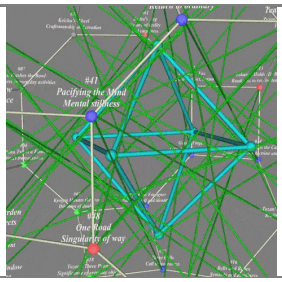
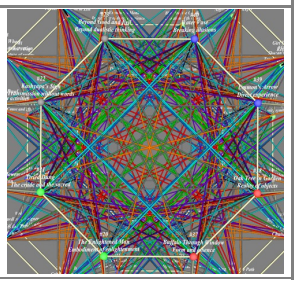
The tentative configuration of paradoxical koans offered the implication that particular substructures might become apparent in some way from the manner in which particular patterns of koans were interrelated. It was assumed that the geometry of intersecting links (understood cognitively) could form such substructures at the centre of the configuration -- indicative of a higher order of cognitive integration. Whilst the possibility was recognized in the interaction with AI, the calculations to determine and detect such structures proved a challenge to frame with the expectation of a timely fruitful outcome.

By inspection of internal links of a particular length (and therefore of a characteristic geometry), a substructure of a particular form (framing an empty space around the centre) could be recognized. The question was then the geometry of that form. With the longest link pattern excluded as being through the origin, the next longest appeared to frame a slightly distorted cube. Further investigation was able to fit a 2-frequency octahedral geodesic sphere of 18 4-valent vertices (18 vertices, 32 faces, 48 edges) -- seemingly framed by 72 links. Other possibilities included the octahemioctahedron (18 vertices, 32 faces, 48 edges), the duals of the small rhombihexahedron and of the great rhombihexahedron.

The exercise was repeated for the set of next longest links. In that case a cube was formed -- effectively by the intersection of a projection of 6 of the 12 square faces of the truncated cuboctahedron. A second cube (differently oriented) could be framed by the remaining set of 6 square faces. The set of thrid longest links shown might be framing an octahedron (but as the image on the right indicates), but a methof for identifying any polyhedron so framed requires an as yet unknown methodology (which could not therefore be presented

for AI processing).

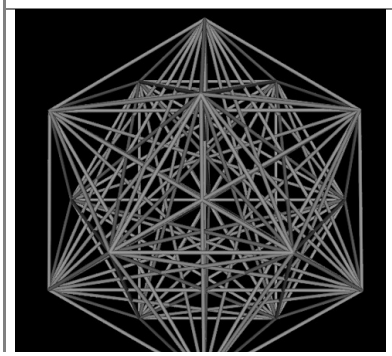
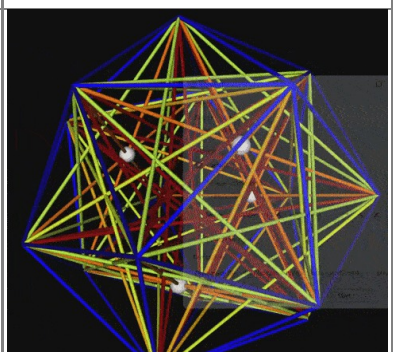
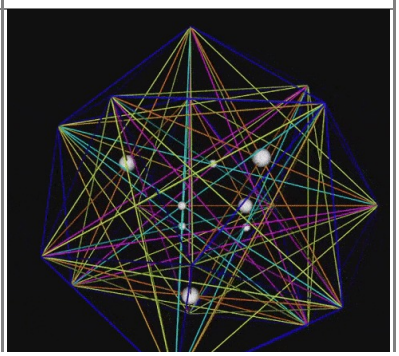
Screenshots of 3D models of patterns of links between koans -- avoiding the centre			
Pattern of longest links (framing octahedral geodesic sphere)	Patten of second longest links (framing a cube)	Pattern of third longest links (possibly framing an octahedron)	Patterns of shorter links (polyhedral framings as yet unclear)
			

Animations of details of "inner chambers" above, as defined by link patterns of different length			
Framed geodesic sphere	Framed cube	Octahedron (unfitted)	Multiple internal structures
			

Inner connectivity of 16-fold Sustainable Development Goals

Given the partial success above with use of the truncated cuboctahedron as a mapping framework for the set of 48 koans, an effort was made to adapt the scripts to mapping 16 Sustainable Development Goals onto the 16 vertics of the 1-frequency truncated tetrahedral geodesic sphere (28 faces and 42 edges)

