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Systemic Coherence of the UN's 17 SDGs as a Global Dream

Rather than merely an arbitrary outcome of political horse-trading

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Introduction

The 17 Sustainable Development Goals (SDGs), as widely promoted by the United Nations, can be understood as the culmination of humanity's best effort to respond in a coherent manner to the longer-term challenges of global governance in this period. Together they are hailed as an achievement for that reason, especially in the light of the lengthy negotiation between 193 countries that the articulation required.

The set of SDGs, whether collectively or individually, continues to evoke criticism. Of obvious concern is whether they will prove to be fit for purpose in the light of other trends which render their scheduled achievement by 2030 improbable (*SDG progress "in danger" of going backwards without change in direction*, *UN News*, 11 September 2019; *SDG Indicators*, *UN SDGs*; *Measuring Distance to the SDG Targets*, *OECD*, March 2020). Will they prove to be as inadequate as the 8 Millennium Development Goals which they were designed to replace -- or Agenda 21 which constituted the precursor?

The articulation of the 169 targets of those goals does not seemingly offer any sense of the pattern of goals in systemic terms -- if pattern there is -- and has attracted specific criticism, although evidence may emerge to the contrary (*The 169 Commandments: the proposed sustainable development goals would be worse than useless*, *The Economist*, 28 March 2015; David Tremblay, et al, *Sustainable Development Goal Interactions: an analysis based on the five pillars of the 2030 agenda*, *Sustainable Development*, 28, 2020. 6).

The focus here is not on such critiques but on whether the set of goals can indeed be considered systemically coherent, as might be necessary for sustainable global governance. Nor is it on the particular concern as to whether the relationships between the goals have been appropriately recognized, designed and articulated to guide the processes of strategic governance with which the goals are associated.

Of greater concern here is how a set of 17 goals can be considered coherent and memorable given that that number does not benefit from the assumed memorability, coherence and credibility of the 12-fold sets of goals that are so widely adopted (*Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources*, 2011). It is indeed a fact that there is no more understanding or interest in why any such set is itself coherent and memorable in that simpler case.

The distinctive exploration here follows from an unexplored assumption. **The set of 17 SDGs can indeed be assumed to be systemically coherent -- despite affirmations to the contrary** by those intimately involved in the process by which they were articulated. Such participation, and documentation of the process, does necessarily reinforce the sense that the outcome was effectively a compromise emerging from a conventional pattern of political horse-trading and influence-peddling. Subsequent efforts at institutional implementation also reinforce that conclusion.

The argument explored here is that global civilization is indeed "unconscious", as argued by John Ralston Saul (*The Unconscious Civilization*, 1995) and otherwise with respect to the collective unconscious by Carl Jung. In that condition it could then be assumed that **the SDGs are best recognized as a collective dream of hidden possibilities and potential** -- much as with the famous assertion of Martin Luther King (*I Have a Dream*, 1963). However, as an inspiring dream, there is the fundamental question of how it is remembered -- despite the sense that it was indeed coherent. As with any dream, a sense of fundamental potential and coherence remains, but the detail of how it might become reality remains elusive for civilization as a whole -- and may well tend to fade (*Societal Learning and the Erosion of Collective Memory*, 1980; *Engaging with Elusive Connectivity and Coherence*, 2018).

It can then be argued that intuitively, and without recognizing the fact, that there is a form of resonance between the "global brain" -- however unconscious its operations -- and its external manifestations like the SDGs. However this resonance can then be understood as deriving from the unexplored higher dimensional organization of the SDGs as discussed in what follows.

Transcending the apparent incoherence of the Sustainable Development Goals

Dreamtime? Understood as an influential "global dream", there is a charming irony to the sense in which the SDGs are experienced as disparately remembered elements of a "Dreamtime" -- a coherent hyperreality. The irony lies in the manner in which the Dreamtime of the early peoples of Australia is deprecated from the reality of a conventional perspective. As in that instance, those for whom the "SDG Dreamtime" is significant struggle to relate it to the perspective of those who have "woken up" -- and vice versa. For the "a-woke", the much invoked set of "human values" can also be seen in this light.

Any such framing as a collective dream then merits comparison with other dreams which have variously lost their original coherence, including Communism, Socialism, Nazism, and Capitalism. Are calls by leadership for countries to be "great again" to be seen as evoking variants of this mode? In this light the Great Reset, currently proposed so enthusiastically by some, might well be understood as a new dream. Its articulation is however perceived by many as so nebulous and entangled in conspiracy that it could held to be a a dark dream -- a collective nightmare (Peter Koenig, *The WEF's Great Reset: euphemism for a WWIII scenario? Global Research*, 27 April 2021; Kevin Smith, *Deleting the Reset: the imminent struggle ahead*, *OffGuardian*, 29 April 2021).

How will the SDGs and the Great Reset engage with each other -- as dreams -- given the complicity of the United Nations in both? Could the set of SDGs also be perceived as a nightmare by some? Such an intepretation could wll follow from any focus on the "horse-trading" origins of the SDGs.

Processes? Another way to understand the SDGs as currently experienced is in the light of the so-call "Belgian compromise" in decision-making, as explored from a cybernetic perspective (*Principia Cybernetica*). Complex issues are settled by conceding something to every party concerned, through an agreement that is usually so complicated that nobody completely understands all its implications. In spite of the apparent inefficiency of these settlements, the compromises do work in practice, because they stop the existing conflicts, and thus allow life to go on without fights or obstructions.

This argument does not focus on the possibility explored in what follows, namely that there are largely unrecognized forces in play which enable forms of coherence essentially beyond conventional understanding. Social cycle theory currently focuses on cycles of 70 to 100 years (Kondratiev wave, Technology life cycle, Hegemonic stability theory). Less evident is their relevance to the rise and fall of empires of the past over even longer periods (Johan Galtung and Sohail Inayatullah, *Macrohistory and Macrohistorians: perspectives on individual, social, and civilizational change*, 1997; Sohail Inayatullah, *Epistemes and the Long Term Future*, *Metafuture*, 2002).

The process can also be explored in terms of enantiodromia, namely the emergence of the unconscious opposite in the course of time from a cultural historical perspective (William Irwin Thompson, *From Nation to Emanation: Planetary Culture and World Governance*, 1982).

Hidden forces? More relevant to this argument is the challenge to the comprehension of the forces in play, as argued by management cybernetician Stafford Beer using an adaptation of *Le Chatelier's Principle* as applied to social systems):

> Reformers, critics of institutions, consultants in innovation, people in short who "want to get something done", often fail to see this point. They cannot understand why their strictures, advice or demands do not result in effective change. They expect either to achieve a measure of success in their own terms or to be flung off the premises. But an ultra-stable system (like a social institution)... has no need to react in either of these ways. It specializes in equilibrial readjustment, which is to the observer a secret form of change requiring no actual alteration in the macro-systemic characteristics that he is trying to do something about. (*The cybernetic cytoblast - management itself*, Chairman's Address to the International Cybernetic Congress, September 1969)

Rather than any such obstructive counterforce, a reverse effect of significance to emergence and evolution could be hypothesized. This would be readily named as "angelic" -- engendering coherence -- in contrast to the "demonic" forces to which reference is now so frequently made at every level of society. The religious framework aside, these evoke hypotheses regarding psychosocial dynamics of higher dimensionality than are conventionally recognized -- possibly commensurate with the explanations of reality by fundamental physics in terms of 10 to 26 extra dimensions. Any Gaia paradigm could well call for understanding in such terms. **Is humanity effectively constrained into coherence by unconscious systemic forces** beyond conventional awareness -- through a corollary to Beer's *Le Chatelier's Principle*?

Psychoanalysis of civilization? As a "collective dream" however, there is the further charm to the possibility that its elusive elements -- as recalled in a waking mode -- invite "interpretation" much as is the practice of psychoanalysis. **How might a psychoanalytic approach to global civilization interpret the SDG dream?** Aspects of this understanding are explored separately (*Dreamables, Deniables, Deliverables and Duende*, 2015).

With respect to the 17 SDGs, the argument here is somewhat consistent with the exploration of the "dream" of the renowned theoretical physicist Wolfgang Pauli. Throughout his life Pauli was preoccupied with the question of why the fine-structure constant, a dimensionless fundamental constant, has a value nearly equal to 1/137. This preoccupation gave rise to a long-lasting exploration with the psychoanalyst Carl Jung, as has been extensively documented by Arthur I. Miller (*137: Jung, Pauli, and the Pursuit of a Scientific Obsession*, 2010; *Deciphering the Cosmic Number: the strange friendship of Wolfgang Pauli and Carl Jung*, 2009) and by Remo, F. Roth (*Return of the World Soul, Wolfgang Pauli, C.G. Jung and the Challenge of Psychophysical Reality [unus mundus]*, 2012).

Prime number closure? Intended as they are to change the world, it is only a mathematician who could comment on the coherence of a prime number set of 17 SDGs in changing the world -- in the light of the coherence of a 17-fold set of equations held to have had a similar function, as claimed by Ian Stewart (*In Pursuit of the Unknown: 17 equations that changed the world*, 2012).

The quest in what follows is for greater understanding of the coherence of 17-fold patterns. Both Stewart and his colleague Marcus du Sautoy (*The Music of the Primes*, 2003) note the unsuspected role of the 17-fold "wallpaper group". This is seemingly one of the very rare ways in which the 17-fold set of UN Goals might be recognized as coherent (Anna Nelson, et al, *17 Plane Symmetry Groups*; Frank A. Farris. *Creating Symmetry: the artful mathematics of wallpaper patterns*, 2015).

A second related possibility is the fact that only 17 distinct sets of regular polygons (triangles, squares and hexagons) can be packed in combinations around a point (*Counting how many regular polygons combinations can form 360 degrees around a point*, *Math StackExchange*, 2019). Understood as a tesselation, this is otherwise expressed in terms of the 17 possible ways that a pattern can be used to tile a flat surface with a common single vertex. Used separately the three polygons make a total of 3

A third lead to any intuited sense of 17-fold coherence in 4 dimensions is offered in by the 64 convex uniform 4-polytopes of which 5 are polyhedral prisms based on the Platonic solids and 13 are polyhedral prisms based on the Archimedean solids. One is however duplicated with the cubic hyperprism (namely a tesseract), reducing the set to 17.

Neuronal connectivity in the global brain? Seemingly unrelated to the abstractions of mathematics are the results of recent neuroscience research of the Blue Brain Project which indicate the remarkable possibility of cognitive processes taking up even up to 11-dimensional form in the light of emergent neuronal connectivity in the human brain:

> For most people, **it is a stretch of the imagination to understand the world in four dimensions but a new study has discovered structures in the brain with up to eleven dimensions** - ground-breaking work that is beginning to reveal the brain's deepest architectural secrets...The appearance of high-dimensional cavities when the brain is processing information means that the neurons in the network react to stimuli in an extremely organized manner. It is as if the brain reacts to a stimulus by building then razing a tower of multi-dimensional blocks, starting with rods (1D), then planks (2D), then cubes (3D), and then more complex geometries with 4D, 5D, etc. The progression of activity through the brain resembles a multi-dimensional sandcastle that materializes out of the sand and then disintegrates. (*Blue Brain Team Discovers a Multi-Dimensional Universe in Brain Networks*, *Frontiers Communications in Neuroscience*, 12 June 2017) [*emphasis added*]

As noted above, it can then be argued that intuitively, and without recognizing the fact, that there is a form of resonance between the "global brain" -- however unconscious its operations -- and its external manifestations like the SDGs. However this resonance can then be understood as deriving from the unexplored higher dimensional organization of the SDGs as discussed in what follows.

Radical recursivity of cognitive implications of topology and geometry

Strategic organization in checklists: The argument here is extensively illustrated below by the use of polygonal and polyhedral forms which are the preoccupation of geometry and topology. Curiously this preoccupation tends to be unrelated to the manner in which strategies of governance are articulated -- or the manner in which institutions and communications are organized. It is however of greater relevance to the organization of computer memory (*Torus interconnect -- as used in supercomputers*, 2019).

Strategies tend to be articulated in checklists strangely constrained by particular numbers -- without any explanation of why a set of any given size "works":

- 8: notably including the Buddhist Noble Eightfold Path, and "imitations" in physics and policy analysis, as well as the Millennium Development Goals of the UN
- 10-foldness: *Habitual use of a 10-fold strategic framework?* (2020)
- 12: *Checklist of 12-fold Principles, Plans, Symbols and Concepts: web resources* (2011)
- 14: *14 Grand Challenges for Engineering in the 21st Century* of the National Academy of Engineering
- 15: 15 Global Challenges of the Millennium Project
- 16: 16 global challenges identified by Policy Horizons Canada (*The Next Generation of Emerging Global Challenges*, *Horizons*, 19 October 2018).
- 17: Sustainable Development Goals of the UN
- 20: *Requisite 20-fold Articulation of Operative Insights? Checklist of web resources on 20 strategies, rules, methods and insights* (2018)
- 30: Universal Declaration of Human Rights

A sense of geometry is recognized in metaphoric references to "institutional geometry" -- but without any use whatsoever of that discipline in the design of institutional architecture (variable geometry, "multi-speed Europe") . The discipline has however some limited applications in table seating arrangements, the design of processions and parades, and military configurations. The elements distinguished in such strategies may well be recognized as "topics" of concern -- but without any reference to the discipline which shares the

etymological origins of the term, namely topology. It is especially ironical in that, as the method of topoi, topics are fundamental to memorability and the techniques of enhancing it (Sara Rubinelli, *Ars Topica: the classical technique of constructing arguments from Aristotle to Cicero*, Springer, 2009; Frances Yates, *The Art of Memory*, 1966)

Mapping strategies in 3D: It is possible to engage in exercises through which any set of strategies can be represented in three dimensions -- mapped onto polyhedra -- rather than in one-dimensional checklists:

- *Identifying Polyhedra Enabling Memorable Strategic Mapping: visualization of organization and strategic coherence through 3D modelling* (2020)
- *Psychosocial Implication in Polyhedral Animations in 3D* (2015)
- *Towards Polyhedral Global Governance: complexifying oversimplistic strategic metaphors* (2008).

To some extent such exercises can be compared to the symbolic preoccupations of sacred geometry or to the insights associated with projective geometry (Olive Whicher, *Projective Geometry: creative polarities in space and time*, 1986). However the concern here is how the well-defined sets of distinctive operations associated with geometry or topology may be understood as having a degree of cognitive correspondence with the disparate elements of a set of strategies or concepts.

