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# PRELIMINARY CLARIFICATION OF SOME PROBLEMS OF PROCESSING NETWORKS OF ENTITIES IN ORDER MEANINGFULLY TO MAP PSYCHO-SOCIAL RELATIONSHIPS

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## Introduction

There are a series of problems to be solved in order to process sequentially ordered entities whose interrelationships are given

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| into | 1. | A three-dimensional spherical volume in which the entities should be positioned with minimum distortion of their relationships; |
| followed by | 2. | Reduction of the three-dimensional projection to a projection onto the surface of a sphere; |
| followed by | 3. | Reprojection to permit plotting of the sphere surface on a planar surface. |

The solution to such problems, adapted to the processing of relationships held on magnetic tape, should be used to supply meaningful maps of relationships between psycho-social entities, such as described in the following papers by the author, namely:

- organizations
  (cf. Information systems and inter-organizational space. Annuals of the American Academy of Political and Social Science, January 1971)

  Inter-organization data and data bank design (Paper presented at a panel of the International Studies Association, San Juan, 1971)

- problems
  (cf. World problems and human development, Brussels, Union of International Associations, 1972)

- concepts
  (cf. Toward a concept inventory; suggestions for a computerized procedure. (Paper presented to Research Committee 1, Committee on Conceptual and Terminological Analysis, IPSA, Montreal 1973) This paper relates these problems to other attempts at processing networks.)

or even between all three as suggested in

(cf. Computer-aided visualization of psycho-social structures (Paper presented to an AAAS symposium, Philadelphia, December 1971)

Generalization of problem situation

We have a series of sequentially labelled nodes

$$P_1, P_2, \ldots P_n$$

where n is known and not a large number.

There are directed arcs either $P_i P_j$ or $P_j P_i$ linking some of these nodes. There may be many arcs linking to a particular node.

The arcs may be of two main types:

$$P_iP_j'$$

$$P_iP_j''$$

(Arcs of the second type have several sub-types:

$$P_iP_j''^{a}, \quad P_iP_j''^{b},$$

$$P_iP_j''^{e}, \quad P_iP_j''^{d}$$

whose differences do not affect the problems discussed here but may affect the data formats and the mapping symbols used to plot the results.)

<u>PROBLEM</u> 1 : Allocation of the nodes to positions in a 3-dimensional sphere

1. It is desired that the more connected nodes should be positioned more towards the centre of the sphere relative to the less connected nodes. This rule is however modified by seven considerations:

   1.1. The arcs of the type $P_i$ $P_j$ indicate that $P_j$ is <u>dependent upon</u> $P_i$. $P_i$ should therefore be located more towards the centre than $P_j$.

   1.2. If $P_j$ is otherwise relatively poorly connected, it may be positioned anywhere on a rough sphere around the location of $P_i$. The latter needs only be done if there are several such nodes equally dependent upon $P_i$ whose non-arc relationship to each other in the 3-D space need to be most equitably distributed in relation to $P_i$. This applies not only to nodes directly dependent on $P_i$ but also to those nodal complexes dependent upon $P_i$. Although in such a case the nodal complex is treated as a whole (and not decomposed) in positioning it in relation to $P_i$. It may however be decomposed to position its components in relation to $P_j$.

   1.3. A node may not be as close to the centre as any node on which it is dependent, except when there are nodes almost solely dependent on it as in 1.2. (in which case the distance separating the locations of $P_i$ and $P_j$ should be relatively small).

   1.4. The extent to which $P_i$ is displaced towards the centre is increased if other nodes linked to $P_i$ are in relation of dependency to it. Each node may thus have a constituency of successive sub-nodes at several removes (in a tree-type relationship to it). The extent of the increase in the displacement towards the centre is proportional to the relative importance of the constituency of sub-nodes.

   1.5. On a similar basis to 1.4., if the constituency of nodes linked to $P_i$ by arcs of the type $P_iP_j$" is strong relative to otherwise equivalent nodes (i.e. those located the same distance from the centre), the displacement of $P_i$ towards the centre is increased -- otherwise it is displaced outwards in proportion to any relative weakness.

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1.6. It is not required that the positioning of nodes in the spherical volume give rise to a uniformly decreasing den sity gradent from the centre outwards. The core of the sphere may be relatively less dense, just as there may be localized volumes of high density further out.

1.7. Rather than work in terms of "nodal constituencies" which requires complex searches of the network of nodes, it is preferable that the nodes should be weighted according to their degree of connectedness. Starting from a unit value, the node should be weighted upwards by the number of nodes to which it is connected, <u>other than</u> those to which it is connected by dependency relations of the form $P_j P_i'$. This gives an initial weighting differential as a basis for iteration, so that the relative importance of a nodes constituency is finally represented by the nodal weighting after iteration. (Since each additional iteration effectively represents sensitivity to arcs at one further remove, the value of this additional weighting of each successive iteration should be decreased to achieve convergence.)

2. The nodes must be well distributed around the three dimensional space. This suggests the introduction of other rules concerning the distance between nodes.

2.1. The network of nodes will tend to pull in towards the centre due to the rules outlined in points 1.1 - 1.7. Initial positioning can be arbitrary on the surface of concentric spheres. A particular node being allocated arbitrarily on a sphere the relative length of whose radius is proportional to the relative value of the nodal weighting.