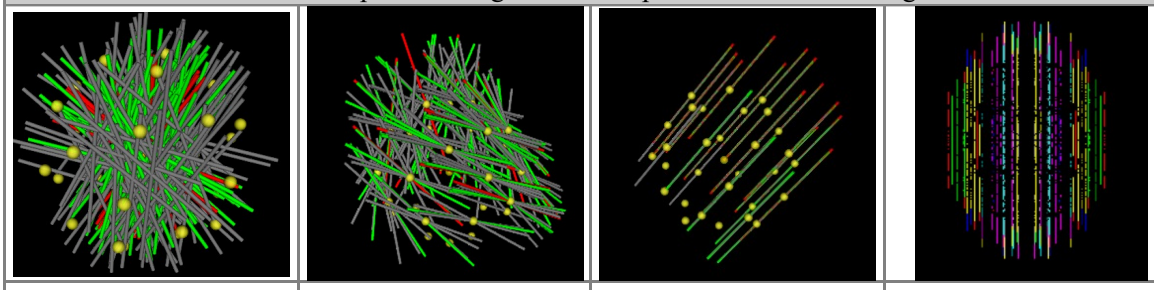
The following configurations in 3D were generated by AI scripts to illustrate attempts to highlight the presence of substuctures of potential interest within that configuration -- as well as the pattern of internal links between the SDGs. The central animation indicates intersections successfully detected by AI and framing a tetrhedron. The animation on the right indicaates addtional intersections (smaller white spheres) detected by visual inspection through rotation of the first set by 180 degrees and reducing its size. Ignoring the difference in scale, the two tetrahedra form an 8-pointed star or stellated octahedron (potentially recalling the 8-fold set of Millennium Development Goals which preceded the set of 16 SDGs).

Animations indicative of the "inner" relationships between the set of 16 Sustainable Development Goals (the 17th Goal being understood as the configuration of the 16)		
Black/White	White spheres marking 4 AI-detected intersections	Smaller white spheres not detected with AI
		

Inner connectivity of 30-fold Universal Declaration of Human Rights

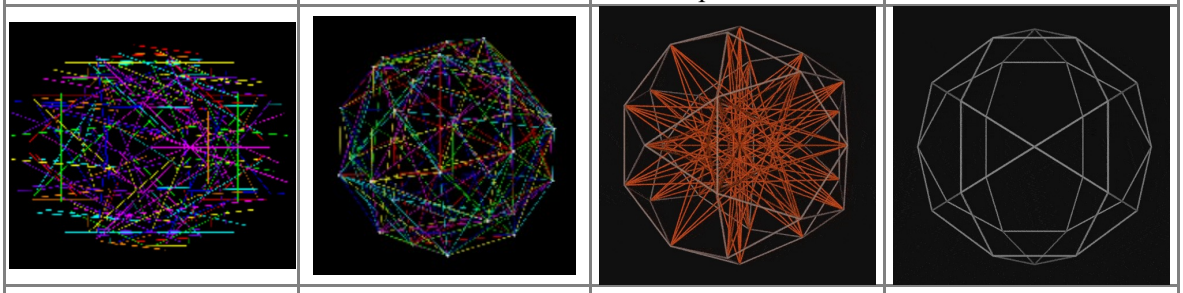
Given the fundamental importance associated with the 30 articles of the Universal Declaration of Human Rights -- typically presented in list form -- the systemic significance of the pattern of internal links merited similar exploration (together with the associated substructures), . Again this called for a relatively simple adaptation of the scripts developed above for any representation in 3D.

Initial results of efforts to display relationships between a set of 30 human rights coherently
visual metaphors of fragmented comprehension of human rights



Progressive experimental improvements in 3D displays of human rights connectivity
(highlighting an elusive "inner chanber")

Preliminary test	Refinded test	Rotation of longest link pattern	Cycling through patterns of links



rBeer 30 syntegrity and paper tensegrity paper ? ***

Patterns of connectivity of keystone and indicator species in ecosystems

Question: Keystone species and indicator species are necessarily vital to the sustainability of an ecosystem. Do you have any trace of efforts to provide lists of such species (or a more general grouping), whether for a specific ecosystem or more generally. It could then be argued that the links between such species merit exploration as a system which might well be mapped onto a polyhedron -- as has been attempted in this exchange with human rights, SDGs and koans.