Radical recursion: This operational approach raises the question as to whether there is a radically recursive approach to engagement with both any topological configuration and any configuration of strategies. This is potentially consistent with the arguments of George Lakoff and Rafael Núñez (*Where Mathematics Comes From: how the embodied mind brings mathematics into being*, 2000). This would then offer the possibility of reframing the comprehensibility, memorability and coherence of complex strategies -- and the cognitive engagement with them.

Recursive has a mathematical significance. From a cognitive perspective it raises the question of degrees of cognitive self-reference -- potentially irrelevant to mathematicians (Hilary Lawson, *Reflexivity: the post-modern predicament*, 1986). Aspects of the argument are developed by Douglas Hofstadter (*I Am a Strange Loop*, 2007) in a sequel to his seminal study (*Gödel, Escher, Bach: an Eternal Golden Braid*, 1979).

For Steven M. Rosen (*What is Radical Recursion? Seed*, 4, 2003):

> Recursion or self-reference is a key feature of contemporary research and writing in semiotics. The paper commences by focusing on the role of recursion in poststructuralism. It is suggested that much of what passes for recursion in this field is in fact not recursive all the way down. After the paradoxical meaning of radical recursion is adumbrated, topology is employed to provide some examples. The properties of the Moebius strip prove helpful in bringing out the dialectical nature of radical recursion. The Moebius is employed to explore the recursive interplay of terms that are classically regarded as binary opposites: identity and difference, object and subject, continuity and discontinuity, etc. To realize radical recursion in an even more concrete manner, a higher-dimensional counterpart of the Moebius strip is utilized, namely, the Klein bottle. The presentation concludes by enlisting phenomenological philosopher Maurice Merleau-Ponty's concept of depth to interpret the Klein bottle's extra dimension

Ole Bjerg (*Accelerating Luhmann: Towards a Systems Theory of Ambivalence*, 2006):

> A radical recursivity is created in which the complexity drop between system and environment can no longer be taken for granted, and in which the very difference between system and environment is in danger of collapsing. This was precisely the problematic I wanted to demonstrate in the Gedankenexperiment above.

Underlying subtlety: The argument here is that the valuable distinctions, so clearly articulated with regard to mathematical operations, are usefully understood as instances of more fundamental cognitive operations which may well be evident in a variety of domains -- under other names. The difficulty is that any effort to name and define such operations within one discipline detracts from the subtler and more general understanding with is thereby denatured. Such definition, however valid on its own terms, is then to be recognized as a form of reification -- a cognitive instance of misplaced concreteness.

The point is usefully made by Christopher Alexander in emphasizing recognition of a "quality without a name" within well-designed environments (*The Timeless Way of Building*, 1979). For Alexander, this quality is objective and precise, but it cannot be named. This is a central quality which is the root criterion of life and spirit in a man, a town, a building, or a wilderness -- inviting discussion in more general terms (*Pattern of transformations as a dynamic quality without a name*, 2012).

Spectrum or spectra? A case could be made for presenting the contrast between objective distinctions (as advocated in the disciplines of science) at one end of a spectrum -- a spectrum of recursivity somewhat analogous to the autism spectrum. At the other extreme would be forms of participative cognitive engagement with reality -- readily deprecated from the other extreme as pseudoscientific and highly subjective (Henryk Skolimowski, 1994. *The Participatory Mind: a new theory of knowledge and of the universe*, 1994; Darrell A. Posey, *Cultural and Spiritual Values of Biodiversity*, United Nations Environmental Programme, 1999). Those imbuing significance in icons would then be closer to the latter -- whereas any sense of meaningful discourse with the features of nature would be even closer.

The difficulty for those claiming preference for either extreme is the extent to which they are obliged to engage in the modality of the other. Thus those preferring subjective engagement are commonly obliged to make objective use of the technology engendered by the other. It is of course common for those depending professionally on an objective perspective to indulge in psychoactive substances in quest of the perspective offered by theo other extreme.

Jones** degrees of reification concentric 7-fold axes of bias

Role of 17 2D tiling patterns in ordering SDGs?

In introducing a study of tiling patterns, Jay Friedenberg comments:

> We are surrounded by geometric patterns. These can be seen in the tiles on floors, in wallpaper patterns on our walls, and in the clothes we wear. They are also found in decorative artwork that adorns the outside of buildings and in signs, advertising, and in graphic and web design.... All major cultures have produced and enjoyed them.... What is the allure of these patterns? Why have we been compelled to create and view them over the course of human history? (*The Perceived Beauty of Regular Polygon Tessellations, Symmetry*, 2019, 11, 984)

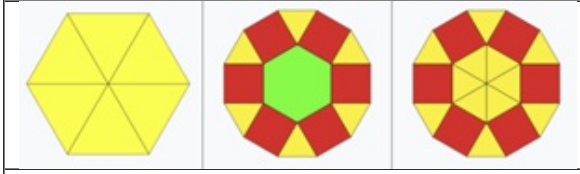
Arguably the allure of such patterns extends, if only unconsciously, to preferences for the arrangement of concepts -- and to the clustering of strategies such as the SDGs.

Friedenberg examines the questions he highlights by looking at a group of tessellations, also known as tilings, whose geometric and mathematical properties have long been well defined (*What are the conditions for a polygon to be tessellated? Mathematics Stack Exchange*). Their properties are used to predict and help explain their aesthetic appeal. Different types of tessellations can be identified. They can be categorized by the types of polygons they contain, the way they are arranged around defining vertices, and their symmetry properties. Mathematics also explores a generalization of 2D tilings into higher dimensional tessellations with a variety of geometries.

Some special kinds of tesselation include Euclidean tilings by convex regular polygons. There are only 15 types that tesselate; no convex polygon with seven or more sides can tessellate. There are only 21 combinations of regular polygons that will fit around a vertex. And of these 21 there are there are only 11 that will actually extend to a tessellation:

- Regular tilings: Every regular tiling is defined by a vertex point. There must be 6 equilateral triangles, 4 squares or 3 regular hexagons at that vertex, yielding the 3 regular tessellations. A regular polygon can only tessellate the plane when its interior angle (in degrees) divides *** (this is because an integral number of them must meet at a vertex). This condition is only met for equilateral triangles, squares, and regular hexagons.

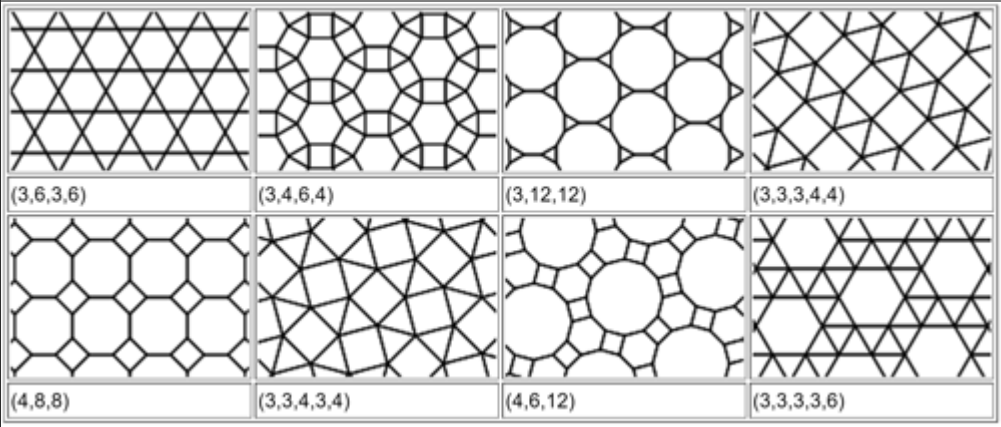
Indication of the 3 Archimedean 1-uniform semi-regular tilings



- Archimedean, uniform or semiregular tilings: In this case regular tiles of more than one shape are combined with an identical arrangement around every vertex. For every pair of vertices there is a symmetry operation mapping the first vertex to the second. This gives rise to 8 uniform tilings.

Alternative indications of the 8 Archimedean 1-uniform semi-regular tilings

Reproduced from *Wikipedia*



Reproduced from *Tessellations by Polygons* (*Math and the Art of M C Escher*)

As noted separately (Uniform tilings, *Wikipedia*), further distinctions can be made between:

- the 11 1-uniform tilings (as above), with their inclusion of 3 regular tilings, and 8 semiregular ones (with 2 or more types of regular polygon) faces.
- the 20 2-uniform tilings
- the 61 3-uniform tilings

- the 151 4-uniform tilings
- the 332 5-uniform tilings and
- the 673 6-uniform tilings

Such patterns frame the question as to whether and how their recognition governs the organization of patterns of concepts and strategies. What is the cognitive role of pattern recognition with respect to sets of concepts? What degrees of order in concept articulation are considered appropriate and "well-formed" -- and why?

There are exactly 17 two-dimensional space (plane symmetry) groups. These are sometimes called wallpaper groups, as they represent the seventeen possible symmetry types that can be used for a wallpaper (as noted below). It is the two dimensional case of a more general problem: the 3D case, for example, can be interpreted as the number of different crystalline structures.

This little known property is suggestive of a set of fundamental cognitive patterns implied by the distinctive nature of each SDG -- offering the possibility of an unusually distinctive image by which it could be identified (reminiscent of the role of Scottish tartans). Unfortunately the term "wallpaper" also invites more cynical interpretation as it might relate to SDGs -- in "papering over the cracks" of global governance.

Yet to be clarified is why such symmetrical patterns are cognitively satisfactory and how the geometric constraint on their formation as a tesselation offers an implication of closure somehow vital to a sense of the completeness and closure of a strategic set. Whereas tesselation requires that only 2π worth of polygonal angles can be arranged around each point, it is unclear how this functions as a constraint on strategic coherence.

3-fold and 4-fold: The questions are given a degree of focus by the widespread preference for 3-fold sets of concepts, as can be variously discussed (*Triangulation of Incommensurable Concepts for Global Configuration*, 2011). This explored the following themes:

Triadic logic?	Interrelating multiple triadic approaches
Triadic dialectics	Integrative thinking
Triadic strategic applications	Enhancing coherence through spherical triangulation
Triadic conceptualization	Navigation of the strategic universe
Triadic education and learning	

An analogous argument could be made for 4-fold sets of concepts, given the extensive use of explanation in terms of quadrants of categories:

- Dana Fusek: *Explaining Four Quadrants of Data Visualization Charts*
- John Lee: *The 4 Quadrant Model: a co-occurring treatment framework* (*Choose Help*, 13 September 2019)
- Daniel Mullins: *The 4 Types of Visualisation and the Role They Play in Market Research* (*B2B International*)
- Marten Kuilman: *The Quadralectic Theory*; *Quadralectic Architecture*.
- *Boston Consulting Group (BCG) Matrix* (*Corporate Finance Institute*)
- *The Four Quadrants of Integral Metatheory: on the value of integrating multiple perspectives* (*Psychology Today*, 8 October 2020)
- SWOT 2x2 matrix: a strategic planning technique

The relation between 3-fold and 4-fold articulations is however a challenge -- perhaps most usefully highlighted by tiling patterns. The relation has notably been a preoccupation for Carl Jung and those influenced by his perspective -- framed by the leitmotiv of the Axiom of Maria. (Marie-Lousie von Franz, *Number and Time: reflections leading toward a unification of depth psychology and physics*, 1986). As noted by Lance Storm:

> In regard to the problem of three and four, the Jungian scholar Marie-Louise von Franz once wrote: "It becomes evident that a psychological problem of considerable importance is constellated between the numbers three and four". (*From Three to Four: the influence of the number archetype on our epistemological foundations*, *Quadrant: the journal of the C G Jung Foundation*, 33, 2003, 1)

5-fold? In the light of the specific exclusion of the pentagonal polygon from the possible tiling patterns described above, the status of 5-fold patterns merits careful consideration in relation to the 3-fold and the 4-fold, especially given its fundamental role in the Hygeia of the Pythagoreans and the *Wu Xing* model of Chinese philosophy (*Memorable dynamics of living and dying: Hygeia and Wu Xing*, 2014).

From a 2D planar tiling perspective, the 5-fold is not "compatible" with 3-fold, 4-fold or 6-fold. It is however curious to note its special navigational role in 3D spherical geometry as the Pentagramma Mirificum (*Global Psychosocial Implication in the Pentagramma Mirificum: clues from spherical geometry to "getting around" and circumnavigating imaginatively*, 2015). Of potential relevance is the particular importance given to 5-foldness by Peter Senge (*The Fifth Discipline: the art and practice of the learning organization*, 1990). This has been used as an exploration a critique of the patterning process (*Patterning Intuition with the Fifth Discipline: critical review of the conclusion of the 5-fold Patterning Instinct*, 2019; *5-fold Pattern Language*, 1984).

6-fold: The mathematics of tiling are closely related to the constraints governing the distinctive symmetries of crystal systems (*Crystal symmetry*)

> The crystal classes may be sub-divided into one of 6 crystal systems 6 crystal systems. Space groups are a combination of the 3D lattice types and the point groups (total of 65). Each of the 32 crystal classes is unique to one of the 6 crystal systems: Triclinic, monoclinic, orthorhombic, tetragonal, hexagonal and isometric (cubic) hexagonal and isometric (cubic) Interestingly,

> while all mirror planes and poles Interestingly, while all mirror planes and poles of rotation must intersect at one point, this point may not be a center of symmetry

The question is then how (and why) such 6-foldness might "translate" into coherent thinking in other domains, as for example:

- John Greathouse: *Six Fundamental Laws That Control Your Business*, (*Forbes*, 5 July 2012)
- Anne Marie Helmenstine: *Six Steps of the Scientific Method* (*ThoughtCo*, 18 February 2020)
- Edward de Bono: *Six Thinking Hats* (1985); *Six Frames For Thinking About Information* (2008)
- Raymond Abellio: *La Structure absolue. essai de phénoménologie génétique* (1965)

Six Pack: The Primary Flight Instruments	Basic operations/primitives giving a language Turing completeness
1. Airspeed Indicator *(Pitot Static)* 2. Attitude Indicator *(Gyro)* 3. Altimeter *(Pitot Static)* 4. Vertical Speed Indicator *(Pitot Static)* 5. Heading Indicator *(Gyro)* 6. Turn Coordinator *(Gyro)*	• Right: Move the Machine's head to the right of the current square • Left: Move the Machine's head to the left of the current square • Print: Print a symbol on the current square • Scan: Identify any symbols on the current square • Erase: Erase any symbols presented on the current square • Nothing/halt: Do nothing
Geoff McKay, *LearnToFly*, 13 March 2010	Jack Copeland, *Turing's O-Machines*, *Alan Turing.net*, May 2000

Language appropriate to distinguishing the underlying nature of any fundamental operations is offered in the case of the 6-fold set of Grothendieck's six operations. This is a formalism in homological algebra which originally arose from the relations in étale cohomology that arise from a morphism of schemes $f : X \to Y$. The basic insight was that many of the elementary facts relating cohomology on X and Y were formal consequences of a small number of axioms. These axioms hold in many cases **completely unrelated to the original context, and therefore the formal consequences also hold**. The six operations formalism has since been shown to apply to contexts such as D-modules on algebraic varieties, sheaves on locally compact topological spaces, and motives. The operations are six functions, usually between derived categories and so are actually left and right derived functors.

