24 October 2022 | Draft

Eliciting Potential Patterns of Governance from 16 Sustainable Development Goals

Interactive exploration of Goal 17 through a polyhedral compound of 16 tetrahedra in 3D

-- / --

Introduction
Enabling pattern detection in a 16-tetrahedral compound
Detection of spheres configured to form tetrahedra
Pathway challenge between multiple tetrahedra
Detection of cylinders configured to form tetrahedra
Reconciliation of distinguished sphere and cylinder colours with face colours
Insights from polyhedral pattern dynamics through "playfulness"?
Learnings in the pattern detection process
Potential significance of fourfold patterning of SDGs
Tetrahedral principles basic to geometry of thinking
Quaternity, quaternions and the unifying goal of partnership?
Requisite complementarity of forms of representation of a unifying goal?
Insights into partnership from tetrahedral representation of SDGs in 4D?
17th Goal and the memorable representation of 16?
References

Introduction

The 17 Sustainable Development Goals formulated by the United Nations can be upheld as the current culmination of reflection on coherent global governance. Whether this is the consequence of intelligent design in systemic terms, rather than merely an arbitrary outcome of political horse-trading, can however be challenged (*Systemic Coherence of the UN's 17 SDGs as a Global Dream*, 2021; Femi Asu, *SDGs: the UN dreams big -- you should too! Your Commonwealth: Youth Voices*, 29 September 2015).

Whether the set of goals is indeed systemically coherent in a recognizable manner, it could also be understood as implying a form of coherence emerging from the collective unconscious. The organization of the set, and the 169 tasks associated with them, therefore invites continuing consideration of how that degree of complexity can be comprehended and rendered memorable. The nature of that challenge has been explored separately (*Cognitive Embodiment of Patterns of Governance of Higher Order*, 2022). As engendered by a set of values, their nature and organization also merits consideration (*Values, Virtues and Sins of a Viable Democratic Civilization*, 2022).

Such previous considerations have focused on a possible 16-fold organization of which the 17th Goal is the coordinating function or perspective -- however that is itself to be understood. This suggests the possibility of representing the configuration of goals in diagrammatic or geometrical form in order to highlight their potential relationships and the patterns they form that are integral to the systemic coherence of the set. To this end SDG iconography has tended to focus on a variety of circular diagrams and tabular arrangements. Understood in that way, any exploration of how the 16 goals might be more fruitfully configured -- as in the following -- can then be recognized as a pursuit of the 17th Goal: Partnership for the goals.

The following argument is however based on the assumption that the current SDG iconography in 2D is inadequate to the challenge of articulating the recognized complexity -- given the questionable performance of global governance. This justifies recourse to eliciting a memorable sense of coherence from representation in 3D -- if not 4D or more. Previous exercises to this end have focused on use of a cubic configuration inviting the kinds of pattern exploration associated with Rubik's Cube (*Interplay of Sustainable Development Goals through Rubik Cube Variations*, 2017). Another approach explored the potential of a 16-fold toroidal configuration using the so-called "simplest torus" (*Framing an operating context of 16 "dimensions"*, 2019; *Functional dynamics of a 16-fold configuration of strategic goals*, 2019). Combining the two in geometrical terms, another exercise made use of the toroidal drilled truncated cube (*Implicate order* *through hypercube and drilled truncated cube?* 2022; *Polyhedral representation of Sustainable Development Goals including "Own Goals"?* 2022).

The following exercise explores a compound of 16 tetrahedra, one of a number of such polyhedral compounds composed of simpler polyhedra sharing a common centre (a perspective usefully symbolized by the 17th Goal). Use of the tetrahedron as the simplest Platonic polyhedron derives from an assumption here that the subtle complexity of any "goal" in cognitive and systemic terms merits representation beyond its common indication in 2D using a "point", a "line", or a "field" (as a spatial framing of a topical focus) -- as typically characteristic of systems diagrams. As the minimal systemic configuration in 3D (of 4 points, 6 edges, and 4 faces), the tetrahedron therefore merits particular consideration. As argued extensively by Buckminster Fuller (and discussed below), the tetrahedron, as a basic vectorial model, is the fundamental structural system (*Synergetics: explorations in the geometry of thinking*, 1975).

This raises the question of the systemic patterns that might then be recognizable in a compound of 16 tetrahedra -- as a representation of the set of 16 SDGs as potentially fundamental to the governance of the global system with its psychosocial implications. A tetrahedral understanding of goal is then suggested below in the light of philosophical and psychological insights into quaternity, most notably as a consequence of the work of Carl Jung (*Aion: Researches into the Phenomenology of the Self*, 1969).

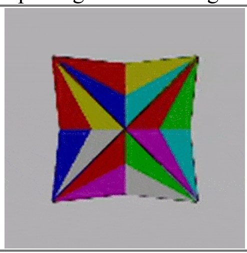
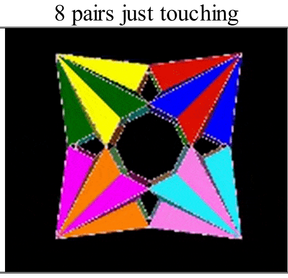
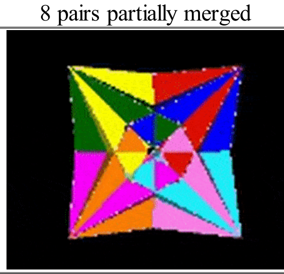
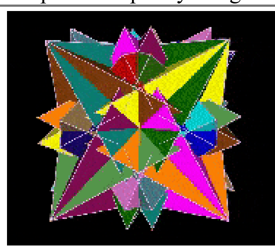
Whilst representations of compounds of polyhedra have long been available as the focus of commentary, there are challenges to eliciting comprehensible patterns from their complexity. The compound of 16 tetrahedra used in this exericse consists of 224 geometrical elements -- potentially of a complexity of the same order as suggested by the 169 SDG tasks. The lengthy process of detecting patterns in that configuration, to enable their memorable representation, is presented here as one of trial and error -- a learning exercise in its own right. This suggests the value of such an exercise in evoking the kinds of engagement evident in the case of Rubik's Cube, or the construction of any mandala (*Eliciting Insight from Mandala-style Logos in 3D*, 2020).

The animations associated with this exercise are presented here as a means of eliciting imaginative ways of "thinking otherwise" about SDGs regarding any coherent response of global governance.

Enabling pattern detection in a 16-tetrahedral compound

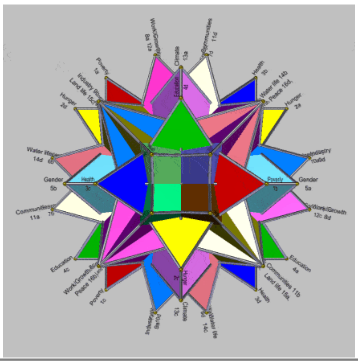
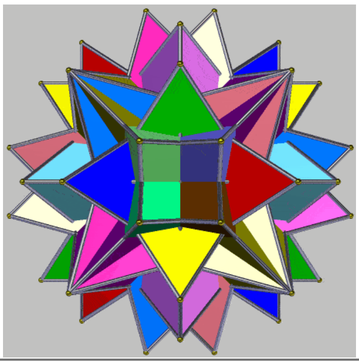
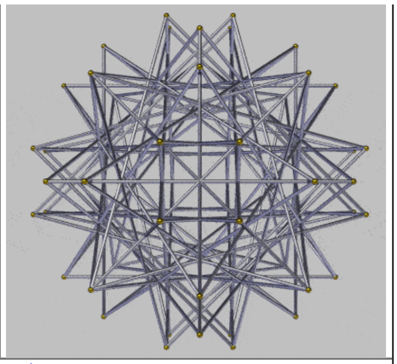
First enumerated in 1976, a uniform polyhedron compound in geometry is a polyhedral compound whose constituents are identical (although possibly enantiomorphous) uniform polyhedra, in an arrangement that is also uniform, i.e. the symmetry group of the compound acts transitively on the compound's vertices. Six compounds of tetrahedra are recognized, none of them being a compound of 16. As noted above, the polyhedra in the compound share a common centre. Variants of such compounds are recognized in 4D, justifying the more general term of polytope compound.

Three approaches appear to have been used to offer a 16-fold configuration of tetrahedra -- considered here as a potential key to the systemic configuration of 16 SDGs. George Hart presents a compound of 16 tetrahedra in virtual reality through rotation of a 2-tetrahedra compound (known as a stella octangula) on each of a cube's 3-fold axes (below left). The Antiprism Polyhedron Modelling Software developed by Adrian Rossiter offers a variety of means of generating and presenting 16-tetrahedral compounds. The "polyhedral kaleidoscope" facility enabled a tetrahedron to be repeated to form a 16-fold symmetric arrangement. As shown below, the 16 are combined into 8 pairs which can be separated or merged to different degrees -- with the result exported into a virtual reality format. The degrees of separation are potentially suggestive of both confusion and integration. The 8-fold pattern features otherwise in the exercise which follows.

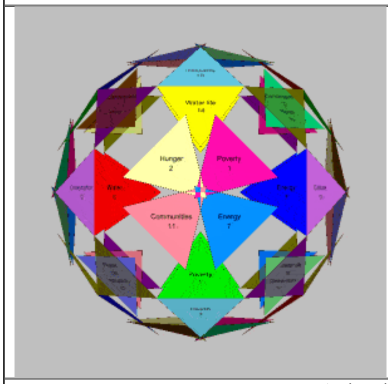
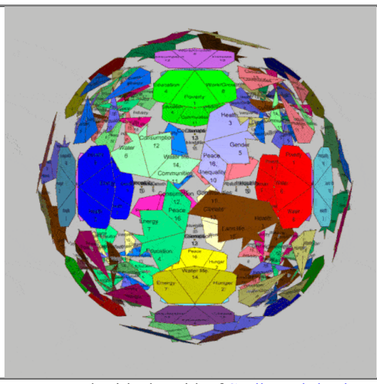
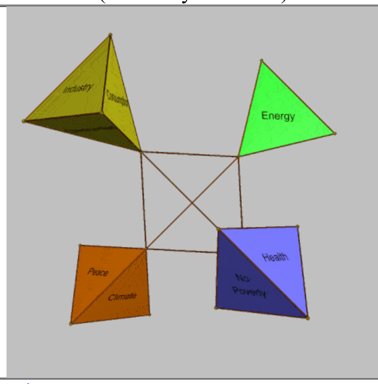
Renderings in 3D of a compound of 16 tetrahedra			
Spinning a stella octangula	8 pairs just touching	8 pairs partially merged	8 pairs completely merged
			
Modification of model of George Hart	Animations prepared with the aid of Antiprism Polyhedron Modelling Software		

The Stella4D application developed by Robert Web combines an 8-tetrahedral compound, with a 6-tetrahedral compound, and a 2-tetrahedral compound to give the compound (shown below). It is this version which is the focus of the following exercise. Without its construction in the Stella4D polyhedral library the following exercise would not have been possible.

Renderings in 3D of a compound of 16 tetrahedra		
As generated by Stella4D (with SDG names added)	Succesively rendering 16 tetra invisible	Stella 4D wireframe rendering

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3">Animations prepared with the aid of Stella Polyhedron Navigator</td></tr>
</table>

The difficulty is that the Stella4D application in which it is presented in 3D makes it difficult to elict and manipulate some patterns which are of potential interest -- despite a variety of ways in which the 16-fold compound can be usefully explored, as shown below.