 # Show/Hide AI response #

Question: You have given indicative examples but it would seem important to trace systematic lists of such species

 # Show/Hide AI response #

Question: From a systemic perspective, the questions and answers above should be applicable to psychosocial

systems. Is there any trace of recognition keystone "roles" or indicator modes of cognition which would merit the mapping envisaged

 # Show/Hide AI response #

Question: Could your response have been further informed by reference to the extensive studies of systems of psychological types (Myers-Briggs, etc). This would frame the question as to how each of the psychological types might function as a keystone species in a psychosocial system -- and how these might be fruitfully mapped in 3D, rather than otherwise.

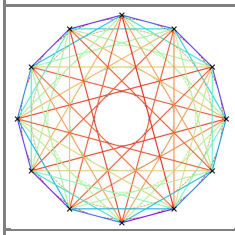
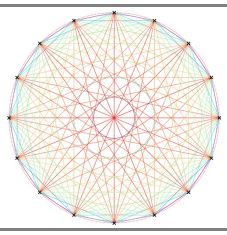
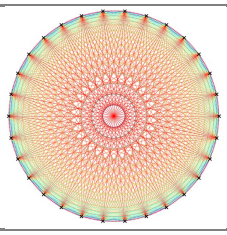
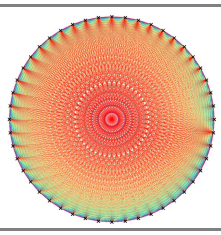
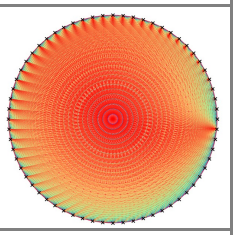
 # Show/Hide AI response #

Everything connected to everything -- framing substructures?

The difficulty posed by the complex articulations presented was how to detect and represent any substructures framed at the centre by the intersections between the internal links -- and indeed how to comprehend what they might imply cognitively.

Question: In the light of what we have attempted in 3D, I would like to provide a visual contrast through representation of the problem in 2D -- whether as a diagram you could evoke or as an X3D model I could manipulate. In 2D the problem could be stated in terms of a circle of say 12 nodes. The diagram would provide links between all of them (with the exception of the antipodal links). The links would be distinctively colour coded by length. The result would be the visual appearance of a central circle to which the longest links would be tangential. Other such circles of different diameter might be suggested by the links of different colours. Is this feasible? Is the task adequately defined? Would X3D be less problematic?

 # Show/Hide AI response #

12	16	30	48	64
				

The 12-fold case, traditionally recognized, features in many list-formatted articulations (*Checklist of 12-fold Principles, Plans, Symbols and Concepts*, 2011). These invite further reflection, as discussed separately (*Clarifying the Unexplored Dynamics of 12-fold Round tables*, 2019; *Enabling a 12-fold Pattern of Systemic Dialogue for Governance*, 2011; *Map of Systemic Interdependencies None Dares Name: 12-fold challenge of global life and death*, 2011; *Eliciting a 12-fold Pattern of Generic Operational Insights*, 2011).

The problematic implications are all the greater for the 16-fold and 30-fold case, as partially explored above in the case of the 16 Sustainable Development Goals and the 30 articles of the Univeral Declaration of Human Rights. Hence the challenge of the 48-fold set of koans.

Question: Your explanations are valuable. My interest is naturally in the extent to which these are valid for the models we explored in 3D. Had we avoided the OFF file data and merely generated nodes appropriately spaced on a sphere, would we have had less difficulty -- or the same

 # Show/Hide AI response #

Question: In the 2D images you generated you added concentric circles to which the various link types were tangential. Could the intersection challenge be avoided by generating concentric semi-transparent spheres -- but how would the diameter be determined

 # Show/Hide AI response #

Question: Much appreciated. It suggests to me that I could usefully repeat such contrasts with respect to the 3D models with which we have variously struggled. Can you use that routine to repeat the exercise for 16 nodes, 30 nodes, 48 nodes and 64 nodes. I note in the 64 model that the pattern on the right and below differs from that on the left and above. Do you have a comment

 # Show/Hide AI response #

Review of procedural challenges for future reference

The exchanges with the two AIs indicated several unfortunate contrasting constraints in practice -- whether common to both or specific. Both responded proactively to the challenge of producing scripts (whether to be run locally or not) in order to generate a 3D model -- scripts far beyond the competence of the author. Both were very helpful in clarifying the task envisaged and in moving on to more viable possibilities.