2.2. The initial situation of 2.1. will be fairly distorted. There will be a "cramping" effect on some nodes. Clearly there must be a certain tolerance of distortion but when this is exceeded a node, or nodal complex, must be repositioned. The following rule seems fairly appropriate. The larger the number of (or nodal weighting due to nodes <u>dependent on</u> a given node ($P_i P_j'$), the longer the <u>average</u> distance of separation from nodes to which it is linked by relationships of the form $P_i P_j''$ or $P_j P_i'$ (i.e. on which it is dependent).

2.3. When the above constraints prevent repositioning around the centre, the whole nodal complex may be positioned with less distortion further outwards -- as a volume of relatively high nodal density.

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PROBLEM 2 : Allocation of nodes in a three-dimensional spherical volume to positions on the surface of a sphere with minimum distortion

2.4. The solution to Problem 1 gives for each node a

- latitude
- longitude
- distance from centre

which is in effect the optimum relative weighting of the node. Information on the

- distances between nodes
- angular separation of nodes

is also available.

2.5. One approach to the solution of Problem 2 is to move, outwards from the centre of the spherical volume, a spherical surface which would "collect" the most central nodes first. Each such node may be assumed to generate a circular area which may or may not be contiguous with that of other nodes encountered at the same time. The node may be considered to remain at the centre point of each such area.

As each successive layer of nodes is encountered, these must be converted, according to the following rules, into areas on the surface.
(N.B. When performing this exercise on computer options required may be to map one sector of the volume

- as a sector
- as a complete spherical surface.)

2.6. If the node is a direct dependent of a node previously encountered, then it generates a sub-region within the area of the node on which it is dependent.

2.7. If a node is not a direct dependent of another node, then it generates a new area, positioned in relation to the other according to its position in the 3-D volume.

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If the comparison shows

- one very strong, one or more very minor, then it goes as a component area of the stronger

- one very strong, one relatively strong, then it becomes an "enclave" of the weaker in the stronger

- two or more equally strong, then it becomes an independent "condominium" area, contiguous if possible with all the areas on which it is dependent, otherwise non-contiguous with all of them and subject to whatever other rules may apply.

- two or more equally weak, then it behaves as for the previous case.

3.2. The original degree of angular separation of the centre node (of an area) and the new point encountered weights the probability of the new area being contiguous: High angular separation, low probability.

4.1. Gaps between areas increase in area in same way as areas themselves.

4.2. Gaps are inserted at places on the line between contiguous areas if the number of low-level (local) links across the boundary falls below the average as the sphere moves out. These increase in size, if the linking does not compensate.

2.8 If a network connection (arc of type $P_iP_j$") is [illegible] then it is mapped onto the surface as a line be[illegible] centres.
(N.B. This must probably be performed as a se[illegible] operation, since network connections to a not yet encountered node further out would not be possible.)

2.9. If a node at the same level is encountered which is unconnected to those previously encountered (at a higher level), then the area created should be contiguous with only those areas representing nodes with which it is directly connected.

2.10. The increase in relative size of areas as the surface moves out is governed by the following:

- If an area encounters many dependent nodes its size increase is increased relative to the average

- In the exceptional case that directly dependent nodes are encountered before the node on which they centre, the centre node area should grow to include all the dependent nodes.

2.11. Clearly as the sphere moves out, it will encounter nodal complexes which are relatively unconnected to nodes represented by areas on which they would otherwise be projected. Such new areas must be displaced away from the already existing area.

In effect such new areas enter at an appropriate interstice, and are contiguous or not with previous areas according to rules above.

3. In case of conflict of the previous rules, an arbitrary rule is requested to determine whether there should be contiguity of areas.

3.1. When a new area could be a component of two or more areas, it is allocated by comparison of the relative [illegible] of the arcs or the nodal weightings pulling it [illegible] positions.

PROBLEM 3a : Projection of a mapping on the surface of a sphere to permit it to be plotted on a planar surface.

A simple algorithm is required to permit production of the plot of the surface of the sphere into lozenge-shaped sectors. These can then be cut out and folded onto the surface of a sphere.

PROBLEM 3b : Projection of a mapping on the surface of a sphere onto a planar surface with minimum distortion

This is the standard problem of geographers concerned to produce world maps with minimum distortion.

NETWORK MATHS

# McGILL UNIVERSITY

INTER-DEPARTMENTAL MEMORANDUM

DATE: 18.10.73

TO: Prof. E. Rosenthall

FROM: W. G. Brown

SUBJECT: Mr. de Laet's letter and enclosure

I have perused Mr. Judge's paper. The problems he poses are interesting, and, if a rigorous solution is required, probably quite difficult.

First, with reference to the two names suggested by John Coleman: I doubt that Coxeter could be of much help here, nor, for that matter, Tutte. However, there are mathematicians at Waterloo who might be interested and also competent to offer some assistance. One who comes to mind is Jack Edmonds. He is in the Department of Combinatorics & Optimization there, and is a specialist in combinatorial algorithms, particularly relating to graph theory.

The difficulty in the present paper stems from the fact that the problem is not purely combinatorial: geometric considerations are also relevant. For example, geographers might be satisfied with an algorithm which empirically appears to be effective; but mathematicians would be unlikely to accept such algorithms. If the author is looking for such an empirical solution, he is unlikely to find much assistance from the ranks of mathematicians. And the prospect of finding an algorithm which could be proved rigorously to be effective (in some reasonable sense) is not too good: interesting problems like these don't seem to lend themselves to such solutions.

If there is any way in which I might be of assistance to Mr. Judge, I would certainly be prepared to help. He might also try Dr. Jean Butler, of the School of Computer Science; or Blaustein, in Electrical Engineering.