<table>
<tr><th colspan="3">Indicative transformations of compound of 16 tetrahedra in Stella 4D representing 16 SDGs
(4 faces of a single tetrahedron may engender multiple labels, not distinguished in these examples)</th></tr>
<tr><td>"Exploding" the compound</td><td>Unfolding the compound</td><td>Configuration of 4 tetrahedral configurations of 4 SDGs
(arbitrarily clustered)</td></tr>
<tr><td></td><td></td><td>
</td></tr>
<tr><td colspan="3">Animations prepared with the aid of Stella Polyhedron Navigator</td></tr>
</table>

The 16-tetrahedra compound can however be exported from Stella4D (into VRML format) enabling it to be imported into the more recent X3D format, using the X3D-Edit application. It is within the latter context that the following process was undertaken. There the 224 elements of the model (64 vertices, 64 faces, 96 edges) can then be edited individually as in any simple text file. The result can then be viewed over the web with the aid of a variety of viewers, whether freely available or otherwise. During the exercise, main use was made of H3DViewer and occasional use of FreeWRL. Access to users can then be provided via X3DOM for HTML5 web pages.

The principal difficulty within Stella4D is that although the pattern of 16 tetrahedra can be variously appreciated (as partially indicated above), the conversion into X3D does not offer any indication in the resulting code of which of the 64 vertices (as "spheres") is associated with which of the 96 edges (as "cylinders") -- and how these are related to any one of the 16 tetrahedra in the engendered compound. It is not apparent which elements corresponded to the 8, 6 or 2-tetrahedral componenets of the compound, for example.

Within this context, in the absence of requisite skills in geometry (or the use of other applications), the simplest approach proved to be one of tedious trial and error. This involved successive modifications of the colour of the 64 individual spheres (or the 96 cylinders) -- then viewed in 3D -- in order to cluster the elements relating to particular tetrahedral configurations.

Of some relevance to the process is the capacity in Stella4D (and in X3D viewers) to switch between solid-face and wireframe (transparent-face) renderings of the 16-tetrahedral compound (as shown above).

Detection of spheres configured to form tetrahedra

As noted above, through a process of trial and error, the 64 vertices of the 16-tetrahedral compound, were coloured progressively to distinguish the 16 individual polyhedra in the compound. A particular difficulty of interest in that process is the constraint on recognition of contrasting colours. Whilst it is relatively easy (for some) to distinguish between an 8-fold set of colours, the difficulty is greater with respect to a 16-fold set (as noted below).

The focus on spheres progressively enabled the following to be isolated:

- stella octangula, namely the 2-tetrahedra pattern composed of one tetrahedron (coloured red) and a second (coloured blue). A particular orientation of the model presented these in the form of the well-recognized Star of David in 2D -- although that orientation disguised the fact that both were tetrahedra as viewed from different orientations in 3D. Implications of a 3D variant of the Star of David are discussed separately (*Framing Global Transformation through the Polyhedral Merkabah*, 2017). The latter focuses on the implicit cognitive cycles neglected in viable complex systems.
- isolation of 8 vertices associated with the 2-tetrahedral stella octangula, which appeared to frame the other tetrahedra in an

interesting manner. These were therefore enlarged and presented as flashing from black to white. Of particular relevance, it is these vertices which were of higher valence than others being at the confluence of 6 edges rather than 3.

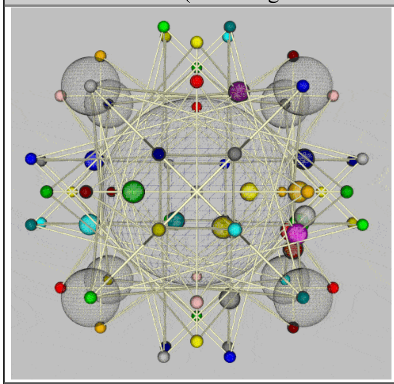
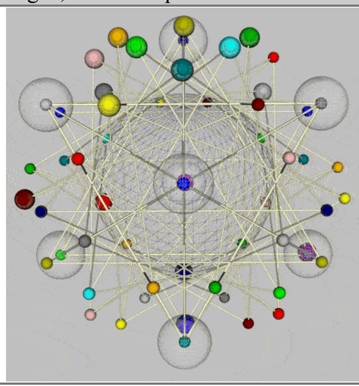
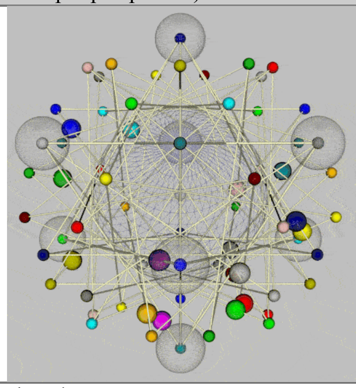
Pathway challenge between multiple tetrahedra

The X3D format allows for the possibility of movement of an element (such as a sphere) between the coordinates of the points defining the locus of the tetrahedral vertices -- namely along the pathway defining the edges of a tetrahedron. The sets of distinctively colour coded spheres enabled the association of a dynamic with each identified tetrahedron.

This facility was used to specify the movement of larger spheres along tetrahedral pathways. However, as discussed below, there is no unique pathway which such a moving sphere can follow around the 6 edges of a tetrahedron. One approach is to have a complementary movement in the opposite direction -- with the movement of each omitting one edge which is traversed by the other.

Using a single direction, it was not immediately possible to achieve a consistent movement for all the tetrahedra. This possibility was temporarily set aside and features in the discussion below.

Alternative views of animated movement of spheres along 16 tetrahedral pathways between 64 vertices
(8 framing vertices are enlarged; a central sphere is included to facilitate depth perspective)

Interactive 3D version in which smooth movement is best viewed

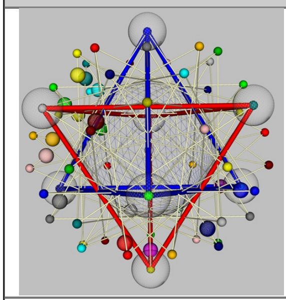
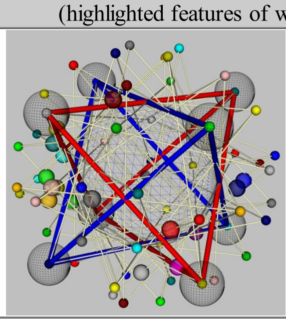
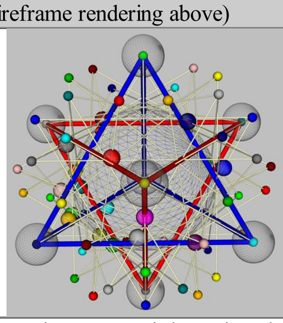
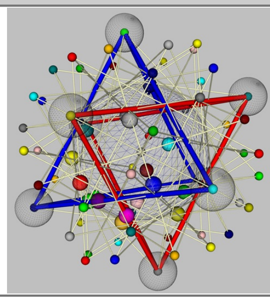
Detection of cylinders configured to form tetrahedra

Attention was then focused on use of the colour coding approach to cluster the 96 cylinders composing the edges of tetrahedra -- indicated by the thin lines in the images above. This process was distinct from the identification above of "pathways" between spheres. Whilst the latter are defined in the geometry of the application by movement, the cylinders enable the edges of the tetrahedra forming those pathways to be distinguished by colour coding.

As noted above, the framing seemingly offered by the 2-tetrahedra stella octangula is of particular interest -- as can be variously viewed.

>

Alternative views of stella octangula configuration of 2 tetrahedra (in a compound of 16)
(highlighted features of wireframe rendering above)

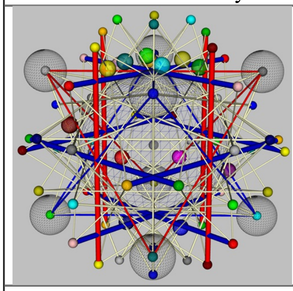
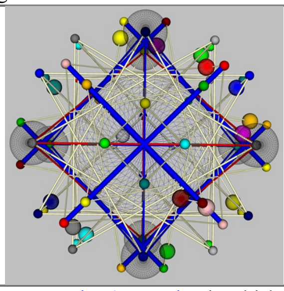
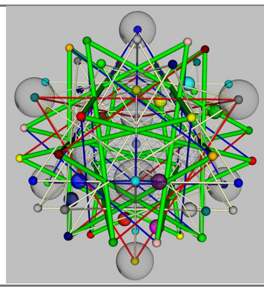
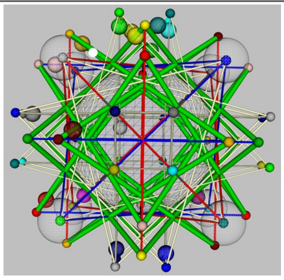
Interactive 3D version in which smooth movement is best viewed

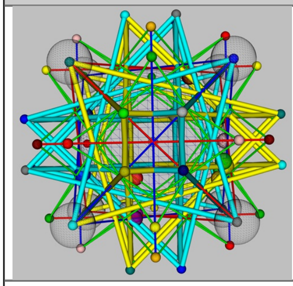
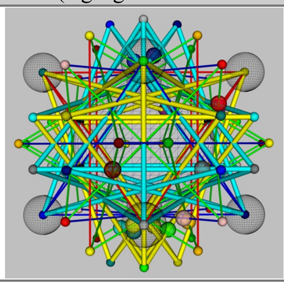
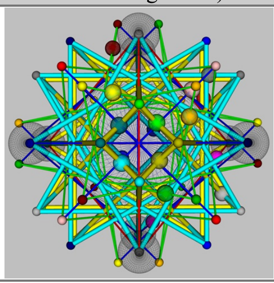
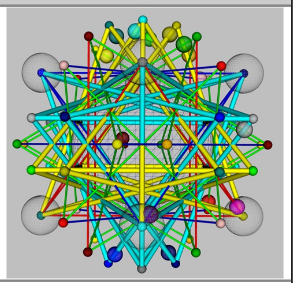
Additional patterns detected included the following (as shown below):

- clustering of mutually orthogonal elements of detected tetrahedra
  - a set of 4 (coded red and enlarged) readily viewed as vertical
  - a set of 6 (coded blue and enlarged) readily viewed as horizontal
- this left a set of "angled" cylinders (coded green and enlarged) from which the following complementary sets were isolated
  - a set of 4 (coded yellow and enlarged)
  - a set of 4 (coded cyan and enlarged)

  although the obvious possibility of subdividing these sets of 4 into 2 groups of 2 was considered but avoided
- this left a remaining set of "angled" cylinders (coded green and enlarged) which appeared to be completed (to form tetrahedra) by those in the mutually orthogonal sets (indicated above in red and blue)

Alternative views of selected patterns in a compound of 16 tetrahedra

<table>
<tr><td colspan="4">(highlighted features of wireframe rendering above)</td></tr>
<tr><td colspan="2">Mutually orthogonal elements</td><td colspan="2">Connecting elements</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Interactive 3D version in which smooth movement is best viewed</td></tr>
</table>

<table>
<tr><td colspan="4">Alternative views of selected patterns in a compound of 16 tetrahedra
(highlighted features of wireframe rendering above)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Interactive 3D version in which smooth movement is best viewed</td></tr>
</table>

Reconciliation of distinguished sphere and cylinder colours with face colours

The trial and error process above was primarily conducted using the wireframe rendering. There was therefore a need to reconcile the results with the colour distinctions for the faces of the 16 polyhedra in the compound -- the face colours being distinctive for each tetrahedron (as exported from Stella4D). The process of colour coding was complicated by the fact that, depending on how the model was oriented in 3D, the shading of the colours blurred the distinctions between some colours.

As noted above, it is relatively easy (for some) to distinguish between an 8-fold set of colours, the difficulty is greater with respect to a 16-fold set. In a web environment particular colours may well be labelled "web safe" in contrast with others rendered variously by different browsers. The 8-fold set of the RGB convention is defined in terms of a ternary bit pattern between 000 and 111.

Of potential relevance to the distinction of 16 SDGs is that the basic web protocol recognizes 16 named colours -- in contrast with a vast array of colours identified by code combinations according to a variety of conventions. Using the ternary RGB colour convention, this extends the range by a further 8 using the range between 0.5 0.5 0.5 and 1 1 1. HTML used to recognize 16 color names ("black", "white", "gray", "silver", "maroon", "red", "purple", "fuchsia", "green", "lime", "olive", "yellow", "navy", "blue", "teal", and "aqua"), but new browsers can recognize 147 CSS3 color names.