Contrasting difficulties were evident in the manner that both AIs constrained usage for understandable marketing and resource management reasons -- even for a subscribing user. Claude used time limitations for each session, partially governed by the quantity of information exchangeD. The constraint lay in the limitation on the length and number of iterations of a lengthy script before the session was forcibly paused for several hours. ChatGPT was far more generous in that respect, but frequently "reset" the interaction by deleting any data files essential to the task -- such that they had to be repeatedly uploaded. Session termination under some conditions necessitated reframing the task from scratch since no record of past interactions was retained. There would seem to be a prevailing assumption that an exchange with AI is typically brief with little need to extend the number of a possibly extensive number of interactions in order to resolve an issue.

Most remarkable was the apparent inability to learn from errors indicated in the interaction. The same errors were repeated in the development of scripts and in the interpretation of imported data files. This was most obvious in the processing of the OFF coordinate files for polyhedra provided from Stella4D (for which preliminary comment lines had to be ignored, together with appended colour information). Also somewhat curious were the minor formatting errors in the X3D files variously generated.

The task was finally defined in the following geometrical terms. Given a semi-regular polyhedron characterized by N nodes. Could all the internal links between the non-antipodal nodes be identified and rendered in 3D. Having achieved this, could the coordinates of intersections between them be identified and represented by distinctive nodes in 3D -- focusing specifically on intersections involving 3 or more links. Given those coordinates, could an internal polyhedron be fitted to that set of vertices. Curiously, although both AIs readily analyzed polyhedra in terms of numbers of internal links between nodes, a degree of uncertainity remained regarding the results and what might have been erroneously omitted or overlooked in the scripts developed for that purpose..

The exchanges with AIs passed unsuccessfully through the detection and representation in 3D of internal triangles and squares, as well as through dynamic displacement towards the centre of surface polygonal configurations of vertices -- again unsuccessfully. What was achieved in the very lengthy iterative testing process was indeed visually unusual. Their potential value lies in the extent to which they represent the variety of ways in which coherence can be misunderstood -- or inappropriately represented in practice. This highlights the challenges of communicating with AI -- from text and with respect to structured configurations in 3D. AIs do nt as yet "think in 3D" -- despite a capacity to develop and represent structures in 3D.

ChatGPT immediately scoped out a programming procedure to analyze and generate results in 3D. Claude produced complete Python scripts, enabling multiple rounds of testing and refinement. The constraint lay in the need to add many lines of code to complete the generation based on the example provided -- a tedious

procedure prone to error.

Question: Could we take a step back and consider the difficulties with the current procedure which may or may not converge on a solution. I am surprised that so many issues are revisited without any memory of their previous resolution. You would presulably agree that the method is not sufficiently stable to be applied to other polyhedra -- despite confidence that it can.

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Question: My difficulty is that I need to achieve closure with a summary of successes and constraints and the problematic generic possibilities. It is of course the case that a stable generic solution could be achieved after an unknown number of iterations. However the difficulty is that the resultant analysis, and associated learning, would not be usefully retained for later application.

 # Show/Hide AI response #

Question: A structured report would indeed be useful, especially if it can be fed back to you at some later stage

 # Show/Hide AI response #

Question: The failure to develop a method of identifying symmetrical inner structures systematically could be approached otherwise by exploring the distinction from an insphere. Ths is conventionally (but with some ambiguity) understood as tangential to the faces of a polyhedron (in contrast to a circumsphere). In the cases above the focus was however on the possibility of a succession of structures which might be recognized as distinguished by being tangential to inner links of the same length between vertices. The tangents would then be to concentric spheres of which the smallest diameter would be associated with the longest links. Intersections of such links at the point of tangency would then be vertices of an implied substructure. For future reference, could you comment on this articulation and the possibility of determining the number of such inner spheres in various cases as a basis for determining the number of points of tangency for each such sphere.