- the direct image
- the inverse image
- the proper (or extraordinary) direct image
- the proper (or extraordinary) inverse image
- internal tensor product
- internal Hom

Role of the 17-fold "wallpaper group" in ordering SDGs

Another way to characterize the tessellations indicated above is by their symmetry properties. There are four basic symmetry transformations that when applied can transform a shape in a tessellation upon itself or into another identical shape. These are translation, reflection, rotation, and glide reflection -- otherwise known as Euclidean plane isometries.

Known as the "wallpaper group", it has been demonstrated that **17 distinct sets of regular polygons (triangles, squares and hexagons) can be packed in combinations around a point** (*Counting how many regular polygons combinations can form 360 degrees around a point*, *Mathematics Stack Exchange*, 2019). Understood as a tesselation, this is otherwise expressed in terms of the **17 possible ways that a pattern can be used to tile a flat surface with a common single vertex**. As noted above, used separately the three polygons make a total of only 3

A wallpaper group (or plane symmetry group or plane crystallographic group) is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art, especially in textiles and tiles as well as wallpaper. Valuable clarifications of the challenge of explaining the wallpaper group are presented in the associated *Wikipedia* pages (*Talk: Wallpaper group*; *Wallpaper group diagrams*; *Wallpaper groups*; *Ebene kristallographische Gruppe*)

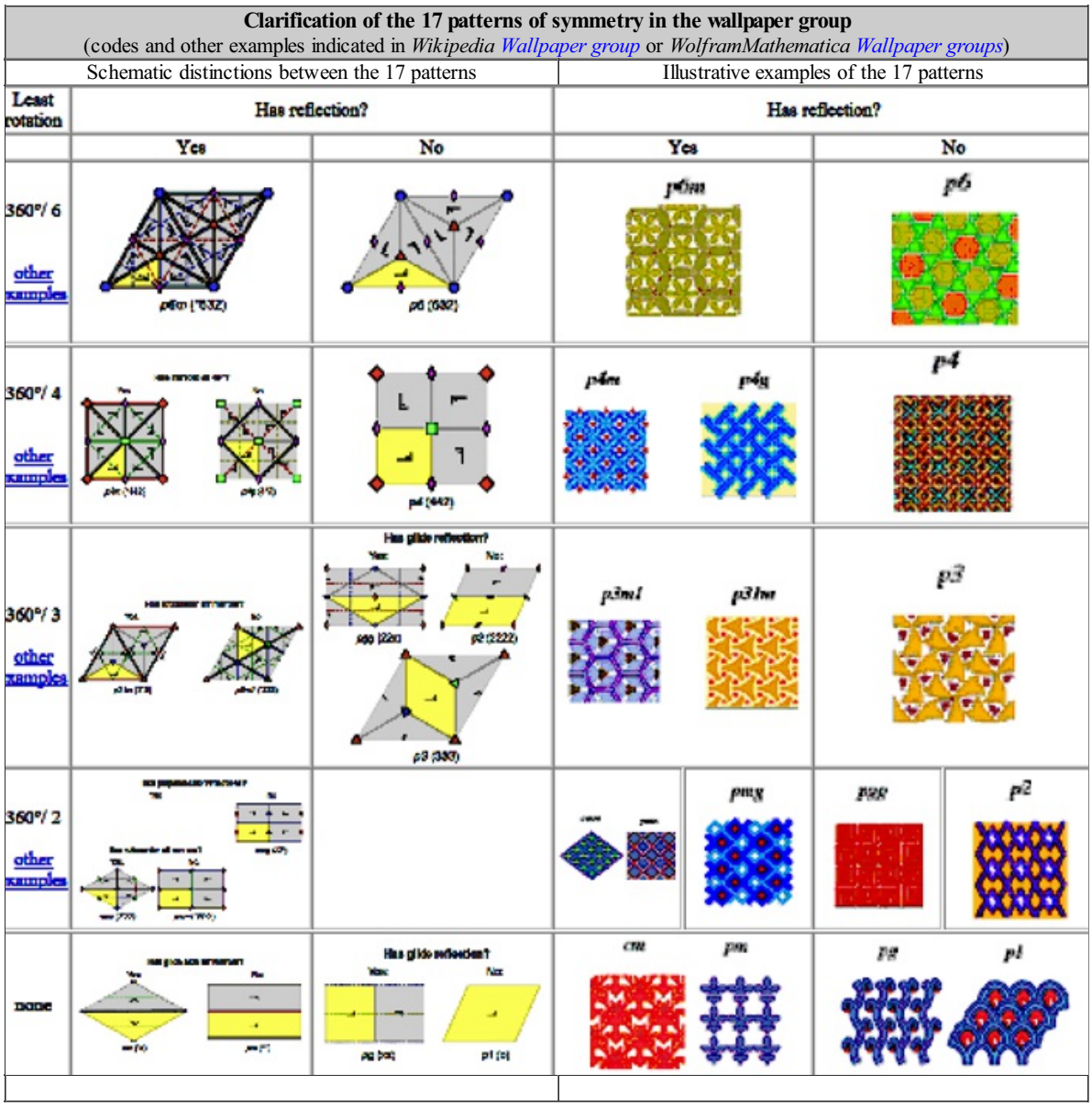
The set of 17 derives from the fact that a graph can be viewed as a polygon with face, edges, and vertices, which can be unfolded to form a possibly infinite set of polygons which tile either the sphere, the plane or the hyperbolic plane. If the Euler characteristic is positive then the graph has an elliptic (spherical) structure; if it is negative it will have a hyperbolic structure; but if it is zero then it has a parabolic structure. When the full set of possible graphs is enumerated it is found that only 17 have Euler characteristic 0, namely the 17-fold wallpaper group.

There are various efforts to render comprehensible, through schematics and examples, the variety of patterns in the wallpaper group:

- Anna Nelson, Holli Newman and Molly Shipley: *17 Plane Symmetry Groups*
- David E. Joyce: *Wallpaper Groups: the 17 plane symmetry groups* (Clark University, 1997)
- Steve Edwards: *Identifying the 17 Plane Symmetry Groups* (Kennesaw State University, 2015)
- Samantha Burns, Courtney Fletcher, and Aubray Zell: *The 17 plane symmetry groups* (Boise State University, 2012)
- Xah Lee: *The 17 Wallpaper Groups* (2021)
- *The Simplest Diagram of the 17 Symmetry Types, Ever* (*Artlandia*, 2017)
- *17 Wallpaper Groups* (based on Doris Schattschneider, *M. C. Escher: Visions of Symmetry*, W. H. Freeman)

As noted by Marcus du Sautoy, the Islamic Alhambra palace in Granada contains examples of all 17 patterns (Maria Francisca Blanco Blanco and Ana Lúcia Nogueira de Camargo Harris, *Symmetry Groups in the Alhambra*). This case has given rise to useful criticism of

how such patterns are detected (Branko Grünbaum, *What symmetry groups are present in the Alhambra? Notices of the American Mathematical Society*, 53, 2006, 6; Klaus Landwehr, *Visual Discrimination of the 17 Plane Symmetry Groups*, *Symmetry*, 3, 2011, 2).



Although no such indication is offered, ironically **this is seemingly one of the very rare ways in which the 17-fold set of UN Goals might be recognized as coherent** (Anna Nelson, et al, *17 Plane Symmetry Groups*; Frank A. Farris. *Creating Symmetry: the artful mathematics of wallpaper patterns*, 2015). Others are variously presented (*Prime Curios: 17*; Tanya Khovanova (*Number Gossip: 17*).

Cognitive operations potentially analogous to generation of tiling patterns

"Cognitive tiling" or "conceptual tiling"? As noted above, the set of 17 tiling patterns is generated by distinctive combinations of so-called Euclidean plane isometries. With respect to the geometry of the patterns, these operations are described in terms of translation, rotation, reflection and glide reflection. The question is to what degree these operations bear significant resemblance to cognitive operations, most obviously as might be perceptible in the discourse by which strategies are engendered.

Ironically, it is appropriate to note that "tiling" may itself be used in a way which is relevant to this argument. Most specific are quotations by Bernardo Kastrup:

- I invite you to try and briefly remove the conceptual tiling that blocks your view of reality, so you can see the raw chunks of perceptual pixels in the medium of mind.
- But Zen is largely an attempt to, during meditation, transcend thought (i.e. the intellect) and the "conceptual tiling" we live in. And, insofar as our whole civilization an culture are based and driven by "intellectual tilings", Zen does try to transcend that too, doesn't it? Naturally, one always returns to the ego state, and so do Zen practitioners. (*The Biggest Error Ever Made in the Name of Science*, *Metaphysical Speculations*, October 2013)

Related understandings are offered by:

- Francis Rousseaux: Like the sacred place, the conceptual model is intended to reassure and protect us against the thought and experience of a singular situation. Can all situations be adapted to such an automatic prescription? Obviously, never, because the conceptual tiling is an integral part of the situation, given in this case as a different and overloaded form, but always allowing the

possibility of a singular interpretation. (*Knowledge acquisition or manifestation of the thought? Knowledge Management and Philosophy*, 2003)

- Martin J Pickering and Simon Garrod: However, in my view, the particular integration of production and comprehension processes that the authors propose is unrealistic in consisting of a complex cognitive tiling of predictive modeling processes... (*An integrated view of language production and comprehension*, *Behavioral and Brain Sciences*, 36, 2013, 4)

Geometrical metaphors in discourse: There is an extensive literature on the role of geometrical metaphors in discourse, although this tends to focus on the use of metaphor in mathematical education. This includes:

- Mathieu Aubry: *Metaphors in Mathematics* (2009)
- Maciej Rosiński: *Metaphor Activation in Multimodal Discourse: case studies on the emergence of geometrical concepts.* (University of Warsaw Repository, 2018)
- Gloriana González: *A geometry teacher's use of a metaphor in relation to a prototypical image to help students remember a set of theorems* (*The Journal of Mathematical Behavior*, 32, 2013, 3)
- Wajeeh Daher: *Students' Positioning and Emotions in Learning Geometric Definition* (*Journal on Mathematics Education*, 11, 2020, 1)
- Carlos Cornejo, et al: *The physiognomic and the geometrical apprehensions of metaphor* (*Culture and Psychology*, 19, 2013, 4)
- J. Rees: *Geometry and Rhetoric: Thinking about Thinking in Pictures* (*Nexus Network Journal*, 12, 2010, 3)
- Nathalie Sinclair et al: *To be or to become: how dynamic geometry changes discourse* (*Research in Mathematics Education*, 10, 2008, 2)
- Barry Stocker: *Pascal and Derrida: Geometry, Origin and Discourse* (*Symposium*, IV, 2000, 1)

For Patricio Herbst:

> The present study, close to Otte's [1983[approach, investigated a way to carry out an internal critique of a text that would allow one to address questions like: how docs textuality as a linear temporal process create its own mathematic? And what are the textual meanings of the mathematical notions which are developed inside a textbook and along that temporal axis? The main theoretical influences on my efforts to conceptualize and analyze text have been Foucault's (1972) archaeology of knowledge, Chevallard's (1991) theory of didactic transposition and Eco's (1979) notion. of model reader and open-closed work. However, their presence is mostly' silent; my understanding of their works guides my way of interrogating and asserting., but they are not responsible for my actual questions and conclusions.(*The Number-Line Metaphor in the Discourse of a Textbook Series* (*For the Learning of Mathematics*, 17, 1997, 3)

The argument can be developed with respect to the simplest features of geometry, namely points, lines, volumes and holes (*Metaphorical Geometry in Quest of Globality -- in response to global governance challenges*, 2009),

Tiling as an instance of cognitive closure? What could be recognized as the central mystery of any recognized tiling pattern is how a sense of satisfaction is associated with the closure of pattern recognition. More generally this is the satisfaction of resolving a riddle or completing, any puzzle (crossword, sudoku, Rubik cube, etc), or solving a problem of mathematics or design. How is such closure most fruitfully understood in generic terms (Hilary Lawson: *Closure: A Story of Everything*, 2001).

Mathematics frames the matter in terms of symmetry in relation to closure, with an extensive literature on closure in set theory and on symmetric closure -- expressed formally through requisite abstractions (Marcin J. Schroeder, *Concept of Symmetry in Closure Spaces as a Tool for Naturalization of Information* 2008). In general, the closure of a relation is the smallest extension of the relation that has a certain specific property such as the reflexivity, symmetry or transitivity. Distinctions are made between difunctional closure, contact closure, reflexive transitive closure, reflexive transitive symmetric closure, equivalence closure and congruence closure.

A seemingly quite unrelated approach to the experience of closure has been developed by Gestalt psychology which offers a numbetr of principles (termed "laws") that now influential in design, including: Law of Closure, Law of Symmetry, Law of Similarity, Law of Proximity. Thus the Law of Symmetry is the gestalt grouping law that states that elements that are symmetrical to each other tend to be perceived as a unified group; with closure, parts are combined to form a simpler whole (Steven Bradley, *Design Principles: Visual Perception and The Principles of Gestalt*, *Smashing Magazine*, 29 March 2014). These insights have been developed in relation to problem solving.

Metaphorical implications from tiling operations: It is curious to note, although consistent with the possibility argued here, that the geometrical terms by which the tiling patterns are distinguished are far from unrelated to those descriptive of discourse and debate -- if only metaphorically and/or via synonyms. Potentially more relevant, however, is the manner in which metaphorical uses of the geometrical terms are employed far more loosely in discourse than they are with respect to the transformations associated with those operations. This may indeed have implications for distinctive formulation of strategies.