Does the standard STD iconography use those 16 named colours -- deemed "safe"? If not, why not, and are these appropriately distinguished in the range of world cultures -- aside from the contrasting significance associated with them (Jeremy Girard, *Visual Color Symbolism Chart by Culture: what different colors mean in different cultures*, *ThoughtCo*, 25 September 2019; Aina Casaponsa, et al, *The way you see colour depends on what language you speak*, *The Conversation*, 16 April 2018; *Different cultures see different colours*, University of Melbourne, 2 October 2017).

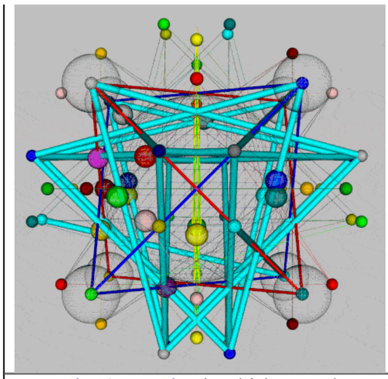
To clarify the reconciliation with the face colours of the 16-tetrahedral compound, it was decided to use the array of 16 named HTML colours, rather than the set exported from Stella4D (although many were necessarily common). However, for aesthetic reasons, substitutes for the colours of white and black were used. The sphere and cylinder colours were then modified to correspond to the face colours of the tetrahedra they framed.

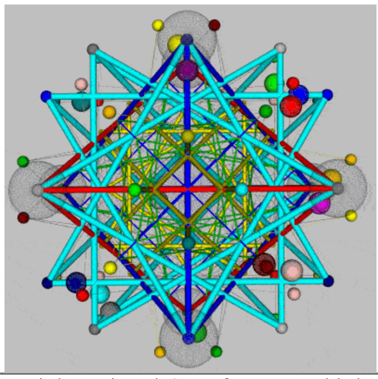
Insights from polyhedral pattern dynamics through "playfulness"?

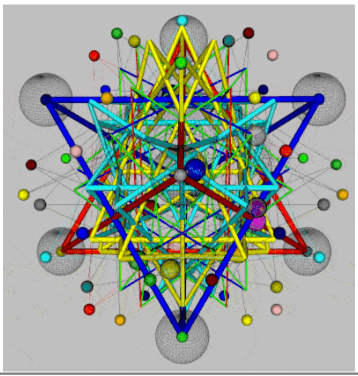
Once a degree of control is achieved over most of the elements composing the 16-tetrahedral compound, web technology enables the complex to be explored experimentally with a degree of interactive playfulness. The engagement follows in part from that with respect to Rubik's Cube and puzzle solving in general (*Envisaging a Comprehensible Global Brain -- as a Playful Organ*, 2019; *Humour and Play-Fullness: essential integrative processes in governance, religion and transdisciplinarity*, 2005).

The screen shots below (of the separately accessible 3D animations) suggest ways in which patterns of interaction between individual SDGs may change dynamically -- namely whether their significance in relation to others is "shrunk" or "englarged". The animations also offer a sense of how the role of particular patterns of systems communication may alternate over time, perhaps according to shifting policies and contextual circumstances.

<table>
<tr><td colspan="3">Animations of screen shots of alternative views of pathway dynamics of 16-tetrahedra complex</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>







Interactive 3D version in which smooth movement is best viewed. Some features enabled with the guidance of Sergey Bederov of Cortona 3D.

Although not explored at this stage, there is every possibility of enlarging or shrinking particular parts of a pattern -- or rendering them effectively invisible. This could be done in the light of the volume of communication traffic between organizational units responsible for implementation of particular goals. With further integration of other technologies, this could be done as a real time animation through which the "health" of a complex system could be monitored.

A question raised by such animation facilities is to how much complexity of the SDG pattern of goals a particular user wishes to be exposed. How is the capacity to "make sense" of complexity challenged? What degrees of complexity are vital to sustainable governance and how should they be represented, to whom, and under what circumsntances? In the case of interactive presentations in 3D, how many distinctive viewpoints should be offered to users, and what other parameters should the user be able to control -- colour, rates of movement, size, etc? When and where should labels be associated with particular features?

The further question is how the elements of such animations can be "translated" into communications between organizational units responsible for implementation. Clearly animations could be augmented to reflect the volume and frequency of such communications. All such questions, together with the current constraints of web technology (and the level of skills in their use), merit consideration in any reflection on the communication of systemic insights and their coherence.

Learnings in the pattern detection process

As indicated above, the point of departure was a very precise set of data defining a complex structure which could be visualized in interesting ways within an existing virtual reality software application. Although accessible in a mode amendable by a conventional text editor application, unfortunately the technical specification of the 160 elements defining the structure (64 spheres and 96 cylinders) -- could not be readily related to what could be so clearly visualized. The puzzle was how to interrelate the elements to enable the patterns within the structure to become evident -- whatever they might be. Aspects of the process could be compared to (re)solving a jigsaw puzzle.

Eliciting the underlying patterns suggested parallels with global governance as puzzle solving (*Global Governance as a Riddle*, 2018; *Cross-system strategic "puzzles": enabling a different form of connectivity*, 2008), and hence the reference above to engagement with Rubik's Cube (*Roman dodecahedron, Chinese puzzle balls and Rubik's Cube?* 2018).

Approaching the challenge with a relative level of geometrical ignorance, the question was the nature of the patterns which would become apparent as the exercise developed. How many distinctive patterns within a complex structure could be detected and highlighted as memorable. The technical constraints and possibilities of virtual reality modelling had to be addressed to discover means of highlighting such patterns and rendering them memorable. Were separate images and animations necessary -- or could they be integrated within a single interactive model?

Clearly as an exercise in design, highlighting patterns involves aesthetic considerations and preferences. These include relative sizes, use of distinctive colours, rates of movement, relative transparency, and shifts in perspective. These are difficult to handle statically in 2D, although simple web animations offer a wider range of possibilities -- despite recording constraints (frame rates, browser constraints, etc). Fortunately web technology increasingly enables interactive engagement with models in 3D via X3DOM -- as offered in particular cases above.

Less evident are the relative advantages and disadvantages -- and for whom -- of static 2D imagery versus dynamically interactive 3D displays. Despite their advantages for the print media, to what extent do screen shots detract from the comprehensibility and memorability of interactive animations?

As a key to comprehending the global complex of 16 SDGs -- potentially the essence of the 17th goal -- recourse to a 16-fold compound of tetrahedra could be challenged. Framed otherwise however, **the challenge could be understood as one of packing memorability of multiple patterns of relevance into a singularly complex structure**. This could be compared to the challenge of mandala creation as cultivated in some cultures and traditions.

It is intriguing to note the advantages and disadvantages of removing one pattern in order to make another evident. This frames the question of how to shift perspective between patterns -- and to facilitate that in an interactive application. Beyond the interactivity it enables, web 3D technology provides for shifts between "viewpoints" offering perspectives on patterns of interest. One issue is how to relate these for a user. Especially intriguing is the possibility of offering perspectives from within a model -- even its centre point -- or from parts of it (as from the 8 framing vertices of the 16-tetrahedral compound).

Insight into the elements defining the structure progressively offered recognition that the crude *ad hoc* detection process could potentially be replaced or complemented by greater mathematical understanding of how they are created.

As evident in other applications using AI, software could be developed to detect and highlight patterns -- and identify interesting viewpoints. This would be especially relevant to their detection in structures in 4D or more. Clearly the complex of SDGs might benefit from exploration as a 4D or 5D structure -- as suggested in the case of NATO (*Envisaging NATO Otherwise -- in 3D and 4D?* 2017). This explored the potentially hidden faces of global strategy highlighted through polyhedra implied by the NATO logo.

Potential significance of fourfold patterning of SDGs

The current bias in expert information presentation for governance contrasts with the early argument in 1968 of the political scientist Harold Lasswell (*The transition toward more sophisticated procedures*, 1968):

> Why do we put so much emphasis on audio-visual means of portraying goal, trend, condition, projection, and alternative? Partly because so many valuable participants in decision-making have dramatizing imaginations... They are not enamoured of numbers or of analytic abstractions. They are at their best in deliberations that encourage contextuality by a varied repertory of means, and where an immediate sense of time, space, and figure is retained. (In: Davis B. Bobrow and J.L. Schinartz (Ed.). *Computers and the Policy-making Community; applications to international relations*. Prentice-Hall, 1968, p. 307-314)

Communication of goal coherence: Experiments such as those above frame the question as to how communications in support of the SDG goals can be enabled and enhanced. What patterns of communication are of value to the integrity of global governance? The question is discussed separately (*Towards a higher order of coherent global strategic organization?* and *Towards a geometry of systemic thinking and its symbolism*) within a more general context (*Time for Provocative Mnemonic Aids to Systemic Connectivity?* 2022; *Memorability, Mnemonics, Maths, Music and Governance*, 2022).

Of some interest, as noted above, is the significance associated with mandala construction in some cultures and traditions (*The Symbolism Behind the Creation and Destruction of the Buddhist Sand Mandala*, *Buddhists.org*, 21 August 2015; Lynn Lum, *Mandala Construction at Longmont Buddhist Temple*, *Harvard Pluralism Project Archive*, 5 July 2007). Such processes in 2D frame the questions as to "why 3D" and "why polyhedra"?

Neuroscience insight: One justification is suggested by discoveries of the neuroscientists of the Blue Brain Project. This indicates the remarkable possibility of cognitive processes taking even up to 11-dimensional form in the light of emergent neuronal connectivity in the human brain:

> Using mathematics in a novel way in neuroscience, the Blue Brain Project shows that the brain operates on many dimensions, not just the three dimensions that we are accustomed to. For most people, it is a stretch of the imagination to understand the world in four dimensions but a new study has discovered structures in the brain with up to eleven dimensions - ground-breaking work that is beginning to reveal the brain's deepest architectural secrets..... these structures arise when a group of neurons forms a clique: each neuron connects to every other neuron in the group in a very specific way that generates a precise geometric object. The more neurons there are in a clique, the higher the dimension of the geometric object. ...
>
> The appearance of high-dimensional cavities when the brain is processing information means that the neurons in the network react to stimuli in an extremely organized manner. It is as if the brain reacts to a stimulus by building then razing a tower of multi-dimensional blocks, starting with rods (1D), then planks (2D), then cubes (3D), and then more complex geometries with 4D, 5D, etc. The progression of activity through the brain resembles a multi-dimensional sandcastle that materializes out of the sand and then disintegrates. (*Blue Brain Team Discovers a Multi-Dimensional Universe in Brain Networks*, *Frontiers Communications in Neuroscience*, 12 June 2017)

Such discoveries suggest the need for a form of cognitive resonance between external and internal representations of patterns of significance to global governance. The question of "why tetrahedra", with the emphasis on 4-foldness, calls for particular clarification as indicated below from a systemic perspective.

Quaternity from a depth psychology perspective: From a contrasting perspective, the fundamental importance of fourfoldness is extensively discussed by Carl Jung (*Aion: Researches into the Phenomenology of the Self*, 1969) and by associates (Marie-Louise von Franz, *Lectures on Jung's Aion*, 2013). Jung offers a highly influential distinction between four complementary functions of consciousness, expanded into eight psychological types. He highlights the effects of tensions between the complexes associated with the dominant and inferior differentiating functions in highly one-sided types.

Jung notes a series of quaternity images from Gnostic texts: *Anthropos Quaternio*, *Shadow Quaternio*, *Paradise Quaternio* (involving the four rivers of the Garden of Eden), and *Lapis Quaternio* (representing the alchemical process whereby the prima materia is split into four elements and then synthesized again into the Rotundum). These four quaternios that Jung elaborated are connected to one another by common images (*Quaternio: Fourfold*, *ARAS: archival research in archetypal symbolism*; *Carl Jung on the Quaternity*, *ARAS*, 20 July 2020).