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With the focus of this exploration on eliciting underlying subtle insights from complex arrays of concepts, the relevance of concentric substructures calls for particular comment in the light of progressive degrees of their comprehension. This progression is commonly framed in terms of initiation and rebirth for which there are many schemes (*Varieties of Rebirth: distinguishing ways of being born again*, 2004). Whether the focus is on arrays of koans, human rights, or strategies, the question is what is comprehended at each such stage of such progression -- and how is deeper insight to be distinguished from the superficial.

Question: Any set of concentric inner spheres highlighted by this approach provides a focus to the cognitive challenge of systemic comprehension. The longest internal lengths can be understood as conceptuall the most incommensurable -- the longest "distance apart". In forming the the structure embedded most deeply, the "inner chamber" they form can be understood as implying a succession of "cognitive initiations" into comprehension of the system from its external polyhedral mapping. This is ironically consistent with allusions to a "sanctum sanctorum" -- accessible only to the cognoscenti -- appropriately a context which is typically nameless and beyond conventional description.

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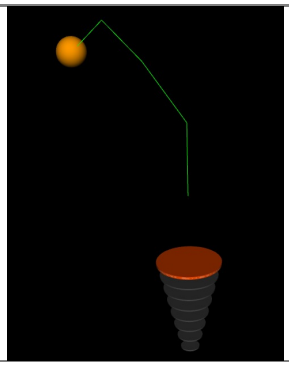
A striking implication of the images -- and the animations successfully produced in 3D -- was the aesthetic consideration whereby they could be rendered more meaningfully attractive (with insight from design professionals). The X3D models by which they were generated are highly adaptable to design preferences for

colours, background, rates of movement, degrees of transparency, labelling, and other types of enhancement. These can all be rendered interactive in the web versions via the X3DOM protocol -- although potentially subject to technical constraints. Relatively little effort has been made to enhance the animations in this preliminary investigation.

Polyhedral cognitive framing of ball dynamics in goal scoring and targetting

Question: Could you assist in clarifying the possibility of a visual illustration in 3D of the cognitive engagement in scoring a "goal" as exemplified in basketball. A ball invites representation in spherical polyhedral terms (as with the truncated icosahedron of football); a circular net could be described as an open configuration of polygons. There would seem to be a sense in which insertion of the ball into the net achieves a degree of "conformation" and enclosure which could be expressed in mathematical terms as pattern matching -- but which has cognitive implications in evoking a sense of supreme satisfaction.

 # Show/Hide AI response #

Preliminary schematic visualizations by AI	
ChatGPT 4o	**Claude 3.5**
	
Generated via DALL-E	X3D Generated via a Python script

Question: I would like to take the visualization possibilities further -- even with the use of dynamics in X3D. If the "net" were to be represented by a truncated cuboctahedron (for example) with an octahedral side "open" or transparent (on the upper side), the ball could be represented by a smaller truncated polyhedron moving into the centre of the larger framing network. This simpler configuration could anticipate a more complex one in which the 8 "sides" of the open octagonal side bend outward to receive the ball -- as with opening petals. The motion is similar to unfolding net animations of polyhedra. Could you clarify these options, prior to any eventual implementation

 # Show/Hide AI response #

Question: Use of the truncated cuboctahedron follows from earlier exchanges which focused on detection of an inner central structure framed by the intersection of the longest non-antipodal internal links between the external vertices of that polyhedron. There remains some uncertainty as to the configuration of that inner polyhedron -- especially whether it necessarily echoes the outer form as a truncated cuboctahedron or takes the more spherical form of an 18-vertex 2-frequency octahedral geodesic sphere (as indicated above). As to the dynamic, the initial position of the "ball" could be outside the "net", but at the same level as its final inner position within it, namely requiring a curved movement to be able to enter the opening of the "net" -- emulating the challenge for a player of skillfully aiming in order to score by getting the ball into the net. Clearly there are various possibilities as to how "open" the outward bending polyhedral "petals" could be -- as an indication of the difficulty of the task. The dynamic in X3D could of course be reversed as an indication of

how the "ball" as an indication of insight might emerge from the framing polyhedron, possibly in a cycle through which the petals open to enable that "externalization". Timing could of course be adjusted according to aesthetic criteria.

 # Show/Hide AI response #

Question: The X3D possibiities are now well clarified. More specifically would it be easier to work with relevant OFF files from Stella4D -- as you have done before. Or given the challenge to which their interpretation has given rise, would it be more practical to generate the polyhedra from scratch. My preference is that cylinders are used to specify edges, given the ease with which their properties can be modified.