Exploration of cognitive implications of operations engendering distinctive "tiles"		
operation	synonym of cognitive significance	metaphors
translation	movement; shift framework; change language; change of perspective	Jessie Chaffee, *36 Metaphors for Translation*, *Words without Borders*
rotation	reorientation; change of orientation; rotation of office	*Rotation and Other Metaphors*; *Multidimensional Word and Sentence Rotation*; *Rotation as Metaphor*; *Crop Rotation as a Metaphor for Interdisciplinary Software Work*; *What Is an Orientational Metaphor?*; *Orientational Metaphors*
reflection	reversal, opposite perspective; mirroring	*Reflecting on the Metaphor and Practice of Reflection in*

		Education; *A critical and functional analysis of the mirror metaphor with reference to the media's responsibility towards society*; *Mirror as a metaphor*; *Human Mirrors: metaphors of intersubjectivity*
glide reflection	segue; nimble?; "revolving door"? "fast footwork"?	*The Revolving Door Metaphor*

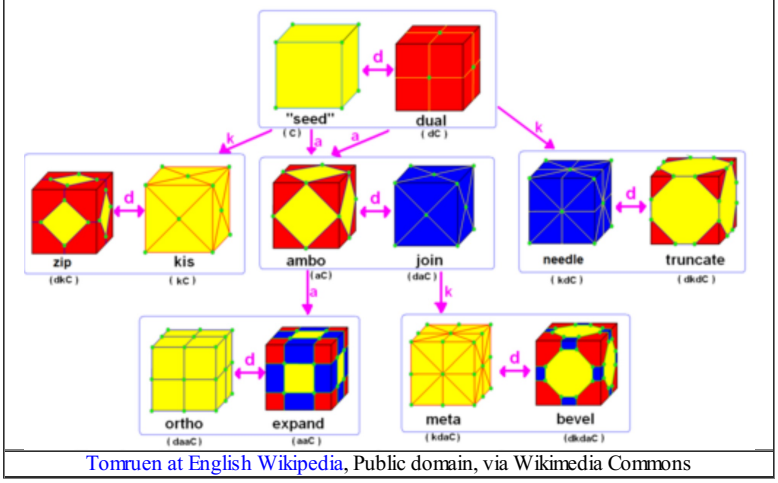
Given the common interpretations of the terms describing the operations in the Euclidean plane, it is to be expected that the metaphors would be suggestive of dynamics that are well recognized in discourse, group dynamics, negotiation and politics. It is however possible that their generic significance would be even more familiar as articulated in dance -- notably through the manner in which dialogue and group dynamics are frequently framed in musical terms or as a "dance":

- Dana Mills: *Dance and Politics: moving beyond boundaries* (2016)
- Anca Giurchescu: *The Power of Dance and Its Social and Political Uses* (*Yearbook for Traditional Music*, 33, 2001)
- Susan A. Reed: *The Politics and Poetics of Dance* (*Annual Review of Anthropology*, 27, 1998),
- Siobhan Burke: *Making (and Seeing) Dance in the Politicized World* (*The New York Times*, 17 November 2017)

Do the dynamics of pattern transformation between "left" and "right" in politics call for reframing in terms of dance? More fundamental are then the cognitive implications of any such dance, as suggested by the work of Mark Johnson (*The Meaning of the Body: aesthetics of human understanding*, 2007) and Maxine Sheets-Johnstone (*The Primacy of Movement*, 1999). One exploration of potential relevance is that of Paris Arnopoulos (*Natural Energy and Social Power: metaphors from physics to politics*, Gamma Institute, 1985).

Cognitive implications of operational modification of polyhedra: Distinctive polyhedra may be created through modification of a seed polyhedron by various prefix operations, as described by the Conway polyhedron notation, and discussed separately (*Topological operations on polyhedra as indicative of cognitive operations*, 2021). The following example shows how 11 new forms can be derived from the cube using 3 operations (named **d**ual, **a**mbo and **k**is). The new polyhedra are shown as maps on the surface of the cube so the topological changes are more apparent.

Conway relational chart
Showing 12 forms created by 3 operations on the cube



Tomruen at English Wikipedia, Public domain, via Wikimedia Commons

The three basic operations which are sufficient for generation of the Platonic and Archimedean polyhedra are:

- **d**ual: replaces each face with a vertex, and each vertex with a face
- **a**mbo: creates degree-4 vertices (otherwise kn own as rectification)
- **k**is: raises a pyramid on each face

Other operations have been distinguished, together constituting a more conventional total of 13; these have been extended in the Antiprism application of Adrian Rossiter to a further set of 18 -- although how they might together be understood as a set remains unclear (*Conway Notation Transformation*, *Antiprism*; *Wythoff-style constructions*, *Antiprism*).

As developed by Rossiter to prodiuce weave patterns on tiled surfaces, the "Wythoff constructive notation" can be used to define "Conway" operations more precisely. According to Rossiter, the number of Conway operator stype patterns is unlimited. raising the question as to whether they can be meaningfully ordered. This would probably depend on the definition of "Conway operation" pattern, and in particular the definition of when the pattern tiles close.

Of primary relevance to this argument is how such operations might be distinguished in cognitive terms with regard to the transformation of any ordered memeplex in the course of discussion -- or in the development of a strategy. What indeed are the distinctive "operations" in discourse? Again some terms identified with respect to transformation of polyhedra offer clues when metaphorically understood: truncate, expand, join. Valuable indications of potential relevance to discourse analysis are to be found in the work of B. R. S. Recht (*Operations on Polyhedra*, *GitHub*, 15 July 2019).

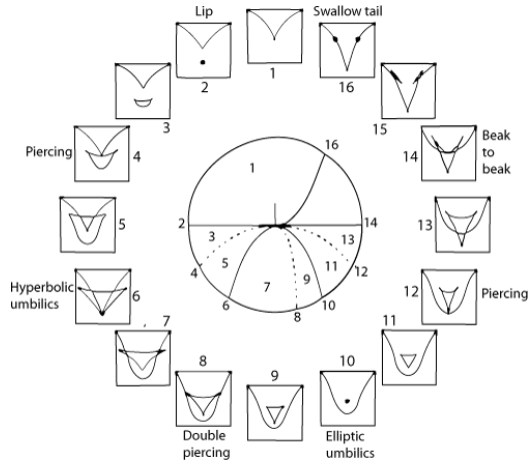
Morphogenetic implications: The generation of a tiling pattern could potentially be understood in terms of morphogenesis as notably articulated by the topologist René Thom (*Structural Stability and Morphogenesis*, 1972), especially given his interest in dance, as noted

below..

There is however little explicit trace of this connection in the literature, with the apparent exception of a study by Mostafa Alani on the hexagonal patterns in Islamic architecture (*Algorithmic investigation of the actual and virtual design space of historic hexagonal-based Islamic patterns*, *International Journal of Architectural Computing*, 16, 2018,1). This found that such designs correlate to each other beyond just the formal dimension and that deep, morphological connections exist between them. The study identifies these connections and presents a categorization system that groups designs together based on their "morphogenetic" characteristics. As indicated there, determining the wallpaper group is important for identifying the "fundamental unit" representing the minimum geometric composition that is being systematically replicated.

From this perspective it is interesting to recognize the "suggestive" nature of the following sequence of 16 images -- possibly to be understood as "visual primitives" characteristic of the attraction dynamics of intercourse, in its specific and general senses. From a cognitive perspective, the question might be asked how Thom's 16-fold identification relates both to the set of Conway operations (above) and to the 16+1 set of SDGs.

Changes: the locus of principal changes of topological type
reproduced from René Thom, *Structural Stability and Morphogenesis*, 1972



1. curve with cusp pointing downward
2. appearance of new point at origin, where lip formation begins --
3. this grows... .
4. pierces the cusp..
5. and crosses it ...to form the phallic mushroom... characteristic of the parabolic umbilic...
6. the cusp meets the lower branch of the lip in a hyperbolic umbilic...
7. and then the two branches cross to form a curvilnear triangle piercing laterally a convex curve
8. the triangle shrinks, first touching the curve
9. and then shrinking inside it
10. to form a hypercycloid with three cusps, and finally vanishes in an elliptic umbilic..
11. reappearing immediately with the same orientation
12. its lower cusp meets the curve
13. and pierces it
14. the curve and upper edge of the triangle touch in beak-to-beak singularity, which separates
15. producing two symmetric swallowtails, reabsorbed into the curve
16. leading to the original configuration

Higher dimensional coherence of SDGs implied by a set of 17 4-dimensional polyhedra?

Given their engagement with time, strategies could be understood as necessarily requiring a 4D perspective -- otherwise they could only be considered as static monuments to possibilities.

A further lead to any intuited sense of 17-fold coherence in 4 dimensions is offered by the 64 convex uniform 4-polytopes of which 5 are polyhedral prisms based on the Platonic solids and 13 are polyhedral prisms based on the Archimedean solids. One is however duplicated with the cubic hyperprism (namely a tesseract), reducing the set to 17.

Known as 4-polytopes (or polychora), there are 64 convex uniform 4-polytopes. This recognized set includes 6 regular convex 4-polytopes, and excludes the infinite sets of the duoprisms and the antiprismatic prisms. The set of 64 itself invites further reflection (as discussed below), With respect to the argument developed here, the 64 are composed of:

- 5 are polyhedral prisms based on the Platonic solids (1 overlap with regular since a cubic hyperprism is a tesseract)
- 13 are polyhedral prisms based on the Archimedean solids

Making a total of 17 (presented visually below), with the remaining 47 being:

- 9 self-dual regular A4 [3,3,3] group (5-cell) family.
- 9 self-dual regular F4 [3,4,3] group (24-cell) family. (Excluding snub 24-cell)
- 15 regular B4 [3,3,4] group (tesseract/16-cell) family (3 overlap with 24-cell family)
- 15 regular H4 [3,3,5] group (120-cell/600-cell) family.
- 1 special snub form in the [3,4,3] group (24-cell) family.
- 1 special non-Wythoffian 4-polytope, the grand antiprism.

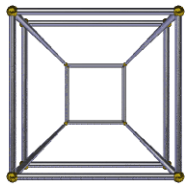
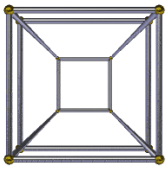
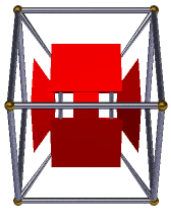
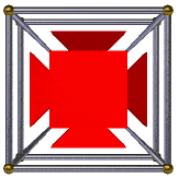
If the SDGs are all about achieving results over time -- with a focus on 2030 -- then **gaining a strategic sense of time as a fourth dimension is a major challenge** (*Strategic Embodiment of Time: configuring questions fundamental to change*, 2010; Four-dimensional requisite for a time-bound global civilization? 2015).

As might be imagined, 4-dimensional structures are a major challenge to comprehension (*Comprehending the shapes of time through four-dimensional uniform polychora*, 2015). If the coherence of the set of SDGs is assumed to be ordered in that respect, there is clearly a need to clarify how such structures might be understood -- especially if the pattern of communications vital to strategic viability can only be recognized in terms of forms of "hyperconnectivity" beyond the conventional sense of patterns of connectivity. There is seemingly a need to engage with "hyperreality" -- especially as the challenges of governance are recognized as increasingly surreal (*Surreal nature of current global governance as experienced*, 2016).

The challenge to comprehension can be visually suggested by the following animations -- given the sense that 4D cannot be represented in 3D except through a dynamic associated with the fourth dimension. The simpler animations on the left render transparent the presence of "cells" presented in red in those on the right. A cell is the three-dimensional analogue of a polyhedral face, and is therefore a polyhedron in its own right.

The animation below is of the tesseract -- included here for illustrative purposes only, since it is excluded from the set of 17 illustrated in what follows. It is excluded, exceptionally, because as a cubic hyperprism it overlaps with the regular tesseract (as noted above). It is however the most familiar form of 4D polyhedron and is of significance to studies of oppositional logic and its associated geometry (*Oppositional logic and its requisite polyhedral geometry*, 2019).

Cubic hyperprism or tesseract
(excluded from set of 17 4D prisms based on Platonic polyhedra)

Animation (without cells)		Simulation of rotation in 4D (showing cells)	
			

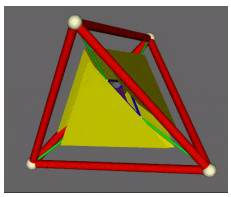
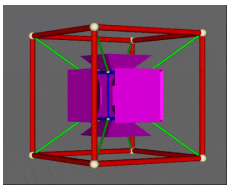
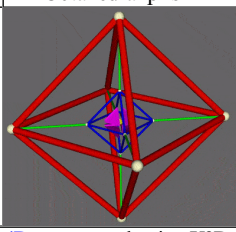
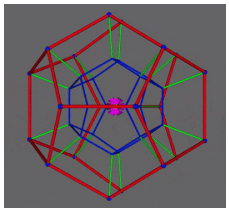
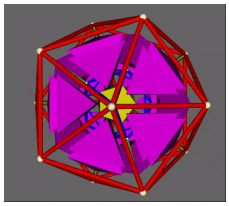
Animations prepared using Stella4D

The above animations and those which follow were all produced via the Stella4D application (freely downloadable), indeed it would have been difficult to imagine how to prepare them otherwise.

4D prisms based on Platonic polyhedra: Of the 17 4D polyhedra of relevance, a first group is based on the Platonic 3D polyhedra. The cubic exception is included here for comparative purposes only. Since it is not possible to represent adequately the complex dynamics of a 4D form, the approach taken here is to animate a 3D projection of the form -- using the animation to highlight the presence and absence of the cells characteristic of the 4D form. The animation avoids the need for producing two separate animations -- one without and one with cells (as conventionally presented statically).

The primary purpose of these animations is to indicate the challenge of comprehending the complex dynamics of strategies -- like the SDGs -- in 4D. It is of course the case that other design options could be used for this purpose (alternative colour choices, rates of movement, size of elements, etc). The quality of the animations is unsatisfactory because of the challenge of recording from the software generating them and displaying the results in web documents.