Other indicative references include:

- David Johnston: *Jung, Philemon and the Fourfold Psyche* (October 2017)
- Maxwell Purrington: *On the Nature of Four: Jung's quaternity, mandalas, the stone and the self* (*Carl Jung Depth Psychology*,

22 November 2019)

- Matthew Fike: *Blake's Fourfold Vision vs. Jung's Visionary Mode* (*Jungian Society for Scholarly Studies*, 2013)
- Aron Monroe Dunlap: *Counting to Four: assessing the Quaternity of C.G. Jung in the Light of Lacan and Sophiology* (Temple University Libraries, 2008)
- Gary Z McGee: *Jung's Four Stages of Character Transformation* (*The New Agora*, 5 January 2022)
- Randall Verarde: *The Four Psychological Functions and the Gospel Drama* (*Jung Journal: Culture and Psyche*, 4, 2010, 1)
- Daniela Boccassini: *At the Roots of Jung's Alchemy Part I: The Red Book's Alchemical Quaternion* (*Jung Journal*, 2022)
- Gerald Sullivan: *A Four-Fold Humanity: Margaret Mead and Psychological Types* (*Journal of the History of the Behavioral Sciences*, 40, 2004, 2)
- Frith Luton: *Quaternity Image as Archetype of Wholeness*
- *The four Functions of the Mind according to Jung* (*Act for Libraries*, 2017)
- *Fourfold Vision and The Tetramorph* (*The Chrysalis*, 5 March 2020)

Tetrahedral principles basic to geometry of thinking

Philosophical logic: Insights from a philosophical perspective have been provided by Arthur Schopenhauer (*On the Fourfold Root of the Principles of Sufficient Reason*, 1913). Research continues on oppositional logic and its geometrical representation in terms of the square of opposition (*7th World Congress on the Square of Opposition*, Leuven, September 2022).

Disparate framings of fourfold integration: Seemingly unrelated preoccupations are of potential relevance to this argument, as previously noted (*Embodiment of Identity in Conscious Creativity: challenge of encompassing "con"*, 2011):

- Arthur M. Young: as designer of the Bell helicopter (and therefore sensitised to complex control issues), highlights the fundamental importance of the four types of action in a learning cycle, including all possible permutations of the relations between knower and object (*The Geometry of Meaning*, 1976). Young makes use of the term quadruplicity. These insights have been adapted, as separately discussed (*Typology of 12 complementary strategies essential to sustainable development*, 1998; *Characteristics of phases in 12-phase learning / action cycles*, 1998). Of interest in relation to understanding of the movement of a free agent, Young points out that a minimum of six observations are required to determine any behaviour. This insight is presumably of relevance to the current issue of appropriate indicators for the operation of any global reserve currency system.

- Edward Haskell: offers a generalization of insights to be derived from the pattern of the periodic table of chemical elements (*Full Circle: the moral force of unified science*, 1972). Of particular relevance is the interaction between controller and controllee in any system, as represented by quadrants and diagonals highlighting the emergence of a coaction cardioid, as discussed separately (*Cardioid Attractor Fundamental to Sustainability: 8 transactional games forming the heart of sustainable relationship*, 2005). Haskell's insights have been notably developed by Timothy Wilken (*The Relationship Continuum*, 2002).

- Ken Wilber offers a synthesis expressed in a 4-fold AQAL system, widely explored through the Integral Movement.

- Through his work on the systematics of J. G. Bennett, Anthony Blake has given focus to the tetrad as one archetypal system. This features more generally in the methodology of *LogoVisual Thinking* (Anthony Blake and John Varney, *LogoVisual Thinking: a guide to making sense*, Centre for Management Creativity, 2004; Brin Best and Anthony Blake, *Making Meaning: Learning through logovisual thinking*, 2005).

- As a pioneer of peace research, Johan Galtung variously argues for eliciting a fourfold pattern from the typical dilemma associated with any conflict of goals:

    - In a simple conflict between two goals -- dilemma when inside a party, a dispute when between parties -- we get a fourfold-table, the Buddhist tetralemma: either this goal, or that goal, neither-nor, both-and, adding compromise with in-between 0,1. From 2 incompatible goals to 5 possibilities it expands our vision; including three possible solutions: neither-nor, compromise, both-and (*Epistemology: On the Use of Dichotomies*, *Hoggar*, 20 October 2014)
    - Expanding the conflict horizon from the dilemma to the trilemma yields compromise, a sometimes adequate way out; expanding to the Buddhist tetralemma yields the both-and and the neither-nor as ways out of a dilemma and can take the more advanced forms of creating new realities as negative or positive transcendencies (*Toward a Conflictology: the quest for transdisciplinarity*, *Handbook of Conflict Analysis and Resolution*, 2008).

- The influential poetry of William Blake is notable for its focus on the fourfold (Cecil Anthony Abrahams, *William Blake's Fourfold Man*, 1978; Neville Goddard, *Fourfold Vision*, 1968). This has been compared to the religious classic of Thomas Boston (*Human Nature in its Four-fold State*, 1720) by David Groves (*Blake, Thomas Boston, and the Fourfold Vision*, *Blake: an illustrated quarterly*, 19, 1986, 4).

Synergetics: Buckminster Fuller: offers especially detailed arguments regarding the design constraints by which an effective system can be constructed in practice. Best known for his implementations of large geodesic domes, the logic underlying such architecture is suggestive of principles of relevance to the design of a global system of governance -- as well as offering an admirable visual metaphor.

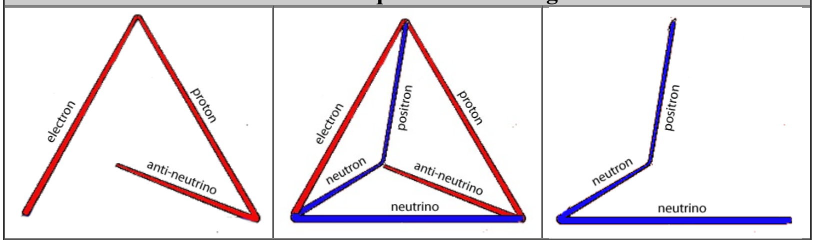
Fuller and his followers have indeed applied such thinking to global distribution of resources, understood in energy terms -- readily to be associated with financial flows. William Shepherd (*Energy Currencies*, 2008) notes that energy currencies were first suggested in 1981 by Buckminster Fuller in connection with his *One World Island Global Electricity Grid*. On behalf of the Single Global Currency Association, a submission was made to the 2009 Buckminster Fuller Challenge by Morrison Bonpasse (*Single Global Currency*, 2009).

Potentially intriguing with this understanding are the tensional integrity principles which underlie the structure of such global systems. Fuller stresses the essential instability of such systems unless the geometry of the constitutive elements is appropriately triangulated -- inferred from the fundamental stability of the 4-fold tetrahedron and how the stability of its development into more complex structures can be ensured

As noted above, Fuller argues extensively that the tetrahedron, as a basic vectorial model, is the fundamental structural system, as presented schematically below (*Synergetics: explorations in the geometry of thinking*, 1975/1979). In his terms:

> By tetrahedron, we mean the minimum thinkable set that would subdivide Universe and have interconnectedness where it comes back upon itself.... The basic structural unit of physical Universe quantation tetrahedron has the fundamental prime number oneness.... The tetrahedron is is the first and simplest subdivision of Universe because it could not have an insideness and outsideness unless it had four vertices and six edges.... With three positive edges and three negative edges, the tetrahedron provides a vectorial quantum module in conceptual array in which the right helix corresponds to the proton set (with electron and anti-neutrino) and the left helix corresponds to the neutron set (with positron and neutrino). The neutron group has has a fundamental leftness and the proton group has a fundamental rightness. They are not mirror images. In the tetrahedron, the two groups interact integrally. The tetrahedron is a form of energy package. (p. 333)

Tetrahedron as vectorial model of quantum according to Buckminster Fuller



Redrawn from Buckminster Fuller, *Synergetics: explorations in the geometry of thinking*, 1975 (Fig. 620.06)

These images are suggestive of implications for creative thinking in the light of theories of quantum consciousness. Appropriately, the images (left and right) are reminiscent of the traditional mythical symbolism in many cultures of divine thunderbolts, notably that of Zeus -- perhaps even more appropriately with the possibility of a curious reciprocity consistent with any confidelity with the divine.

Despite the subtitle of his *magnum opus*, the relationship of his articulation to the thinking that is the concern here is elusive, as separately argued (*Geometry of Thinking for Sustainable Global Governance*, 2009; *Geometry, Topology and Dynamics of Identity*, 2009).

The Archimedean forms have acquired particular significance in oppositional logic, namely the discipline which focuses on the organization of logical relations as they feature in discourse and in computerized search algorithms. Unfortunately the coherent representation of these logical operations with polyhedra has as yet translated only to the most limited degree into relevance to the challenges of governance, as indicated separately (*Oppositional Logic as Comprehensible Key to Sustainable Democracy*, 2018). Ironically, opposition to wider application of those insights has not evoked any form of self-reflective inquiry by that discipline.

Strategic comprehension: The challenges of the times are curiously exemplified by a strangely unquestioned relation to numbers (*Comprehension of Numbers Challenging Global Civilization*, 2014). History may see it as bizarre that a global civilization should be so preoccupied with 1.5 -- being, however coincidentally, both the key to the response to the pandemic through social distancing, and as the target cap on global warming (*Humanity's Magic Number as 1.5? Dimensionless constant governing civilization and its potential collapse*, 2020). Claire Lemercieris and Claire Zalc ask whether history is a matter of individual agency and action, or of finding and quantifying underpinning structures and patterns (*History by Numbers*, *Aeon/Psyche*, 2 September 2022).

In the engagement with complexity, it is appropriate to note the value asssociated with quadrant analysis, based on a 2×2 decision-making matrix (*Quadrant Analysis for Strategic Decision Making*, *MeetingSift*; *What is quadrant analysis? Survature Help Center*; *Time management strategies for busy people using the 4-quadrant method*, *Priority Matrix*; *2×2 Prioritization Matrix*, *Product Plan*). Curiously it has been suggested that there are four approaches to decision-making (J. D. Meier, *4 Decision Making Methods*, *Sources of Insight*; Conor Neill, *The 4 Methods of Decision Making*, *Moviong People to Action*, 16 May 2019). A fourfold pattern of influences on choice-making has been proposed by Daniel Kahneman (*Thinking, Fast and Slow*, 2011).

Potentially of particular relevance to the strategic focus implied by the 17th Goal is the 4-fold articulation notoriously presented by Donald Rumsfeld. It continues to be cited for his prescience in strategic and security circles due to his succinct articulation of the challenge of what may be known with any confidence in a world of increasing uncertainty. His formulation famously took the form of a "poem" -- on *The Unknown* -- presented during a Department of Defense news briefing on 12 February 2002.

The insight has, for example, been used in the analysis by Nathan Freier (*Known Unknowns: Unconventional 'Strategic Shocks' in Defense Strategy Development*. Strategic Studies Institute, U.S. Army War College, November 2008). The 4-fold pattern evokes other interpretations (*Unknown Undoing: challenge of incomprehensibility of systemic neglect*, 2008).

In the context of a 16-fold framework, it could however be asked whether that 4-fold framework implies a "4-squared" framework and calls for its exploration. This is justified by recognition that Rumsfeld's articulation fails to highlight the perception of those variously "knowing" or "failing to know", namely those recognizing or failing to acknowledge issues of strategic relevance. As some form of denial or cultivated ignorance, it could be understood as a "just ignorance theory" analogous to "just war theory". This 16-fold pattern then helps to identify in systemic terms the range of perceptions of both individuals and of the collectives with (corporate) cultures justifying those perspectives. This could give rise to a 16-fold framework as follows -- identifying the manner in which each of the 16 might distinguish their percreption of the others.