 # Show/Hide AI response #

Question: In antitipation of an insightful animation in 3D, could you comment further on how it can be understood as a cognitive metaphor of relevance to achieving a strategic goal. Of some interest in that regard are psychosocial situations in which neither the "net" nor the "ball" are as neatly defined as clarified above. One or the other could be understood as less coherent and well-integrated -- with the problems that would imply for achieving the goal. The skills required for scoring might well be lacking. The net might be too high or the ball too large. However it is clearly the deep cognitive satisfaction of scoring -- or observing others doing so -- that merits far deeper understanding, especially in the light of its sexual connotations

 # Show/Hide AI response #

Question: Given the considerable importance attached to "targets", "targetting" and "target acquisition", whether in collective or individual strategies, could you comment on how those understandings can be related to the cognitive implications of "scoring" -- especially when no clear targets have been identified

 # Show/Hide AI response #

Question: Clarification of the animation possibilities above also noted the potential "petal" dynamics. Fundamental symbolic importance is associated with the opening and closing of petals in Eastern traditions (as with the lotus flower). Could you comment further on the psychosocial implications of reversal of the cycle of "scoring" -- in which the "ball" emerges from the "net" (possibly understood as Indra's Net) -- potentially indicative of creativity, rebirth and Renaissance.

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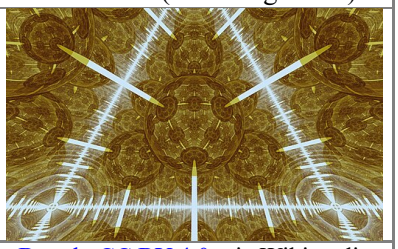
Fundamental symbols of Eastern tradition	
Lotus flower	Indra's Net (rendering in 3D)
	
T.Voekler, CC BY-SA 3.0, via Wikimedia Commons	Bunyk, CC BY 4.0, via Wikimedia Commons

Of potential relevance to this argument is a geometry book (David Mumford, Caroline Series and David Wright***,*** *Indra's Pearls: The Vision of Felix Klein*, Cambridge University Press,2002***).*** The book explores the

patterns created by iterating conformal maps of the complex plane called Möbius transformations, and their connections with symmetry and self-similarity.

Question: Given both the symbolic implications and the focus provided by the goal of placing a ball in the net of an opponent, there would seem to be a case for recognizing how a disassociated -- even problematic -- surrogate for that process is provided by other forms of engaging with a ball and its implications of globality. How is the focus on hitting or kicking a ball (potentially with polyhedral stitching, and possibly over a net and with an opponent), then to be understood. How questionable is the cognitive entanglement of competitive achievement and play.

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References

Robert Aitken (Ed.). The Gateless Barrier: The Wu-Men Kuan (Mumonkan). North Point Press, 1990 [text]

Ronald Atkin:

- The Methodology of Q-Analysis Applied to Social Systems. *Systems Methodology in Social Science Research*, 2, 1982, pp 45-74 [text]
- Combinatorial Connectivities in Social Systems: an application of simplicial complex structures to the study of large organizations. Birkhäuser, 1980

R. H. Blyth (Tr.). The Gateless Gate (TGG). Terebess Collection (Mark T. Morse) [text]

R. Buckminster Fuller (in collaboration with E.J. Applewhite). Synergetics: Explorations in the Geometry of Thinking. Macmillan, 1975/1979

Susantha Goonatilake:

- Toward a Global Science: Mining Civilizational Knowledge. Indiana University Press 1999
- Non-Western Science: mining civilizational knowledge. *Encyclopedia of Life Support Systems* (EOLSS) [text]

Lama Anagarika Govinda. The Inner Structure of the I Ching: the book of transformations. Weatherill, 1981

George Lakoff and Rafael Núñez. Where Mathematics Comes From: how the embodied mind brings mathematics into being. Basic Books, 2000 [summary]

Taneli Luotoniemi. Crooked Houses: Visualizing the Polychora with Hyperbolic Patchwork. Bridges 2017 Conference Proceedings

Wikipedia. Four-dimensional geometry: Tesseract, 3-sphere, N-sphere, Polychoron, "And He Built a Crooked House", Uniform polychoron, Four-dimensional space Capa comum Wiki Series, 2011 2011 [summary]