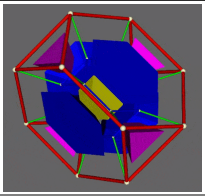
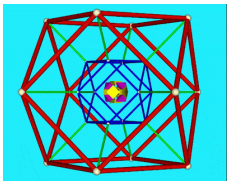
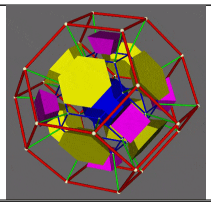
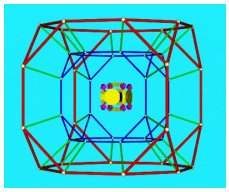
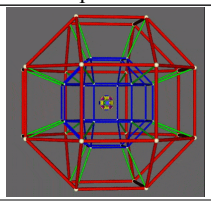
Animations of 3D representations of 4D Prisms based on Platonic polyhedra
(indicate presence and absence of cells)

Tetrahedral prism	Cube prism	Octahedral prism	Dodecahedral prism	Icosahedral prism
				

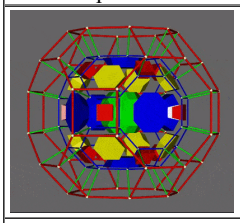
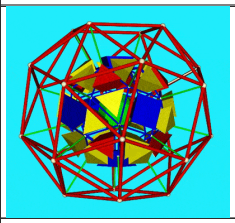
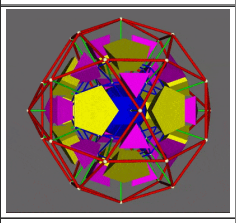
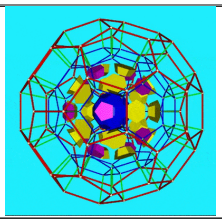
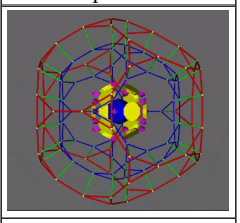
Animations (above and below) prepared using Stella4D, augmented using X3D (with the guidance of Sergey Bederov of Cortona3D)

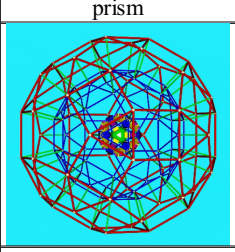
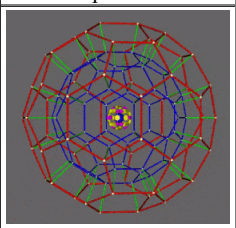
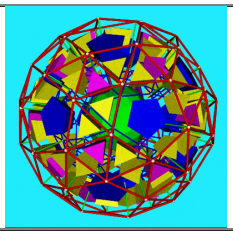
4D prisms based on Archimedean polyhedra: The remaining 13 4D polyhedra of the set of 17, are based on the 13 Archimedean solids. Their animations as 4D prisms are presented below in successive clusters, in each case with juxtaposition of no-cell, with-cell and unfolded variants to facilitate comparison.

Animations of 3D representations of 4D Prisms based on based on Archimedean polyhedra (1-5 of 13)
(indicating presence and absence of cells)

Trunc. tetrahedral prism	Cuboctahedral prism	Trunc. octahedral prism	Trunc. cube prism	Rhombicuboctahedral prism
				

Animations of 3D representations of 4D Prisms based on based on Archimedean polyhedra (6-10 of 13)
(indicating presence and absence of cells)

Trunc. cuboctahedral prism	Snub cube prism	Icosidodecahedral prism	Trunc. icosahedral prism	Trunc. dodecahedral prism
				

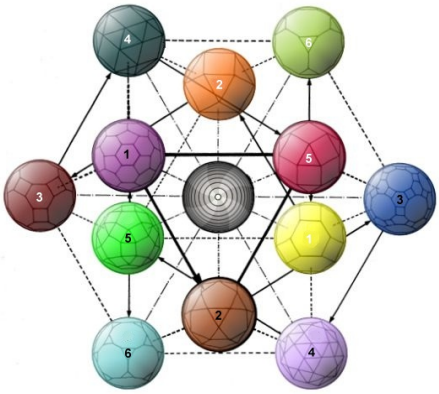
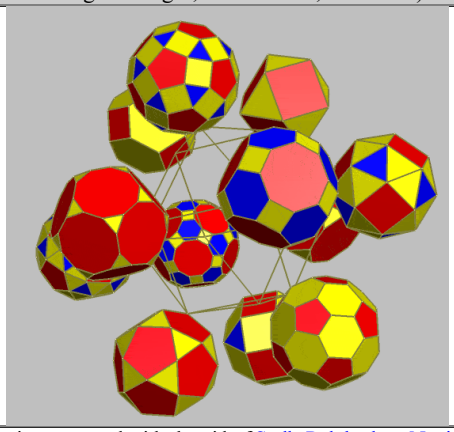
Animations of 3D representations of 4D Prisms based on based on Archimedean polyhedra (11-13 of 13) (indicating presence and absence of cells)				
	Rhombicosidodecahedral prism	Trunc. icosidodecahedral prism	Snub dodecahedral prism	
				

Individually each of the representations of 4D forms above could be understood as indicative of the requisite complexity through which the individual SDGs merit recognition in endeavouring to respond to the challenges of governance with which they are individual associated.

Interrelationship of 17 SDGs modelled by 17 regular polyhedra in 4D

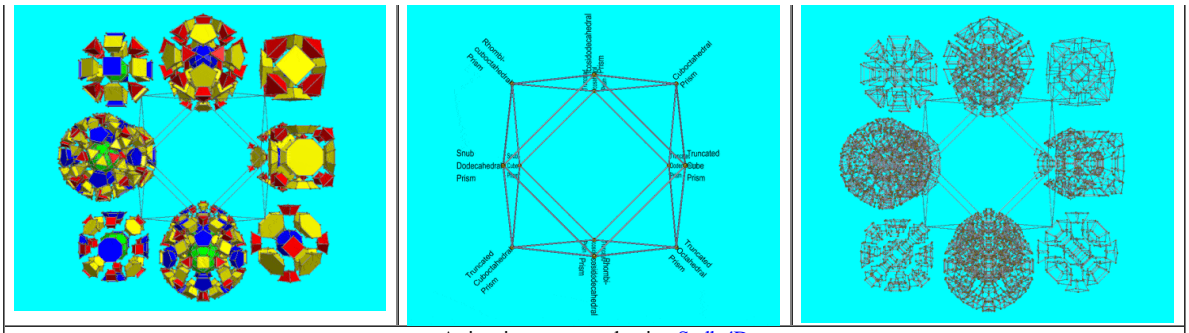
Potentially of far greater relevance are the relations between those forms as being indicative of the coherence of the set of SDGs as a whole in responding to the global problematique as a whole. It is the case that there are geometrical transformations -- or pathways -- between the polyhedra in their 3D forms (as indicated in the schematic below). These are suggestive of lines of communication more conventionally understood. How these merit recognition in a 4D hyperreality is another matter calling for further reflection and visualization.

A point of departure is the closest packing configuration of 12 Archimedean polyhedra around the 13th (image below left), namely the truncated tetrahedron as clarified by Keith Critchlow (*Order in Space: a design source book*, 1969). As discussed separately, this cuboctahedral array can be partially represented by the following animation (*Packing and unpacking of 12 semi-regular Archimedean polyhedra*, 2015)

Arrangement of the 12 Archimedean polyhedra in their most regular pattern, a cuboctahedron	
Arrangement as presented by Keith Critchlow with inclusion of truncated tetrahedron at centre	Rotation of cuboctahedral array of 12 Archimedean polyhedra (around an omitted 13th at the centre; totalling 984 edges, 558 vertices, 452 faces)
	
Adaptation from Order in Space	Animation prepared with the aid of Stella Polyhedron Navigator;

As noted above, the 17 4D polyhedra presented there are based on the 5 Platonic polyhedra and the 13 Archimedean polyhedra. It could then be assumed that the 3D-projections of the 4D variants could potentially be configured in a similar manner as shown below -- if only for mnemonic purposes. The quality of the animations is especially problematic because of the size of the images.

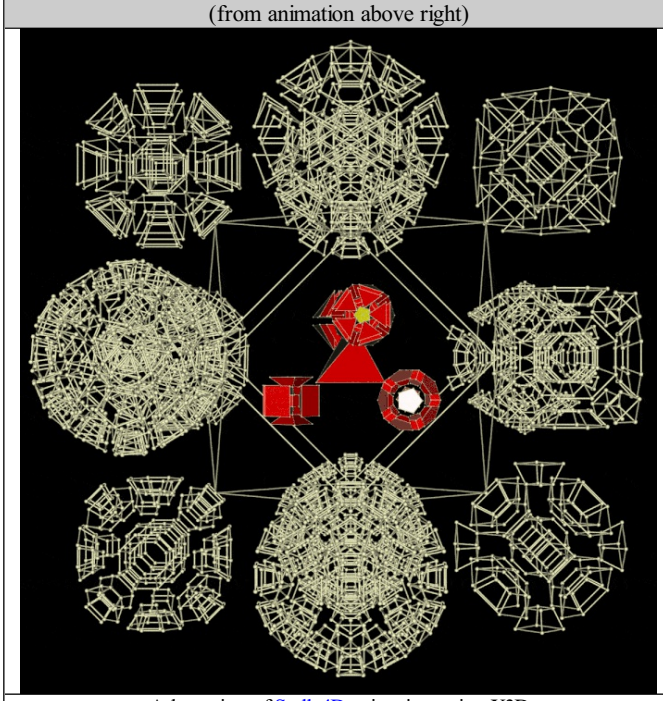
Cuboctahedral array of 12 3D-projections of 4D configurations based on Archimedean polyhedra		
Animation showing cells	Animation with labels only	Animation without cells



Animations prepared using Stella4D

Similarly for 4 of the 3D-projections of the 5 4D Platonic variants, these could be configured around the tetrahedral 5th. The Platonic array could then be nested within the Archimedean array as shown below. The tetrahedron and truncated tetrahedron omitted from the centre of the combined array would be fused in some manner -- and for purposes of visualization are both omitted here.

Animation of tetrahedral array of 4 3D-projections of 4D Platonic polyhedra centered within a static cuboctahedral array of 3D-projections of 12 Archimedean polyhedra
(from animation above right)



Adaptation of Stella4D animation using X3D

These displays frame discussion of how they might be understood with respect to the 17 SDGs and their interrelationship -- given the argument that all 17 4D polyhedra are indicative of a coherent global configuration. Questions could focus on:

- how the above display of 17 makes special cases of the 2 tetrahedral forms potentially "fused" at the centre, and of the cuboctahedral array
- if one tetrahedral form is rendered visible, as in the animation above, this seemingly offers a total of 16 as corresponding to 16 SDGs
- how this consideration relates to the exceptional coordinating status of the 17th SDG Goal

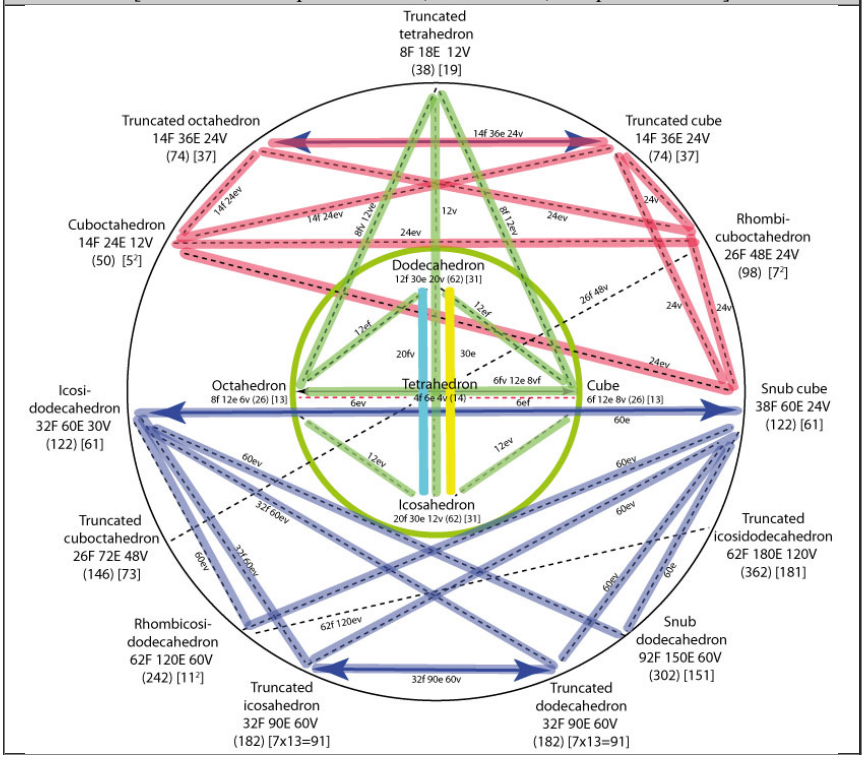
The geometry of the cuboctahedral array invites other questions, given that with 14 faces its dual (the rhombic dodecahedron) necessarily has 14 vertices. Curiously it is this dual which is used in mapping the 16 Boolean connectives (logical operations on two variables) of basic logic in discussion of oppositional logic and its geometrical representation (*Reframing forms of connectivity through the logic of oppositional geometry*, 2020). The distorted mapping from 16 to 14 is conventionally framed in terms of tautology and antilogy -- but potentially to be challenged as a "fudge" to avoid the challenges of 4D comprehension, as discussed separately (*Governance beyond the logical focus on true vs false?* 2019; *Questionable confusion in configuring strategic frameworks: "fudging" self-reflexivity?* 2019). As noted below with respect to the music equivalent of the wallpaper group, this is also limited to 14 (rather than 17).

A similar problem is evident in the reduction from 18 to 17 4D forms (as noted above) consequent on recognition of the cubic hyperprism overlapping with the regular tesseract. It is however that tesseract which is the most familiar form of 4D polyhedron and is of significance to studies of oppositional logic and its associated geometry.

Pattern if connectivity between SDGs? The cuboctahedral configurations (above) of the 4D projections can only serve to indicate potential patterns of hyperconnectivity. It is appropriate to assume that an approach to the nature of that pattern can be explored through that between the Platonic and Archimedean forms in 3D -- setting aside the complexity of the 4D connections.

The following simplified map was developed in an earlier discussion (*Pathway "route maps" of potential psychosocial transformation?* 2015). It offers a sense of particular transformational pathways between patterns of order -- in which prime numbers appear to play a determining role as indicated in that discussion. The colouring of the "routes" serves to highlight pathways of contrasting significance. Arguably some of the features derive simply from design choices, although the degree of symmetry calls for future comment.

Map highlighting distinctive relationships pathways between spherically symmetrical polyhedra (regular and semi-regular)
F=faces, E=edges, V=vertices (Total of these in parenthesis)
[Total reduced to prime number, other than 2, in square brackets]



Archetypal cognitive confusion with strategic implications?