Enhancement of 4-fold to 16-fold pattern in terms of degrees of recognition of strategic challenges and vulnerabilities (tentative in anticipation of more meaningful descriptors with examples)				
	Known knowns	***Known unknowns***	***Unknown knowns***	***Unknown unknowns***
Acknowledged recognition	Unquestioned knowledge	Acknowledged doubts and queries	Hypothetical queries -- "beyond the radar"	Understood generically -- in principle only (if at all)
Acknowledged non-recognition	Recognition of contrary views set aside	Known doubts and queries ignored	Acknowledgement of ignorance	Acknowledgement of avoidance of the unknown
Unacknowledged recognition	Ignored contrary views	Unrecognized queries on known issues	Unconscious recognition?	?
Unacknowledged non-recognition	Failure to attach credibility to alternatives	Avoidance of questions and doubts	Unconscious denial?	"cloud of unknowing"? "brain fog?"

Given the importance of game theory to strategic analysis, it is intriguing to note the emphasis in practice on 2-party games -- perceived as inherently more comprehensible (*Destabilizing Multipolar Society through Binary Decision-making: alternatives to "2-stroke democracy" suggested by 4-sided ball game*s, 2016). As ball games, these may however be played out with teams of up to 16 players -- with many teams of 12 or less. With chess and go as the archetypal strategic games, the 8x8 pattern of the chess board is the pattern over which 32 pieces are played -- 16 a side.

Extending the Rumsfeld pattern to 4x4 as a strategic framework then invites the question as to the development of 4x4 chess and the relevant strategic insights it offers. Known as one variant of minichess, it is the the focus of extensive analysis (Kirill Kryukov, *4x4 Chess*, 2011). Arguably opposing players are obliged to acknowledge the 4 variants of the Rumsfeld pattern, whether in exploiting the failure of awareness of the opponent or acknowledging their own potential failure. This could prove helpful in distinguishing between the "integration" offered by a 17th perspective, and the "confusion" which may be associated with the attempts to achieve it -- and premature assumptions in that regard (*Variety of System Failures Engendered by Negligent Distinctions*, 2016).

Quaternity, quaternions and the unifying goal of partnership?

There is a curious sense in which the nature of a goal, especially the 17th Goal of the SDGs as the "Partnership of the Goals", is especially elusive -- despite the ease with which reference is made to it (*Engaging with Elusive Connectivity and Coherence*, 2018). Particularly problematic is a form of premature closure and definition associated with its identification, most notably through use of "target" (*Health and sustainability misleadingly framed as target acquisition*, 2012; *Reframing a fundamental attractor as a target*, 2014 *Enhancing Sustainable Development Strategies through Avoidance of Military Metaphors*, 1998). This is equally true of the many references to unity in a global context -- and the desirability of it.

Missing in such usage is recognition that the objectivity with which "goal" may be variously represented tends to obscure the subtlety of which it may be indicative, as exemplified by circular configuration of targets, dartboards, mandalas, codons and hexagrams (*Objectively understood configurations indicative of fundamental cognitive implication*, 2021; *Tabling a motion according to rules of order in debate*, 2016). This may well have contrasting aesthetic dimensions too readily set aside, as in the appeal of liminality (*Liminality of betwixt and between*, 2011).

The considerations of quaternity noted above, especially from a depth psychology perspective, can be readily recognized as equally subtle and elusive -- to be understood progressively (if at all) -- and individually rather than collectively. Somewhat ironically, this could also be considered the case with respect to insight into the degree of mathematical abstraction encompassed by quaternions.

Given their etymology, it is therefore somewhat surprising to find little exploration of how these disparate perspectives might be related. One exception is the brief commentary by Herb Klitzner (*Welcome to the Culture of Quaternions: past, present, future*, *The Culture of Quaternions: the Phoenix Bird of Mathematics*, 18 January 2015; *Quaternions and the World of Carl Jung and His Followers*, 1 March 2015; *Quaternions, Cognition, Music, and 4D: a detailed exploration*, May 2015).

Curiously there is frequent reference in a management context to "quarterly goals" (Jay O'Donnel, *Quarterly Goals: How To Set Them And Why They Work*, 2021; Tan Shirley, *Visualizing Success: How To Set Up Effective Quarterly Goals*, *Business.com*, 29 June 2022; Lee Garrett, *The Power Of Quarterly Goals*, *Productivityist*, 2022). The sense of "goal" does not however seem to feature in the other references to quaternity and the fourfold from the integrative psychological perspective (as noted above). The sense of a fourfold goal has however been presented as fundamental to the Christian mission (Joseph Babij, *The Four-Fold Purpose of the Church*, Calvary Community Church, 2019; *The Fourfold Gospel*, Reidsville Alliance Church). The *Fourfold Gospel* is the Christological summary on which the core values of the Christian and Missionary Alliance is based.

It is therefore ironic to discover that "goal" does feature in some applications of the algebraic abstractions framed by quaternions in robotics. The challenge in that context is how "goals" can be attributed through programming to robots and to artificial intelligence more generally (Ilian Bonev, *How to Use Quaternions in Industrial Robotics*, *Mecademic*, 18 February 2022). The focus could raise the question of possibilities of new insights into how goals can be attributed to humans collectively, or elicited from them -- as implied by techniques of motivation. Intriguingly Bonev uses the term "end-effector" rather than goal, suggesting another way of understanding the

17th SDG goal -- whether desirable or inherently questionable.

The nature of quaternions may be understood to a limited degree through comments such as:

- In mathematics, the quaternion number system extends the complex numbers, but in four dimensions instead of just two.
- Quaternions are hypercomplex numbers with 4 dimensions that can be used to represent 3D rotations -- used in graphics programming as a compact representation of the rotation of an object in three dimensions (Kenwright, *Dual-Quaternions and Computer Graphics*, 2020).
- Quaternions are used in pure mathematics, but also have practical uses in applied mathematics, particularly for calculations involving three-dimensional rotations, such as in three-dimensional computer graphics, computer vision, and crystallographic structure analysis. They have been applied in many areas including control theory, signal processing, number theory, flight dynamics and navigation systems of aircraft, orbital mechanics, bioinformatics, molecular dynamics, quantum mechanics, among others
- Quaternions are often the best choice whenever rotation or attitude representations are required. This includes robotics, aerospace engineering, video games, and the like (*Understanding Quaternions*, *3D Game Engine Programming*, 25 June 2012). They are of particular use in optimal control or state estimation scenarios: they are often the representation of choice for the attitude of an object.

With its extensive use of computer graphics, the exercise above invites the question as to the extent to which it could be considered to involve quaternions -- ironically unbeknownst to the developer of the animations presented. References to quaternions make explicit reference to their relation to the rotational symmetry group of the regular tetrahedron -- the focus of the exercise.

Whether reference is made to quaternity, quaternions or polyhedra, there is clearly a challenge to representation of the subtle coherence associated with "goal" and its "unifying" function. In one sense, any preferred representation may well be essentially misleading. The corollary is however that recognition of the complementarity of disparate forms of representation may itself be a challenge.

There is even the curious possibility that correspondences between them may merit recognition (and deprecation) as in the case of the "moonshine theory" through which the "monstrous" nature of the most fundamental forms of symmetry -- the "monster group" -- has been discovered and explored in mathematics (*Potential Psychosocial Significance of Monstrous Moonshine*, 2007). The engagement with "correspondences" which this required is indiciative of the exceptional form of symmetry which may be characteristic of any Rosetta stone for cognitive frameworks (*Theories of Correspondences -- and potential equivalences between them in correlative thinking*, 2007). As "partnership of the goals" of global governance, the "monstrous challenge" of SDG Goal 17 would appear to merit corresponding appreciation -- avoiding tendencies to misleading oversimplification.

Requisite complementarity of forms of representation of a unifying goal?

In the light of the above argument, potential correspondences can be speculatively presented for consideration. These are inspired by the tendency of seemingly unrelated disciplines to articulate their unifying insights in fourfold patterns with which understandings of "goal" may well be associated. Quaternity is indeed recognized as having unifying psychosocial implications, but without it being evident how these may be related to any sense of "goal" -- other than what may be implied by a process of evolution and individuation. A particular challenge with respect to the representation of any unifying goal for governance lies in its comprehensibility and memorability, and the risks of reductive simplification, as argued separately (*Memorability, Mnemonics, Maths, Music and Governance*, 2022; *Comparable Modalities of Aesthetics, Logic and Dialogue -- in the light of correspondences between their polyhedral representation*, 2021).

Quaternary representation: Rather than any implication that tetrahedra, quaternity, quaternions or fourfold patterns are separately adequate to indication of unifying subtlety, the challenge would appear to be one of evoking recognition of how they are complementary ways of indicating a goal-related insight for which no single formal representation is adequate ("necessary but not sufficient"). There is therefore a need for the confrontation of disparate perspectives (*Dynamics of N-fold Integration of Disparate Cognitive Modalities*, 2021; *Global Coherence by Interrelating Disparate Strategic Patterns Dynamically*, 2019).

From a representational perspective, and aside from considerations of preferred fourfold symbols of sacred geometry, the concern here is the potential correspondence between:

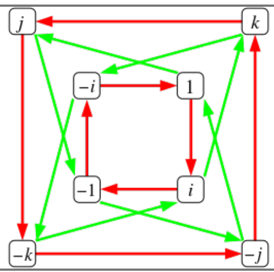
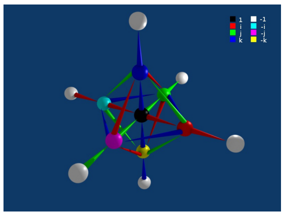
- tetrahedral representation, especially in 3D computer graphics, exemplified by the code underlying the animations above
- algebraic representation of the quaternion pattern, especially given its application to rotation in computer graphics -- and the specification of attitude
- the set of 16 Boolean logical connectives, notably fundamental to computer operation and truth tables
- oppositional logic, as represented in oppositional geometry -- most obviously in the square of opposition
- use of polyhedra to represent the pattern of logical connectives -- whether Archimedean (as with the rhombic dodecahedraon) or by reference to the hypercube (as with the Logic Alphabet tesseract)
- use of Venn diagrams as a means of illustrating logical connectives, notably with respect to the set of SDG goals (*Venn diagram: path to sustainable development* *ConceptDraw*)

It is noteworthy that these may be variously recognized as tending to have recourse to a square in 2D, or derivatives of a cube in 3D -- with the implication of analogues of higher dimensionality. Such forms are indeed "re-presentations", thereby avoiding the challenge of the psychosocial implications of relevance to integrative comprehension of any goal. This is most obvious in the case of the extensive literature on the square of opposition and the papers it evokes (*7th World Congress on the Square of Opposition*, Leuven, September 2022).

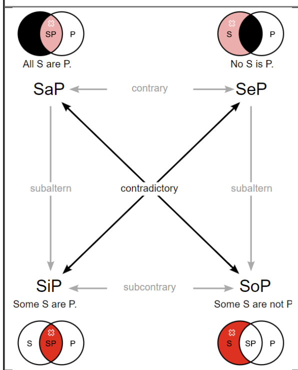
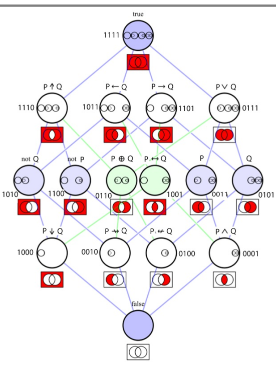
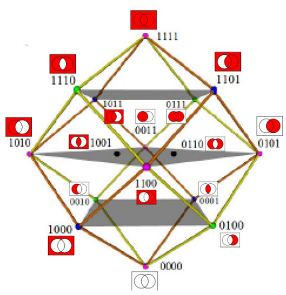
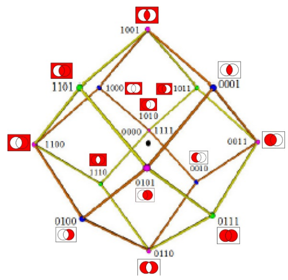
Tetrahedral dynamics and quaternions? As noted above, the challenge of the initial trial and error process was to identify the coordinates of the 4 vertices associated with each tetrahedron in the 16 composing the tetrahedral compound as a representation of the 16 SDG goals. The subsequent phase was to define a pathway for the movement of larger spheres along the edges of each. It is assumed that the dynamics of that movement constitute a form of container for each goal, with the compound acting as a container for the set of goals as a whole -- with the implication that their "partnership" is then to be understood as a dynamic indication of the 17th Goal.

The process was undertaken with little insight into the geometry, but it became apparent that the coordinates defining the edges of such a pathway in each case took some variant of the following form (below left), as a consequence of each tetrahedron sharing a common centre (and hence the coordinate 0) and with coordinates on the compound's circumsphere. The pattern may be seen by inspection of the coding of the interactive 3D animations above.

As an exercise in computer graphics, the pattern focuses discussion as to the relation to any tabular presentation of quaternions (below). A Cayley graph of the pattern is presented with red, green and blue arrows representing multiplication by **i**, **j**, and **k**, respectively. A 3D representation of quaternions is indicated below right

<table>
<tr><th colspan="4">Potential correspondence between tetrahedral pathway definition and quaternions?</th></tr>
<tr><td>Pattern of coordinates in computer graphics of a pathway defining edges of tetrahedron, excluding rotation
(as in a 16-tetrahedral compound)</td><td>Quaternion multiplication table
Non commutativity is emphasized by coloured squares</td><td>Cayley graph of Q8</td><td>Three-dimensional graph of Q8.
Multiplication by negative numbers are omitted for clarity.</td></tr>
<tr><td>
<table>
<tr><th>x coord.</th><th>y coord.</th><th>z coord.</th></tr>
<tr><td>-a</td><td>b</td><td>0</td></tr>
<tr><td>a</td><td>b</td><td>0</td></tr>
<tr><td>0</td><td>-b</td><td>-a</td></tr>
<tr><td>0</td><td>-b</td><td>a</td></tr>
</table>
</td><td>
<table>
<tr><th>×</th><th>1</th><th>i</th><th>j</th><th>k</th></tr>
<tr><td>1</td><td>1</td><td>i</td><td>j</td><td>k</td></tr>
<tr><td>i</td><td>i</td><td>−1</td><td>k</td><td>−j</td></tr>
<tr><td>j</td><td>j</td><td>−k</td><td>−1</td><td>i</td></tr>
<tr><td>k</td><td>k</td><td>j</td><td>−i</td><td>−1</td></tr>
</table>
</td><td>
</td><td></td></tr>
<tr><td></td><td>Reproduced from *Wikipedia*</td><td>Cayley_graph_Q8 Sullivan.t.j, CC BY-SA 3.0, via Wikimedia Commons</td><td>Nielmo, CC BY-SA 4.0, via Wikimedia Commons</td></tr>
</table>

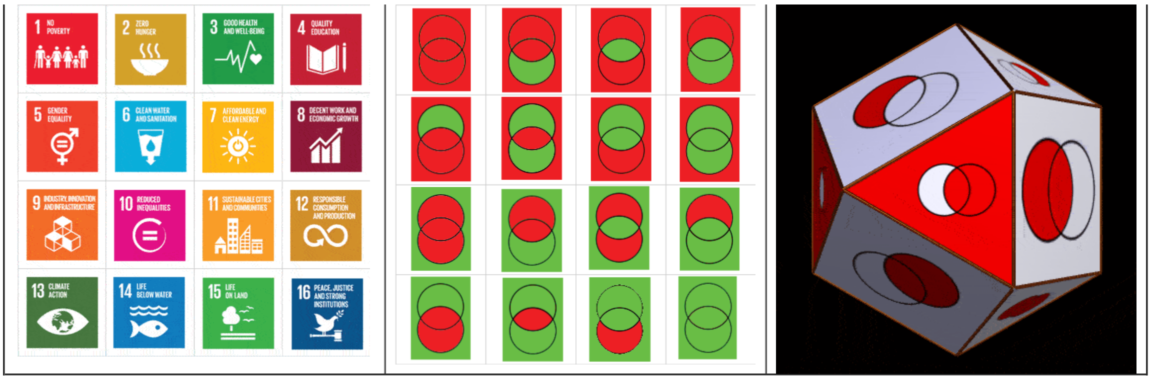
Logical connectivves and Venn diagrams: (*Oppositional logic and its geometry -- 16 minus 2 connectives?* 2021)

<table>
<tr><th colspan="4">Alternative representations of arrays of logical connectives</th></tr>
<tr><td>Representation of square of opposition</td><td>Hasse diagram
with binary codings added</td><td>Hasse rhombic dodecahedron with indicative Venn diagrams added</td><td>Aristotelian rhombic dodecahedron
with indicative Venn diagrams added</td></tr>
<tr><td>
</td><td></td><td></td><td></td></tr>
<tr><td>"Tilman Piesk", Public domain, via Wikimedia Commons</td><td>Augmented from Watchduck (a.k.a. Tilman Piesk), Public domain, via Wikimedia Commons</td><td colspan="2">Augmented from Lorenz Demey and Hans Smessaert (*Geometric and Cognitive Differences between Logical Diagrams for the Boolean Algebra B_4*)</td></tr>
</table>

A fundamental question is how the 16 SDGs reflect a coherent systemic perspective -- despite their apparently disparate focus. Given the significance attributed to the Venn diagram representation, in relation to the logical connectives, it might be asked whether each SDG could be understood as an exemplification of one such distinction -- an "orientation" or a "mode of dialogue". The challenge is presented below through juxtaposing the iconography of the set of SDGs with that of the connectives.

As noted above, the 17th coordinating goal is excluded from the pattern, but there is also the question of whether the 16 include 2 which are meta-systemic in some way -- justifying the reduction in representation to 14 (*Oppositional logic and its geometry -- 16 minus 2 connectives?* 2021). It is in that sense that the 14 Venn diagrams associated with the 14 connectives are mapped onto a cuboctahedron as presented above. Again this poses the question of how they might best be configured to exemplify their systemic relationships.

<table>
<tr><th colspan="3">Towards a memorable systemic representation of disparate SDGs</th></tr>
<tr><td>Conventional SDG iconography</td><td>Systemically significant use of Venn diagram representation</td><td>Venn diagram of logical connectives mapped onto cuboctahedron</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>



Attitudes, goals and orientations: With the explicit recognition that quaternions are used in computer graphics, in order to define rotations in 3D, there is a tantalizing possibility that their role in robotics (in specifying the "attitude" or "goal" of the robot) offers pointers to exploring each of the 16 contrasting tetrahedra in those terms -- inspired by the analytical language of quaternions (David Corfield, *Quaternionic Analysis*, *The n-Category Cafe*, 6 May 2008).

Are the 16 tetrahedra representing the SDGs to be appropriately recognized in terms of their contrasting "orientations" in a global "knowledge space" whose dimensionality is yet to be fully understood? The language of Bryant Julstrom offers one pointer to such a possibility:

> This module raises the general question of representing rotations in three dimensions. It motivates quaternions by referring to Hamilton's search for a four-dimensional system that embeds the real and the complex numbers, and approaches quaternions in three dimensions through the corresponding vectors and matrices. Quaternions are applied to rotating shape representations in computer graphics. (*Using Real Quaternions to Represent Rotations in 3D*, UMAP)

From 16 to 64: With respect to the 16-tetrahedral compound (of 8+6+2 tetrahedra), the animations above highlighted the 64-fold pattern notably featured in the previous use of the drilled truncated cube for mapping purposes (*Polyhedral representation of Sustainable Development Goals including "Own Goals"?* 2022). With respect to the framework offered by quaternions, it is therefore intriguing to note observations with regard to its generalization and recognition of an 8+6+2 pattern:

> There are a total of 64 possible multiplication rules that can be defined starting with the generalized imaginary units first introduced by Hamilton [in 1843]. Of these sixty-four choices, only **eight** lead to non-commutative division algebras: **two** are associated to the left- and right-chirality quaternions, and the other **six** are generalizations of the split-quaternion concept first introduced by Cockle. We show that the 4×4 matrix representations of both the left- and right-chirality versions of the generalized split-quaternions are algebraically isomorphic and can be related to each other by 4×4 permutation matrices of the C2×C2 group. As examples of applications of the generalized quaternion concept, we first show that the left- and right-chirality quaternions can be used to describe Lorentz transformations with a constant velocity in an arbitrary spatial direction. (Hong-Yang Lin, Marc Cahay, Badri N. Vellambi and Dennis Morris, *A Generalization of Quaternions and Their Applications*, *Symmetry* 14, 2022, 3) *[emphasis added]*

Computer calculations by Stuart Hoggar have determined the vertices of a polytope in quaternionic 4-space (*64 Lines from a Quaternionic Polytope*, *Geometriae Dedicata*, 1998, 69; *Two Quaternionic 4-polytopes*, *The Geometric Vein*, Coxeter Festschrift, 1981)

Should the "64 possible multiplication rules" (termed "choices" above) be recognized as related to traditional intuition regarding the pattern of 64 decision-making situations with which the 6-fold hexagrams of the *I Ching* have so long been associated -- clustered as they are into 8 "houses", with the binary (chiral) distinction between *yin* and *yang*? (*Memorable navigation of viable global pathways from 4-fold to 64-fold and beyond*, 2022).

It is intriguing that traditional commentary on that 8-fold distinction also makes specific reference to orientations -- expressed metaphorically in terms of those of any compass rose vital to navigation (*Ways of looking distinguished in terms of "compass" orientation*, 2021). Again, could the 16 SDGs be usefully associated with distinctive cognitive "orientations" -- if only for mnemonic purposes?

Imagining a unifying goal: Of particular value in relation to any understanding of "goal" is the manner in which quaternions offer explicit recognition of the "imaginary" in secular scientific terms -- through their combination of real and imaginary numbers. This is despite the argument that there is nothing "imaginary" about "imaginary numbers".

Arguably insight into the integrative nature of the 17th SDG goal indeed calls for consideration in terms of some form of imaginary dimension -- articulated to a higher degree than evoked above in terms of a collective "dream". Any collective vision of the future necessarily involves this dimension, as variously argued (Julianne Schutlz and Brendan Gleeson, *Imagining the Future*, *Griffith Review*, 52, 2016; J. Russell Boulding, *The Dynamics of Imaging Futures*, 1978; Elise Boulding, *Why Imagine The Future?*).