This argument has used the set of 17 SDGs as a fundamental global reference (as agreed by 193 countries) and confronted that set with geometrical and topological constraints which are a long-studied feature of mathematics. The exercise can be understood as avoiding the traps of numerology and any excessive enthusiasm for proportions traditionally inferred from symbolism and sacred geometry.

The approach has drawn attention to other strategic articulations, notably sets ranging from 10 to 20 -- and beyond. It has also highlighted patterns of numbers in that same range which merit greater attention in constituting a context for recognition of 17 as one "island of stability" -- understood as a cognitive analogue to use of that term in relation to the periodic table of chemical elements. It remains quite unclear why strategic sets of other sizes achieve a degree of closure -- esteemed collectively satisfactory by some -- however questionable they may appear to be in terms of coherence and memorability.

This confusion frames a need for future inquiry -- for which the following indications may prove to be of relevance.

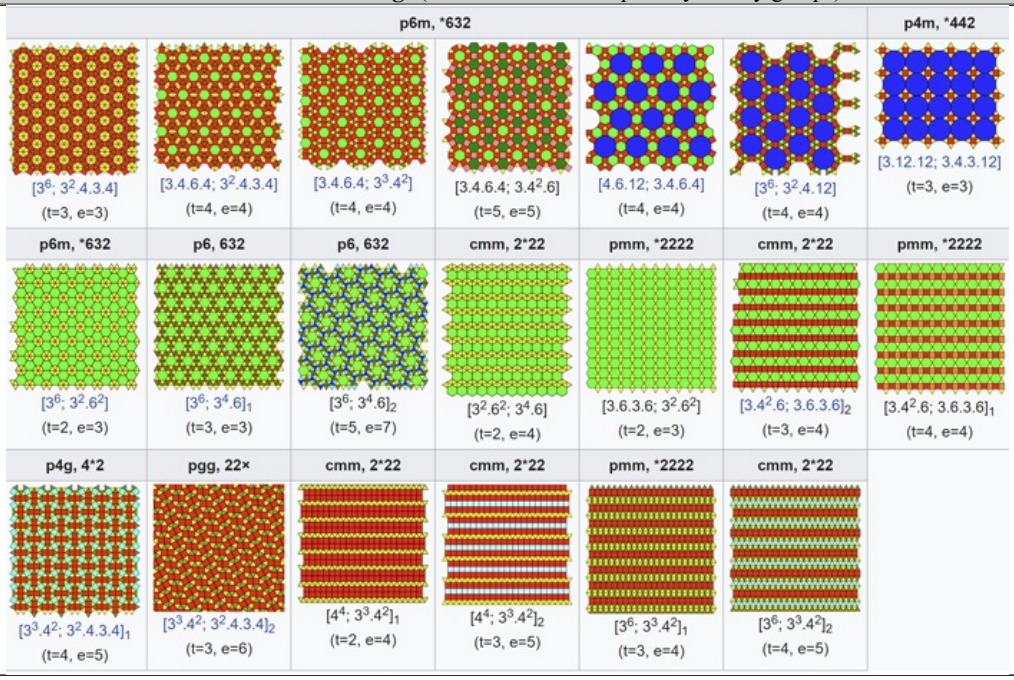
- 64-fold pattern considered fundamental to:
  - the set of there are 64 convex uniform 4-polytopes (as noted above)
  - the array of genetic codons (DNA and RNA codon tables)
  - the cultural framings traditionally offered by the *I Ching* and the *Kama Sutra*, as discussed separately (*Reframing the Dynamics of Engaging with Otherness: triadic correspondences between Topology, Kama Sutra and I Ching*, 2011)
- 17-fold pattern:
  - as a subset of the 64 convex uniform 4-polytopes (as noted above)
  - its relation to the wallpaper group and its role in design, music and dance (as noted below)
  - global consensus on identification of 17 SDGs
- 20-fold pattern:
  - as the array of amino acids encoded by 61 of the 64 genetic codons. Exceptionally, three of them (known as stop codons) do not code for an amino acid but instead signal the release of the nascent polypeptide from the ribosome
  - the 20 2-uniform tilings (noted above) and their relation to the 17 plane symmetry groups (as indicated in the table below)
  - the apparent preference for 20-fold arrays of strategic relevance (*Requisite 20-fold Articulation of Operative Insights? Checklist of web resources on 20 strategies, rules, methods and insights*, 2018)
  - the possibility of a 20-fold pattern of a psychosocial significance (*Memetic Analogue to the 20 Amino Acids as vital to Psychosocial Life?* 2015)

Mnemonic aids? Rather than assuming the absence of any significant relationship, these patterns merit clarification with respect to the significance variously attributed to 12-fold, 14-fold, 15-fold, 16-fold, and 18-fold patterns noted above. The point is developed separately

in the light of unexplained strategic enthusiasm for the 12-fold pattern (*Time for Provocative Mnemonic Aids to Systemic Connectivity? Possibilities of reconciling the "headless hearts" to the "heartless heads"*, 2018). Rather than the analytic approach framed by the common expression just "follow the money", a higher degree of strategic coherence might emerge from an interpretation with which it is too readily confused, namely "follow the numbers".

Arguably, for example, the following images can be understood as offering a mnemonic indication of the set of amino acids, especially given the argument that: *The polar parts of the D- and L-amino acids are related by pseudo glide-plane symmetry in all complexes except L-Leu:-D-Nle, in which parts of the two amino acids are related by pseudo-inversion* (Bjørn Dalhus, et al, *Molecular aggregation in selected crystalline 1:1 complexes of hydrophobic D- and L-amino acids*, *Acta Crystallographica Section C Crystal Structure Communications*, 55, 1999, 9)

The 20 2-uniform tilings (with indication of 17 plane symmetry groups)

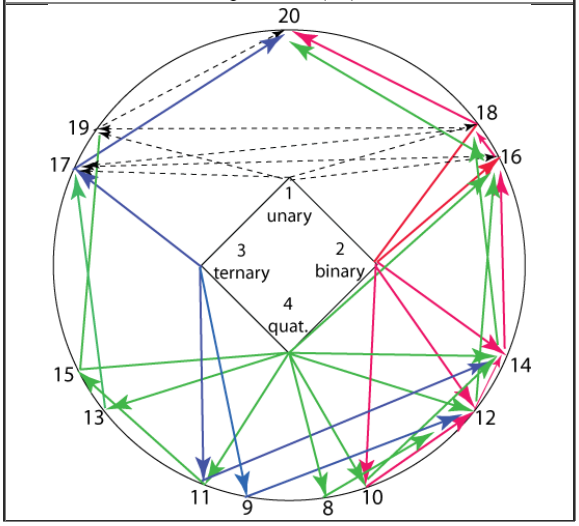


Reproduced from *Euclidean tilings by convex regular polygons* (*Wikipedia*)

Mapping cognitive transformations? On the assumption that cognitive operations are most readily understood as unary, binary, ternary or quaternary in nature, it is appropriate to experiment with ways that sets of concepts and strategies with different numbers of elements might be related and visualized. The focus in the schematic is on those in the range 10 to 20 strategic elements (with some links omitted to simplify the diagram). The question raised is what cognitive operation separates the Gestalt of a 16-fold set from that of a 17-fold set -- given the example of the 17 SDGs, or the 8-fold set of MDGs which preceded it.

Simplistic representation of cognitive operations relating sets of different dimensions

red = binary operations (+2); blue = ternary operations (+3); green = quaternary operations (+4); black dashed (unary operations (+1)



The schematic above could be enriched by interpretation in terms of arity -- understood as the number of arguments or operands taken by a function or operation in logic, mathematics, and computer science. Examples distinguished include: nullary, unary, binary, ternary, quaternary, quniary, senary, septenary, octonary, novenary, and denary. These suggest a degree of relationship to the lemmas of

increasing rarity by which discourse and governance may be challenged: dilemma, trilemma, tetralemma (or quadrilemma), pentalemma (or quintilemma), hexalemma, heptalemma, and octalemma (*Cognitive glass ceilings impeding integrative comprehension*, 2019).

Distinguishing a domain of hyperreality as the appropriate SDG context?

The conventional preoccupation with the "real" in many cases constitutes a denial of the "surreal" and the "hyperreal" -- however the surreal is to be understood (*Surreal nature of current global governance as experienced*, 2016). To a strange degree this denial echoes that of the manner in which the objective is challenged by the subjective (*¡¿ Defining the objective ∞ Refining the subjective ?! Explaining reality ∞ Embodying realization*,, 2011; *A Subjective Objection: Objecting to Subjection: interplay of questions enabling transcendence of fundamental dilemmas?* 2016).

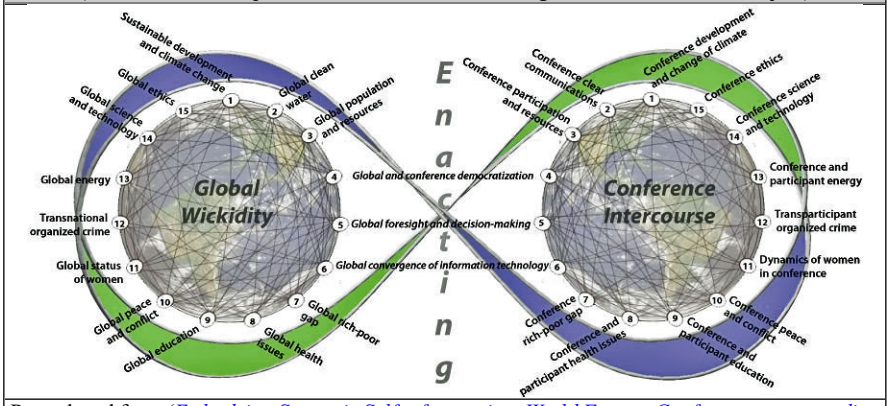
In practice this interplay is most evident in the manner in which the objective dynamics of convention are matched in practice by recourse to use of lpsychoactive substances -- whether legal or illegal. Little effort is made to explore that cognitive transition -- other than by appreciatively engaging in it. The realm of psychoactive experience is of course curiously related to the sense of dreaming -- as evoked above with respect to the SDG Dreamtime. It is also in this sense that strategies and the forces in play may be characterized as "angelic" or "demonic" -- especially by leaders with religious commitments and constituencies.

The relation between the angelic and demonic domains of tradition can therefore be readily framed in this light (*Joe Biden promises to lead the forces of light over the forces of darkness*, *Blog for Arizona*, 21 August 2020; *The Great Controversy -- between a connection to the forces of light or the forces of d, darkness*, *Global Communications Alliance*; *Radical Disaffection Engendered by Elitist Groupthink? Democratic rehearsal of the final battle between the Forces of Light and Darkness*, 2016).

It is in this sense that the traditional framework offers a memorable means of engaging with hyperreality and its patterns of connectivity, as explored separately (*Engaging with Hyperreality through Demonique and Angelique? Mnemonic clues to global governance from mathematical theology and hyperbolic tessellation*, 2016; *Variety of System Failures Engendered by Negligent Distinctions: mnemonic clues to 72 modes of viable system failure from a demonic pattern language*, 2016).

One approach to this possibility is offered by the recognition that an explicit strategic articulation is usefully recognized as paradoxically complemented by a an implicit subjective sense of "hyperreality" (David Bohm, Wholeness and the Implicate Order, 1980). Visually this can be illustrated by an "adaptation" of the pattern of 15 global strategic challenges identified by the Millennium Project. In the schematic below, the articulation of those challenges in the left-hand portion of the diagram is complemented by an implicit variant in the right-hand portion of the schematic. Their paradoxical relationships was suggested by setting them within a Möbius strip in a "creative" review of the 17th International Futures Conference on *Tackling Wicked Problems: where futures research, education and action meet* (Turku, 2015),

30 Future Global and Conferencing Challenges for Humanity
(self-referential adaptation of the 15 Global Challenges of the Millennium Project)

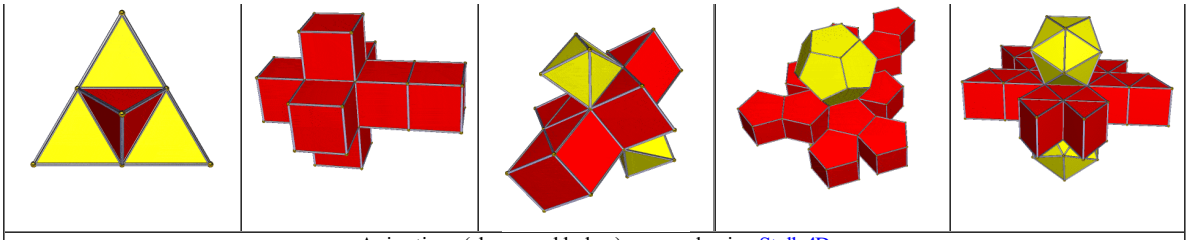


Reproduced from (*Embodying Strategic Self-reference in a World Futures Conference: transcending the wicked problem engendered by projecting negativity elsewhere* (2015)

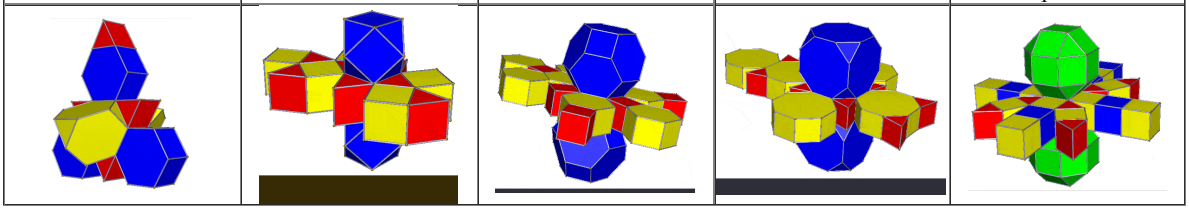
The distinction between the explicit and implicit realms suggested by the above schematic can then be used as a means of interpreting the 4D animations presented above. A greater degree of comprehension of the above structures is obtained by unfolding them into a 3D form. In fact some vertices of the unfolded structures are shared (namely coincident). in the folded 4D form, although this is not apparent.