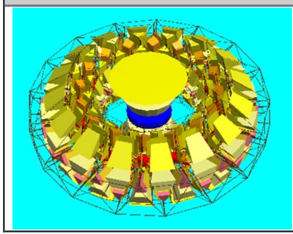
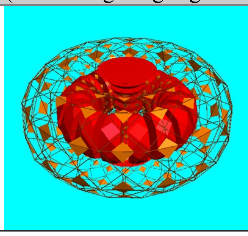
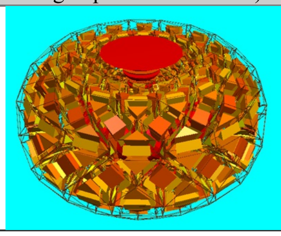
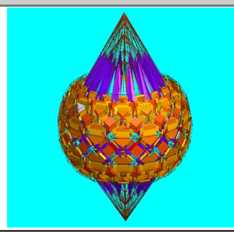
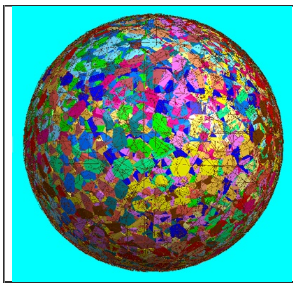
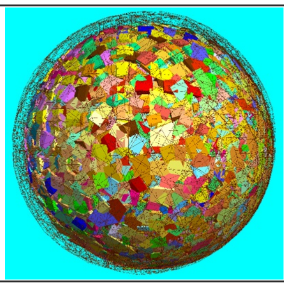
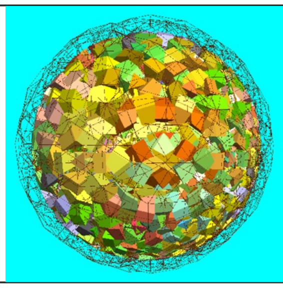
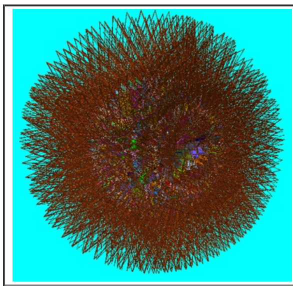
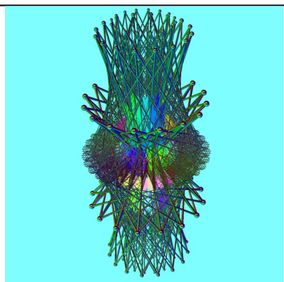
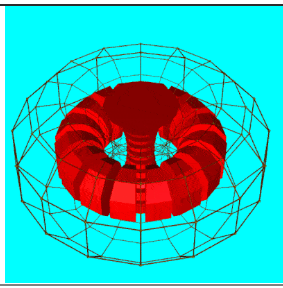
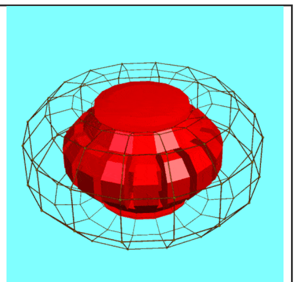
In explicitly combining the "real" and the "imaginary", quaternions could then be recognized as unique in offering insights into a reconciliation of the two-culture dynamic between the so-called "heartless heads" and the "headless hearts" (*Challenge of the "headless hearts" to the "heartless heads"?*, 2018; C. P. Snow, *The Two Cultures and the Scientific Revolution*, 1959).

Insights into partnership from tetrahedral representation of SDGs in 4D?

It is obviously the case that conventional methods of presenting integration in 2D, or even in 3D, are less than helpul in enabling insights into the potential nature of unity or partnership -- if it is better understood in 4D (if not 5D). Since the Stella4D application used in developing the animations above provides for a form of representation in 4D (as explained in the manual), there is a case for reproducing examples of the 16-tetrahedra pattern in 4D.

As indicated in the manual, the point to be stressed is however that these images are necessarily projections of 4D models into 3D -- effectively a 3D "cross-section" of a 4D pattern. The following are therefore presented in a spirit of evoking interest in new ways of exploring and understanding integration -- given the challenge of the 17th perspective in the case of 16 SDGs and the nature of their "partnership".

The first images (below left) are a provocative reminder that it may the case that the archetypal roundtable now needs to seat 16, rather than 12 -- and that any "Last Supper" of global governance calls for other considerations with respect to visualization of patterns of sustainable discourse between systemic archetypes (*Clarifying the Unexplored Dynamics of 12-fold Round tables*, 2019; *Increasing the dimensionality of the archetypal Round Table?* 2018).

Indicative 3D projections of 4D configurations of a 16-tetrahedral pattern of SDGs (contrasting images generated through options of Stella4D)			
			
			
			
Images generated with the aid of Stella Polyhedron Navigator			

Given their engagement with time, strategies could be understood as necessarily requiring a 4D perspective -- otherwise they could only be considered as static monuments to possibilities. If the SDGs are all about achieving results over time -- with a focus on 2030 -- then **gaining a strategic sense of time as a fourth dimension is a major challenge** (*Strategic Embodiment of Time: configuring questions fundamental to change*, 2010; *Four-dimensional requisite for a time-bound global civilization?* 2015).

Potrentially vital to the credibility of any 17th Goal is the attractive power of the iconography through which it can be represented. In a period of rapid innovation in visualization, images such as those above call for evaluation in terms of the "interestingness" which is now a focus of research (Jianfeng Gao, et al, *Modeling Interestingness with Deep Neural Networks*, October 2014; Claire-Hélène Demarty, et al, *Visual Content Indexing and Retrieval with Psycho-Visual Models*, *Predicting Interestingness of Visual Content*, October 2017; Suzanne Hidi and William Baird, *Interestingness: a neglected variable in discourse processing*, *Cognitive Science*, 10, 1986). As currently envisaged, is the 17th Goal inherently "boring"?

Known as 4-polytopes (or polychora), a further lead to any intuited sense of 17-fold coherence in 4 dimensions is offered by the 64 convex uniform 4-polytopes of which 5 are polyhedral prisms based on the Platonic solids and 13 are polyhedral prisms based on the Archimedean solids -- with one understood as duplicated by the cubic hyperprism (namely a tesseract), reducing the set to 17 (*Higher dimensional coherence of SDGs implied by a set of 17 4-dimensional polyhedra?* 2021; *Interrelationship of 17 SDGs modelled by 17 regular polyhedra in 4D*, 2021).

17th Goal and the memorable representation of 16?

SDG iconography and coherence: This poses the question of the design criteria for a memorable configuration of 16 goals -- one that is both indicative of their distinctiveness as well as of their part in the 16-fold pattern as a whole. It is the latter which is lacking in current efforts at such iconography. However, whilst their distinctiveness is indeed ensured, the imagery is in no way indicative of how they form into a pattern together -- a coherence of systemic significance. The iconography is therefore less than helpful in indicating the challenge of the partnership of the goals. Arguably the challenge could be presented metaphorically as one of designing a memorable jigsaw puzzle in which the pieces have systemic significance for the viability of the system as a whole.

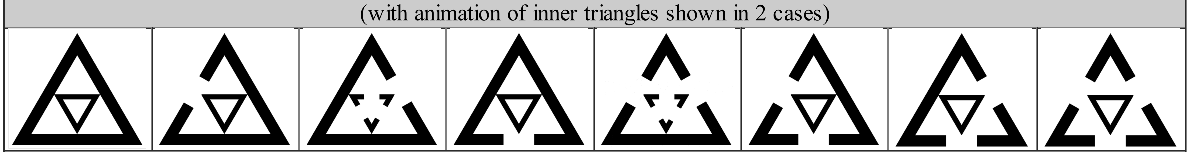
The challenge has been addressed with the pattern of 64 through the use of hexagrams -- complemented by extensive commentary in metaphhorical terms. Traditoonally this has been configured as both a square and circular pattern -- known to have inspired the work of Gottfied Leibnitz (Mary von Aue, *How the 'I Ching' Inspired His Binary System*, *Inverse*, 2 July 2018). It was presented to him 1701 in the form of the Shao Yong circle of 64 hexagrams as the Western orginator of binary codiing. A reduced version of that pattern has been addressed through a pattern of 8 framed metaphorically as "houses", each clustering 8 of the 64, as noted above. Traditionally the pattern of 8 is however represented as the circular *BaGua* pattern of trigrams which features widely in the iconography of some Eastern cultures -- and variously interpreted in metaphoric terms.

It is therefore intriguing to explore how a systemic pattern of 16 could be configured using iconography inspired initially by the hexagram or the trigrams -- at least to some degree. The argument above suggests the value of imbuing a fourfold (if not tetrahedral) structure to the design, potentially in 3D, as well as implying a dynamic.

The difficulty lies in how to present such a pattern, especially given the requirements for its reproduction in the 2D formats favoured by the publishing industry and its technology. This constraint could well be interpreted as a communication constraint worthy of extensive analysis with respect to the cognitive challenges of governance. How indeed is complexity to be appropriately reduced to a 2D representation? That challenge has has long been evident in efforts to map the globe for navigation and other purposes (*List of Map Projections*, *Wikipedia*). Some of these take the form of polyhedra which can be unfolded from 3D to 2D, as with Fuller's Dymaxion Map.

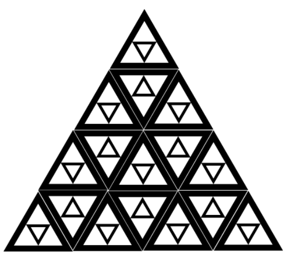
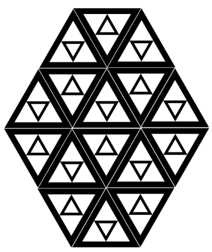
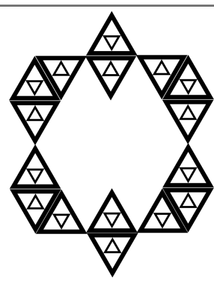
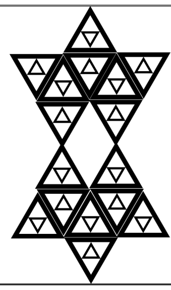
Some kind of animation could indeed be presented with each of the 16 tetrahedra of the compound alternating between four conditions rendering it distinctive by its dynamics. With each tetrahedron defined by 6 lines, these could be presented as an alternative representation of the traditional hexagram, whether the "upper trigram" is presented as circumscribing the "lower trigram", or vice versa.

Tiling design metaphor: One starting point is evident from the following set of images, in which broken and unbroken (yin and yang) lines are wrapped into triangular form in 2D -- suggestive of the tetrahedral pattern emphasized above. The 8 images correspond to the "houses". The reference to the tetrahedron can be taken further by incorporating an inner triangular form within each. This could be as shown, or with 3 lines from the central point to the vertices of the outer triangle. A possible dynamic is shown in two cases.

8-fold pattern of distinctions between triangles (with animation of inner triangles shown in 2 cases)


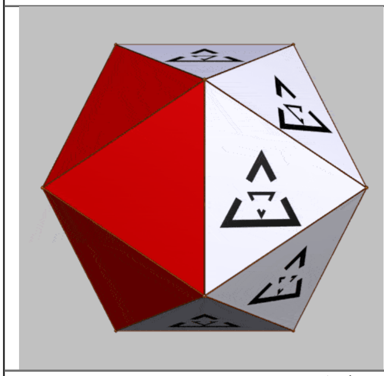
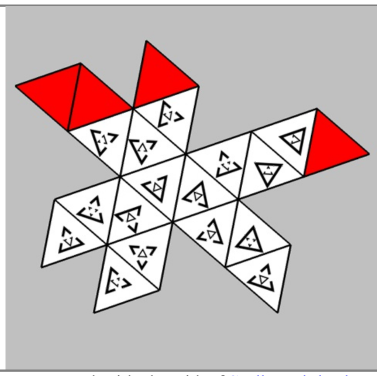
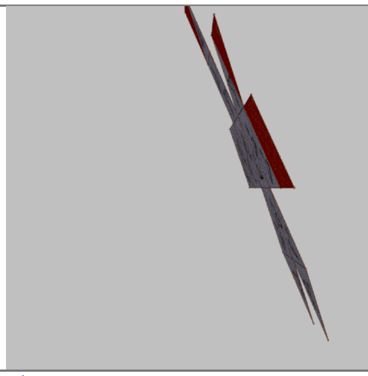
Whilst the above is indicative of an approach to an 8-fold pattern, it does not directly address the challenge of a 16-fold pattern -- nor of how the components might be understood as fitting together, from a 17th perspective. The form of the triangle invites other considerations, partially inspired by the importance of the triangulation of the sphere which is so fundamental to the geometry of polyhedra, and to mapping the globe. The 16-fold configuration in 2D could be framed as a tiling problem, one extensively studied in geometry.

One such pattern is suggested by Michael Beeson (*Triangle Tiling*, San Jose State University, 22 January 2019), as shown below left. Others are shown (centre and right) with the possibnility of others to be envisaged (see *Triangular tiling*, *Wikipedia*). In those presented patterning of 16 is achieved by reversing the outer triangle in the set of 8 shown above, although the triangles in the two images are not distinguished in this way. As in any jigsaw puzzle, they could potentially be configured together so that broken and unbroken lines matched according to some convention. The various configurations invite contrasting "stories" from a systemic perspective.

Alternative tiling patterns of 16 triangles -- from a 17th perspective (broken/unbroken line distinctions between outer triangles not shown)			
10 upright; 6 reversed; 1 central	8 upright; 8 reversed; 2 central	8 upright; 8 reversed; empty centre	8 upright; 8 reversed; empty centre
			

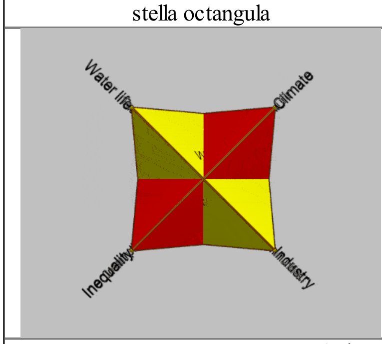
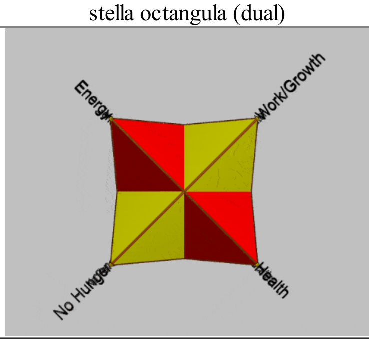
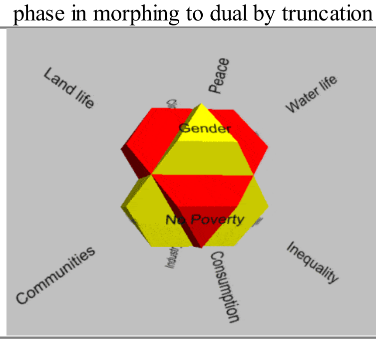
A further constraint is however desirable in order to emphasize the implied tetrahedral form in 3D and the dynamics by which each of the 16 is distinguished. The inner triangle cycles through 8 conditions (as illustrated in the two animations in the set of 8 above) -- whereas only four are required to render each of the 16 distinct. This could be achieved by freezing the change to one line in the inner triangle, perhaps to reflect the distinction in the surrounding triangle.

Systemic analogue to Dymaxion Map: Of potential interest, in the light of the arguments of Buckminster Fuller for the Dymaxion Map, is the placement of the 16 triangular images onto 16 of the triangular faces of of the 20-faced icosahedron used (as variously illustrated below). In this initial experiment, no effort has been made to position the triangles in relation to one another, to enlarge them to fill the triangular space, or to relate them to the 4 unused positions. Of potentially related interest for such a purpose, is the use of the pentakis dodecahedron as dual of the truncated icosahedron (with 24 of its 60 triangular faces then without associated images).

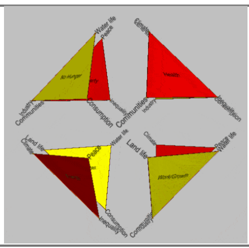
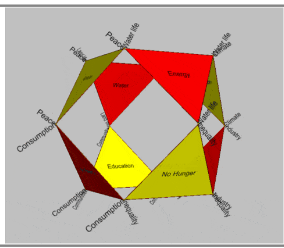
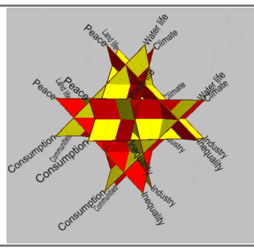
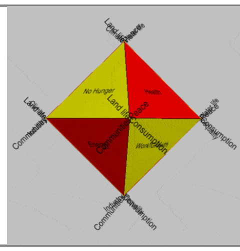
Placement of triangular distinctions between 16 tetrahedra onto faces of isosahedron (following the global systemic arguments for the Dymaxion Map of Buckminster Fuller)		
		
Animations prepared with the aid of Stella Polyhedron Navigator		

Use of 2-tetrahedra stella octangula: Another design approach is through exploitation of the mapping facilities of the 8 faces and 8 vertices of the stella octangula which featured in the exercise above. This is potentially more compact and comprehensible in some respects. Together the compound of 2 tetrahedra clearly offers 16 mapping loci, but with the questionable distinction between use of faces versus use of vertices. However a feature of the compound is that its dual is also a stella octangula. Face-attributions and vertex-attributions are swapped between the two representations in a manner which is potentially interesting in its own right -- especially given the many distinctive morphing transformations between them.

A peculiarity of attribution of 8 SDG labels to the 8 faces of the stella octangula is that the text on the 4 faces of one tetrahedron is effectively hidden by the other tetrahedron of the compound (in the application used) -- a feature of potential mnemonic value in its own right. A phase in the morphing from one to the other (below right) renders both visible.

Alternative views of mapping of 16 SDGs keyword labels onto stella octangula (with 8 SDGs mapped onto 8 faces in one variant -- swapping with 8 vertices on the dual)		
stella octangula	stella octangula (dual)	phase in morphing to dual by truncation
		
Animations prepared with the aid of Stella Polyhedron Navigator		

The SDG labels can also be made variously apparent by "exploding" (and "imploding") the compound or "unfolding" it (as below). Each then invites distinctive comment in terms of perception of the system as a whole.

Screen shots of phases in "exploding" of stella octangula with 16 SDGs keyword labels variously mapped (with 8 SDGs mapped onto 8 faces with 8 vertices on the dual)			
			
Animations prepared with the aid of Stella Polyhedron Navigator			

Coherence as "mysterious" and "magical"? It is appropriate to note that Chinese culture has traditionally embodied the fourfold into the lesser known, and more complex, pattern of 81 ternary tetragrams as the *T'ai Hsüan Ching* (*Tai Xuan Jing, Canon of Supreme Mystery* /*The Great Dark Mystery*) of Yang Hsiung. Some relationships to the 64-fold pattern of that culture are discussed separately (*Hyperspace Clues to the Psychology of the Pattern that Connects in the light of the 81 Tao Te Ching insights*, 2003; *9-fold Higher Order Patterning of Tao Te Ching Insights: possibilities in the mathematics of magic squares, cubes and hypercubes*, 2003). How this might offer useful indications for patterning 16 is a question for the future. This might well constitute a visualization of the "4×4 permutation matrices of the C2×C2 group" indicated above.

As one of the Founding Fathers of the US, and deeply involved in the articuation of its Constitution, it is curious to discover the widely acknowledged expertise of Benjamin Franklin with regard to magic squares (*Magic square integrity and implications for the US Constitution*, 2015). Franklin called his 16x16 magic square ***the most magically magical of any magic square ever made by a magician*** -- with which many mathematicians and mystics would now be held to agree (Peter Loly, *Franklin Squares: a chapter in the scientific studies of magical squares*, University of Manitoba, 2006; William H. Richardson, *Ben Franklin's Amazing Magic Square* [including animation], Wichita State University; *Ben Franklin's 8x8 Magic Square*, Wichita State University).

Equally curious, and potentially relevant to understanding the 16 SDGs as a pattern, is recognition that a polyhedron can have one of 17 types of symmetry. These are the collections of ways in which a polyhedron can be repositioned without being able to distinguish the new position from the original position, as described separately with illustrative examples by Adrian Rossiter (*17 Types of Symmetry*, *Antiprism*, 2010).

A constraint with potentially similar cognitive implications is recognition that the variety of patterns in 2D is limited to 17 distinct groups of planar symmetries, as well as recognition . As understood by mathematics, the so-called wallpaper group is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art, especially in textiles, tessellations and tiles as well as wallpaper. Is this indicative of some sense in which cognitive organization of strategic goals is similarlry constrained? Ironically this is seemingly one of the very rare ways in which the 17-fold set of UN Goals might be recognized as coherent (*Role of the 17-fold "wallpaper group" in ordering SDGs?*, 2022). This contrasts with more conventional representations of SDGs (*Sustainable Development Wallpapers*, *Wallpaper Cave*).

It could be asked how the "mysterious" experience of such 16-fold "magic" is related to the coherence sought in terms of the 17th SDG Goal. Given the current challenges of global governance, it could be appropriately suggested that collective comprehension of the 17th Goal of the SDGs is indeed a "supreme mystery" in the Chinese sense.

References

Cecil Anthony Abrahams. William Blake's Fourfold Man. Bouvier, 1978 [access]

Keith Albar. The Language of Pattern: an enquiry inspired by Islamic decoration. Thames and Hudson, 1974

Stafford Beer. Beyond Dispute: the invention of team syntegrity. John Wiley, 1995

J. G. Bennett and Anthony Blake. Systematics: a new technique in thinking. Institute for the Comparative Study of History, Philosophy and the Sciences, 1966 [text]

Brin Best and Anthony Blake. Making Meaning: Learning through logovisual thinking. Chris Kington Publishing, 2005

Anthony Blake and John Varney. LogoVisual Thinking: a guide to making sense. Centre for Management Creativity, 2004

David Bohm. Wholeness and the Implicate Order. Routledge and Kegan Pual, 1980

Thomas Boston. Human Nature in its Four-fold State. Kessinger Publishing, 2010 [access]

John H. Conway and Derek A. Smith. On Quaternions and Octonions: their geometry, arithmetic, and symmetry. CRC Press, 2003 [review]

Keith Critchlow:

- Order in Space: a design source book. Thames and Hudson, 1969
- Islamic Patterns: an analytical and cosmological approach. Thames and Hudson, 1976
- The Hidden Geometry of Flowers: living rhythms form and number. Floris Books, 2011

R. Buckminster Fuller with E. J. Applewhite:

- Synergetics: explorations in the geometry of thinking. Macmillan, 1975 [text]
- Synergetics 2: further explorations in the geometry of thinking. Macmillan, 1979 [text]

Susan George. Variable Geometry to Design Positive Outcomes. Transnational Institute, 2005 [text]

Anagarika Govinda. The Inner Structure of the I Ching: the Book of Transformations. New Holland Publishers, 1996

Charles Grant:

- Variable Geometry. *Prospect Magazine*, July 2005 [text]
- Can Variable Geometry Save European Enlargement. *CER Bulletin*, October/November 2005, Issue 44 [text]

Alfred Koryzbski, Science and Sanity: an introduction to Non-Aristotelian Systems and General Semantics. Institute of General

Semantics, 1994

Jeremy Lent. The Patterning Instinct: a cultural history of humanity's search for meaning. Prometheus, 2017 [contents]

Ernest McClain:

- Myth of Invariance: the origins of the gods, mathematics and music from the Rg Veda to Plato. Nicolas-Hays, 1976
- The Pythagorean Plato: prelude to the song itself. Nicolas-Hays, 1978
- Meditations Through the Quran: tonal images in an oral culture. Nicolas-Hays, 1981

Ann E. Moyer. The Philosophers' Game: rithmomachia in Medieval and Renaissance Europe. University of Michigan Press, 2001

Anthony Pugh. Polyhedra; a visual approach. University of California Press, 1976.

Robert D. Romanyshyn. Technology as Symptom and Dream. Routledge, 1989

C. P. Snow:

- The Two Cultures. Cambridge University Press, 1959 [summary]
- The Two Cultures and a Second Look: an expanded version of *The Two Cultures and the Scientific Revolution*. Cambridge University Press, 1963

Alexander Wendt. Quantum Mind and Social Science: unifying physical and social ontology. Cambridge University Press, 2015

Robert Williams. Natural Structures: toward a form language. Eudaemon Press, 1972

Frances Yates:

- The Art of Memory. Routledge and Kegan Paul, 1966
- Giordano Bruno and the Hermetic Tradition. University of Chicago Press, 1964