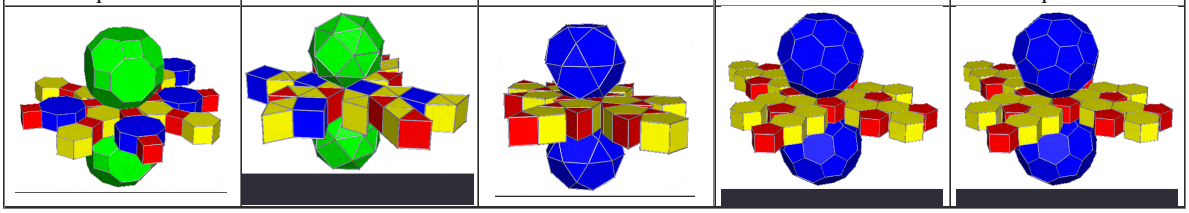
Especially intriguing is the manner in which the unfolding of some the structures gives rise to matching spheres -- separated from each other -- which could be understood as an artificial separation of the explicit and the implicit realms.

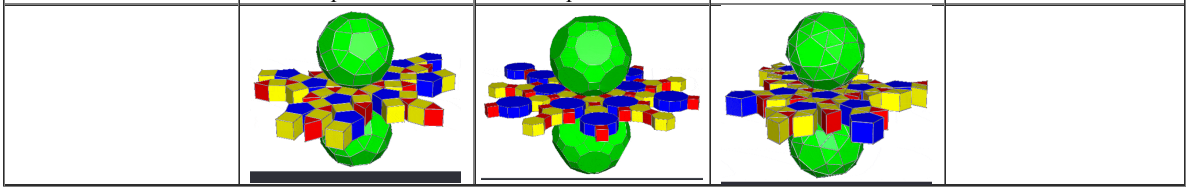
Animations of unfolding of 3D representations of 4D Prisms based on Platonic polyhedra with visualization of cells				
Tetrahedral prism	Cube prism	Octahedral prism	Dodecahedral prism	Icosahedral prism



Animations (above and below) prepared using Stella4D

Animations of unfolding of 3D representations of 4D Prisms based on Archimedean polyhedra (1-5 of 13)				
Trunc. tetrahedral prism	Cuboctahedral prism	Trunc. octahedral prism	Trunc. cube prism	Rhombicuboctahedral prism
				

Animations of unfolding of 3D representations of 4D Prisms based on Archimedean polyhedra (6-10 of 13)				
Trunc. cuboctahedral prism	Snub cube prism	Icosidodecahedral prism	Trunc. icosahedral prism	Trunc. dodecahedral prism
				

Animations of unfolding of 3D representations of 4D Prisms based on Archimedean polyhedra (11-13 of 13)				
	Rhombicosidodecahedral prism	Trunc. icosidodecahedral prism	Snub dodecahedral prism	
				

The hyperreality framing invites related explorations:

- *Imagining Order as Hypercomputing: operating an information engine through meta-analogy* (2014)
- *Hyperaction through Hypercomprehension and Hyperdrive: necessary complement to proliferation of hypermedia in hypersociety* (2006)

The "angelic" metaphor also invites related explorations:

- Rupert Sheldrake and Matthew Fox: *The Physics of Angels: exploring the realm where science and spirit meet* (2014)
- Gregory Bateson and Mary Catherine Bateson: *Angels Fear: towards an epistemology of the sacred* (1987)
- H. S. M. Coxeter: *Angels and Devils* (*The Mathematical Gardner*, 1981)
- John Choi and Nicholas Pippenger: *Counting the Angels and Devils in Escher's Circle Limit IV* (*Journal of Humanistic Mathematics*, 5, 2015, 2)

Higher dimensional patterns of intuited significance in 2D?

The presentation in 3D animations of the 4D polyhedra above offers some striking imagery suggestive of a degree of relationship to more familiar symbols in 2D. For this static reason screen shots of particular phases in the 3D animation are reproduced below (despite the duplication).

The images frame a speculative consideration of any dialogue "round table" if it were to take 3D or 4D form, or multiple tables were to be understood as interrelated -- as in the case of breakout sessions of a conference (*Spherical Configuration of Interlocking Roundtables: internet enhancement of global self-organization through patterns of dialogue*, 1998). Little is of course said of the patterns of dynamics within a conventional 2D round table (*Clarifying the Unexplored Dynamics of 12-fold Round tables*, 2019). This considered the following:

Visualizing a 3D round table of requisite variety
Distinguishing the three "dimensions" of a round table of "global" significance

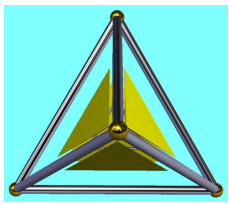
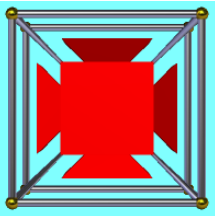
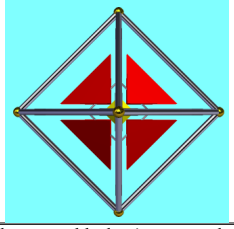
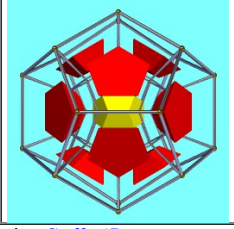
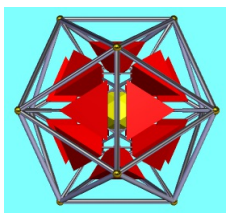
Contrasting 3-fold articulations relevant to dynamics of a "global table"
Reframing a 12-fold pattern of discourse in 3D

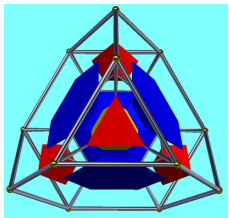
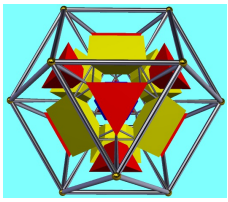
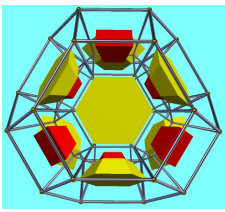
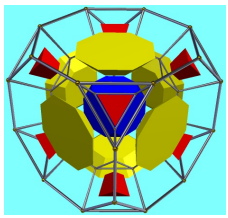
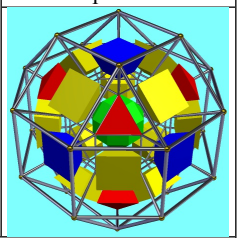
Dynamics of a "global table" rather than a "round table"?
Indication of integrative insight engendered in a "global table"
Conceptual integration via simple polyhedra and their hemipolyhedral equivalents?

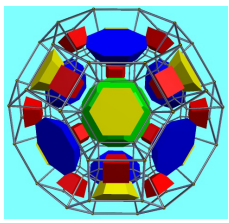
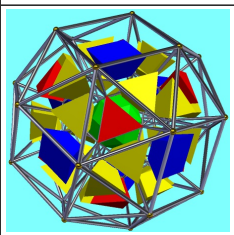
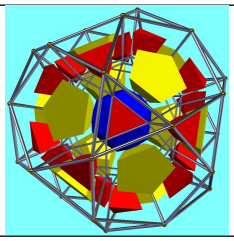
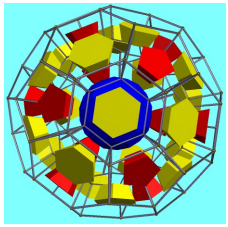
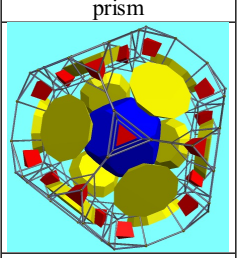
Global table: enabling comprehension, credibility, communicability and memorability

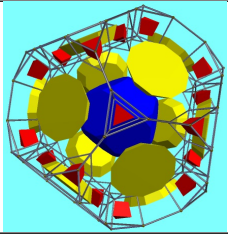
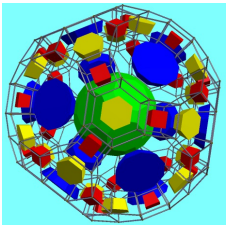
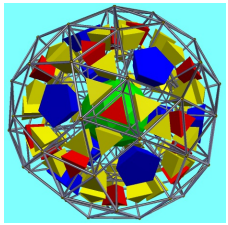
The question would then be whether the "hyperconnectivity" suggested by the following screen shots is indicative of the requisite connectivity of such "global tables" in 4D, and the part played by tesselation (*Envisaging the global dynamics of a "hyperbolic round table" through tessellation*, 2016).

As schematics, beyond negotiation "table arrangements" (and somewhat ironically), the images are also suggestive both of dance patterns (as discussed below) and those envisaged in military confrontations. The 3D and 4D nature of the latter have become all the more relevant now that the superpowers are actively envisaging "space warfare" encounters of space forces (*List of space forces, units, and formations*, *Wikipedia*). This justifies the case for exploring NATO as a "4D institution" -- if not the UN itself (

Screen shots of 3D representations of 4D Prisms based on Platonic polyhedra with indication of cells				
Tetrahedral prism	Cube prism	Octahedral prism	Dodecahedral prism	Icosahedral prism
				
Screenshots (above and below) prepared using *Stella4D*				

Screen shots of 3D representations of 4D Prisms based on Archimedean polyhedra with indication of cells (1-5 of 13)				
Trunc. tetrahedral prism	Cuboctahedral prism	Trunc. octahedral prism	Trunc. cube prism	Rhombicuboctahedral prism
				

Screen shots of 3D representations of 4D Prisms based on Archimedean polyhedra with indication of cells (6-10 of 13)				
Trunc. cuboctahedral prism	Snub cube prism	Icosidodecahedral prism	Trunc. icosahedral prism	Trunc. dodecahedral prism
				

Screen shots of 3D representations of 4D Prisms based on Archimedean polyhedra with indication of cells (11-13 of 13)				
	Rhombicosidodecahedral prism	Trunc. icosidodecahedral prism	Snub dodecahedral prism	
				

Coherence of hyperreality through aesthetic intuition and embodied cognition?

The mathematical abstractions evoked above are readily experienced as alienating and divorced from any sense of coherence -- even when represented visually. It is therefore useful to explore how aesthetics might enable other forms of engagement with strategic issues of governance as exemplified by the set of SDGs. This possibility has been separately explored in more general terms (*A Singable Earth Charter, EU Constitution or Global Ethic?* 2006; *Aesthetics of Governance in the Year 2490*, 1990; *From Information Highways to Songlines of the Noosphere: global configuration of hypertext pathways as a prerequisite for meaningful collective transformation*,

1996).

It is of course the case that those promoting a "serious" methodology in response to the strategic challenges of the SDGs will in their turn deprecate aesthetic considerations as recognized by the influential early argument of C. P. Snow (*The Two Cultures and the Scientific Revolution*, 1959). This could now be understood as reframed as the challenge of the engagement between the "heartless heads" and the "headless hearts" .

A remarkable degree of bridging between the two extremes, notably in the light of geometrical arguments like those above, is provided by the many contributions to the annual conference of The Bridges Organization whose proceedings from 1998 are readily available (Bridges Archive). Insightful overviews from such perspectives are notably summarized by Martha Senger (*The Iconic Revolution*. 2015).

Games: Game-playing is of considerable significance to the exploration of strategy -- as traditionally recognized with respect to chess and its equivalents. The emergence and implementation of the SDGs could be therefore be understood as exemplifying such game-playing, as can be variously explored (*Playing the Great Game with Intelligence: authority versus the people*, 2013). Beyond the articulation by the Club of Rome of a global "problematique" and a corresponding "resolutique", it can be argued that there is a complementary "imaginatique" and "ludique", as separately discussed (*Imagining the Real Challenge and Realizing the Imaginal Pathway of Sustainable Transformation*, 2007). It is the "imaginatique" and the "ludique" which are celebrated in many approaches to game-playing.

"Gaming the SDGs" is the focus of various initiatives:

- Christina Myers: *Gaming Sustainable Development* (*UN Sustainable Development Blog*, January 2019)
- *Gaming for good: How Playmob connects gamers to the SDGs* (*ITU News*, 27 January 2021).
- The *2030 SDGs Game* is a multiplayer, in-person, card-based game (playable online) that simulates taking the “real world” into the year 2030.
- *What's Missing?* is an educational game designed to help raise awareness for the goals and to provide players with a range of ways in which they personally can help achieve the goals.
- *Go Goals!* is a game for children designed to inform while motivating them to actively pursue the SDGs (see also *Trainers' Manual on Games and Activities for Sustainable Development Goals and Climate Change for Children and Youth*, 2020).

Despite the 17-fold pattern of the UN Goals, it is curious to note the extent to which worldwide enthusiasm for Rubik's Cube has been interpreted in that light (*Recognition of Rubik's Cube as a relevant strategic development metaphor*, 2017). The device can be readily adapted to the earlier set of 8 Millennium Development Goals (*Mapping 8 Millennium Development Goals onto the 3x3x3 surfaces of Rubik's Cube*, 2017).

The 17 SDGs could be understood in relation to the properties of a a particular magic square with magic constant of 34, namely 2x17 (*Magic square of sustainability: from 3x3 to 4x4?*, 2017). Given the exceptional status of the 17th Goal, the other 16 could be applied to a 4x4 variant of the Rubik Cube, namely Rubik's Revenge, as discussed separately (*Beyond the superficial: engaging other sides as the hidden challenge of sustainability*, 2017).

Far greater is the challenge of the 17x17x17 Stickerless Magic Cube (*CuberSpeed YuXin HuangLong*), the world's largest mass-produced NxNxN puzzle, featuring 1947 individual pieces and 17 layers (video). The iconic images of the 17 SDG Goals could be easily printed on the interlocking surfaces to offer a sense of the challenge to be solved through the puzzle dynamics.

The Eastern board game corresponding to the chess of Western culture is the game of go -- now a focus of major challenges to AI through Computer Go. Although played on a 9x9 and 13x13 gobans, it is normally played on a 19x19 goban -- and a 17x17 goban variant is known. This suggests an adaptation of AI of relevance to the strategic challenge of the 17 SDGs in engaging with the corresponding problematique. Calculations relevant to the 17x17 goban are reported (Peter Shotwell, *John Tromp and the Big Numbers of Go: the possible positions, games and the longest*, 2016).

Seventeen is the minimum possible number of givens for a sudoku puzzle with a unique solution. Framed in this way, given the conceptual challenge of sudoku, global governance merits recognition as a puzzle -- as continues to be the case in reference to the legendary Gordian Knot. Remarkably, an unusual analysis by CitiBank of the interrelation between SDGs has been presented in matrix form under the heading "Sustainability Sudoku" (*United Nations Sustainable Development Goals: pathways to success -- a systematic framework for aligning investment*, 2018, pp. 20-22).

Art: The possibility *Revisiting the Sustainable Development Goals through Art* is a project of Genève Internationale: Peace, Rights and Human Rights. Some 150 distinctive posters have been produced (*SDG Posters and Art Prints*, *Barewalls*).

With respect to the argument here, of most obvious relevance is the renowned art of M. C. Escher (*Tessellations by Polygons*, *Math and the Art of M C Escher*). Provocatively, given the promoted iconography of the SDGs, it might be asked whether they might somehow tesselate in a suggestively coherent manner (presented otherwise in the form of tiles). This could be a possible extension of the *Tiles IoT Inventor Toolkit* which produces 100+ brainstorming cards that can be combined to invent solutions for the UN’s SDGs.

Given the puzzling features of Escher art, the merit of developing puzzles appropriate to the SDGs is clearly an opportunity, as recognized by UNDP Brazil (*Design a puzzle about the Sustainable Development Goals*, 2016). This is an undertaking not to be confused with the UNDP ART Initiative (focused on territorial partnerships for implementing the SDGs at local level). The case for highlighting the interrelationship between the SDG "tiles" has been made through their presentation in the form of jigsaw pieces (Johanna Kohl, *Sustainable Development Goals (SDG) are more than just nicely coloured icons – a puzzle? Luke*, 17 December 2018).

Poetry: A case can be made for the vital role of poetry in the articulation of policy (*Poetry-making and Policy-making: arranging a marriage between Beauty and the Beast*, 1993; *Poetic Engagement with Afghanistan, Caucasus and Iran: an unexplored strategic*

opportunity? 2009).

Epic poems demonstrate their enduring significance over millennia -- as with the *Epic of Gilgamesh*, the *Mahabharata*, and the *Kalevala* (*List of epic poems*). It could be asked why society could not be periodically challenged to articulate the SDG themes and its 169 targets in such a memorable poem -- a poetic analogue to the Eurovision Song Contest.

The art of *haiku* is especially interesting in this respect given its importance to the second Secretary-General of the United Nations (*Ensuring Strategic Resilience through Haiku Patterns: reframing the scope of the "martial arts" in response to strategic threats*, 2006). In relation to the argument above with respect to the UN's SDGs, it is intriguing to note that traditional Japanese *haiku* consist of three phrases that contain a *kireji*, or "cutting word", in 17 *on* (a type of Japanese phoneme) in a 5, 7, 5 pattern.

Curiously the articulation of the 17 SDGs was celebrated in a *haiku* competition among some of those involved (*Sustainable Development Goals Haiku winner and runners-up announced*! 22 July 2014). The Twitter hashtag of **#SDGhaiku** has since emerged to frame a form of advocacy. In that spirit UNDP has enabled a publication of 82 haiku (*Inspired by Nature: Celebrating Biodiversity with Haikus*, UNDP, 22 May 2017)

A comparable device in evident in the commentary evoked in the Western literary tradition by the enigmatic poem by Wallace Stevens (*Thirteen Ways of Looking at a Blackbird*, 1917) as is evident from its use by Michael Dylan Welch (*Seventeen Ways of Looking at a Haiku*; *Thirteen Ways of Reading Haiku*) and by Ron Padgett (*Thirteen Ways of Looking at a Haiku*). As an enigma, any engagement with it (or with comment about it) is itself problematic. It effectively calls for a creative way of looking at "ways of looking" and the imaginative responses it might evoke -- as is arguably the case with respect to the UN's SDGs.

It has been noted that in his poem Stevens' description of a *Turdus* in a snowy autumn landscape alludes to the Cubist painting tradition of observing subjects simultaneously from numerous viewpoints to present a novel perspective. There is clearly a strong case for exploring more complex understanding of "ways of looking" in the current period, with active discussion of more (world) wars to come. (*Engaging with a memespace of paradoxical complexity*; *Ways of looking at ways of looking -- in a period of invasive surveillance*; *Post-modern challenge to simplistic binary framing of the other*, 2014)

One helpful approach, especially relevant to current framing of global strategy, is the commentary of Nüzhet Akin (*From '0', the Logic of Imagination, to 'Ground Zero', the Imagination of Logic: the enigmas of Wallace Stevens' 'Blackbird' and current US action*. *Journal of Arts and Sciences*, Sayi, 12 Aralik, 2009). For Akin the difficulty of interpreting the poem arises at the point where the reader seeks to find a logical answer to the nature of the blackbird. Umberto Eco might be said to offer an example (*Eternal Fascism: Fourteen Ways of Looking at a Blackshirt*, *New York Review of Books*, 22 June 1995, pp.12-15).

Dance: Given the undoubted role and appeal of dance, it could be asked how it might be relevant to the SDGs and the popular appeal it is assumed that they might evoke. Dance can be recognized as a necessarily fundamental challenge to the dependence on words (Jeffrey Escoffier, *Figures of Dancing: the thinking body and its metaphors*, 1999). As noted in a comprehensive review of choreographic notation by Laurence Louppe:

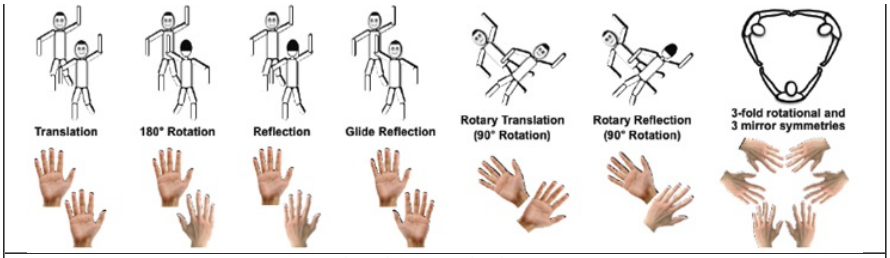
> Dance can have no recourse to the sign, for its essence is to forego the detour that leads there. The access to dance, whether it be perceptual or interpretative, is a direct access that surges up from the heart of matter, from the heart of emotion, above all in contemporary dance. Dance is lived and traversed as a living present; it has, in appearance, no need for a symbolizing system that would be incompatible with experiential givens... (*Traces Of Dance: Choreographers' Drawings And Notations,* Dis Voi, 1994, p. 9)

The latter study includes perspectives regarding a variety of notations and notably features an interview with the topologist René Thom of relevance to the argument above regarding cognitive functions:

> The isomorphism existing between embryonic development and phylogeny has been treated in the works of Haeckel-Muller, which draw us into a controversial field of biology. One aspect of his work in particular which has always surprised me is the problem of the embryology of functions. These are abstract entities, which are nonetheless characterized by a certain unity of biological operations which can often be easily spatialized, either in physical space, or in more abstract spaces like biochemical spaces.
>
> These functions favor a topological interpretation for spaces which contain both spatial position and internal biochemical activities. I personally like to think that a function has a "soul". Functions are always somewhat cyclical. They end up at the point where they started by revolving around a fixed center, and this determines the soul of functions. However, this cyclical aspect of functions often slips from our memory, because it is generally said , for example, that the eye's "function " is to see, whereas in principle, seeing is not a circular function because light IS taken in by the eye, but not given off. But, on the whole, such a circular plan IS essential, and souls form the heart of cycles. However, t his is a very abstract vision of physiology in general, and it is still far from being fully understood even by specialists (p. 76-77)

Karl Schaffer notes the many of types of symmetry in dance and other movement arts, including skydiving (*Three-Dimensional Symmetries in Dance and Other Movement Arts Bridges 2019 Conference Proceedings*). Recognized (as noted above) as isometries in transformations of Euclidean space, translation, rotation, reflection, and glide reflection have previously been considered as symmetries between dancers in the plane.

Two and three dimensional dance symmetries



Reproduced from Karl Schaffer (2019)

Separately Schaffer shows how choreography can allow dancers to move smoothly from one symmetric position to another without moving through positions of asymmetry. This allows sequences exhibiting all seven of the frieze symmetries and all 17 of the wallpaper symmetries to be so portrayed (K. Schaffer, E. Stern, and S. Kim, *Math Dance with Dr. Schaffer and Mr. Stern*, MoveSpeakSpin. 2001; K. Schaffer. *Dancing Deformations. Bridges Conference Proceedings*, 2014).

Given his renowned interest in symmetry, of some relevance is the consideration given by the mathematician John Conway to frieze symmetries in dance movement -- but seemingly knot to wallpaper groups (*Frieze Pattern Dance Steps*).

Is there insight to be gained from Dunmur's performances of the *17-step Lakeland Clog Routine* at traditional step dance festivals (*Lakeland steps Ian Dunmur 2002 Performance*; *Norman Robinson: 17 Step Routine*, *Instep*, 11 January 1984)?

Music and sonification: Little needs to be said about the capacity of humans to detect patterns in music -- a focus of widespread appreciation and numerous studies. Of notable relevance is the increasing importance of sonification in order to enable recognition of patterns in complex sets of data. Music is of obvious symbolic importance as exemplified by the status of national anthems, and that of international institutions (*Reversing the Anthem of Europe to Signal Distress Transcending crises of governance via reverse music and reverse speech?* 2016). This discussed the possible *Sonification of the European problematique and resolutique?* -- a consideration which could apply to the UN's SDGs.

Of potentially fundamental significance in terms of both the string theory of physics and music is the role of 17 with respect to the mathematical understanding of orbifolds. In the mathematical disciplines of topology, geometry, and geometric group theory, an orbifold (for "orbit-manifold") is a generalization of a manifold. It is a topological space (called the underlying space) with the structure so named. The compact 2-dimensional connected orbifolds that are not hyperbolic include the 17 parabolic orbifolds which are the quotients of the plane by the 17 wallpaper groups. Reference is also made to an orbihedron. The role of orbifolds features in the music theory of Dmitri Tymoczko (*A Geometry of Music: harmony and counterpoint in the extended common practice*, 2011; *The Generalized Tonnetz*, *Journal of Music Theory*, 56, 2012,1).

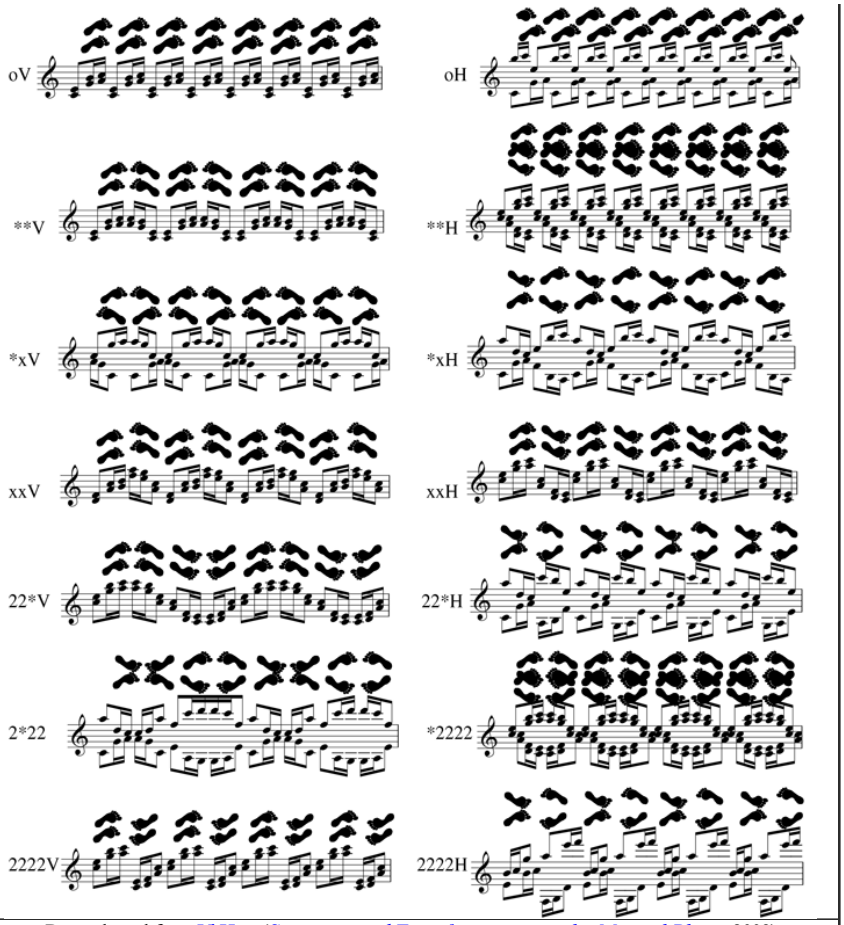
Of most relevance to the argument above is the presentation by *Vi Hart (Symmetry and Transformations in the Musical Plane, Proceedings of the 12th Annual Bridges Conference: Mathematics, Music, Art, Architecture, Culture,* 2009). Hart explains:

> The musical plane is different than the Euclidean plane: it has two different and incomparable dimensions, pitchspace and time, rather than two identical dimensions. Symmetry and transformations in music have been studied both in musical and geometric terms, but not when taking this difference into account. In this paper we show exactly which isometric transformations apply to musical space and how they can be arranged into repeating patterns (frieze patterns and variations of the wallpaper groups). Frieze patterns are created intuitively by composers, sometimes with timbral color patterns or in sequence, and many examples are shown. Thinking about symmetry in the musical plane is useful not just for analysis, but as inspiration for composers.

Hart notes that the 17 wallpaper groups of symmetry groups of patterns that repeat in two dimensions cannot all be naturally mapped to the musical plane because many contain rotations other than 180° or have mirror and glide-reflection lines which meet at angles other than 90°. As illustrated below:

> The following eight patterns (in orbifold notation) do work: o, **, xx, *x, *2222, 2222, 22*, and 2*22, because they can be mapped onto the musical plane with all mirror lines and glide-reflection axes going either horizontally or vertically, and do not contain rotations other than 180°. Interestingly, just as with the Euclidean isometries, this limitation also splits some wallpaper groups into two unique musical wallpaper groups, because they can be mapped onto the musical plane in two different ways at 90° from each other. *2222 and 2*22 fit just one way each. 22*, xx, *x, and ** find two distinct ways to map themselves to the staff. Patterns o and 2222 can both fit at any angle, though they fit into two classes: the way that makes repetitions play in parallel, and the infinite possible offsets. To give these pairs distinct names, we add a “V” for vertical mirror, glide-reflection axis, or parallel repetitions, and add “H” for horizontal mirror, glide-reflection axis, or nonparallel repetitions. Figure 13 illustrates swatches of the 14 groups,

The 14 musical wallpaper groups (showing just one vertical repetition, and using the Conway footprint analogy)



Reproduced from Vi Hart (*Symmetry and Transformations in the Musical Plane*, 2009)

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