# REPRESENTATION, COMPREHENSION AND COMMUNICATION OF SETS

## - the role of number

Examines the cognitive and other factors which tend in practice to limit the number of elements distinguished in a set, particularly for sets fundamental to social science and policy formation, such as: human needs, values, principles, problems. It is argued that the number of elements so distinguished influences significantly both the relationships perceived between the elements and the qualitative characteristics manifested by them, irrespective of the content of the set. Such effects are important in the case of the more abstract sets for which the ambiguity of verbal descriptors creates considerable problems of comprehension and communication, especially when the set of elements is used as the basis for the elaboration of a group of cooperating institutions. The representation of such sets in traditional symbol systems and in modern 2 and 3-dimensional forms, is reviewed as a source of constraints on set formulation and in terms of comprehension of the more complex sets through which the problems of society can be better contained.

Paper presented at the 3rd GPID Network Meeting (Geneva, October 1978) as a contribution to the sub-projects on Networks, Forms of Presentation and Methods of Analysis.

First published in International Classification, 5, 1978, 3, pp 126-133; 6, 1979, 1, pp 15-25; 6, 1979, 3, pp 92-103. Subsequently distributed as a GPID working paper (HSDRGPID-22/UNUP-133)

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## 1. Introduction

There is a widespread tendency to formulate insights, proposals or principles in point form, namely as made up of a specific number of items usually presented as a list. Such items will be considered here as the elements of the set which they collectively constitute in any particular case.

This paper is therefore concerned with problems relating to the representation and comprehension of such sets – whether the elements in any given case are *basic*: human *needs*, human *values, principles, concepts, problems*, human *rights*, human *responsibilities* or components of a policy.

The paper explores the possibility that (irrespective of the nature of the elements in any such case) there may be different kinds of constraints on the distinctions and relationships between the elements, depending upon the total number of elements in the set. Clearly, the total number of elements in the set also affects the manner in which the set can be represented, communicated and comprehended.

Briefly, therefore, the paper argues that consensus on a 5-element set of human needs (or a 5-point programme) for example, implies certain kinds of distinctions and relationships between the 5 elements, depending solely on the number (e.g. in contrast with a 3-element or 10-element set). These may not have been met in a given case because the elements are either (a) inappropriately defined, or (b) appropriate to a 4-element or 6-element set (with the consequence that there are elements in excess or missing from the set). Inadequacies of this kind are of importance in themselves but also affect the representation and communicability of the set, and ultimately its role and viability in the psycho-social domain.

## 2. Context

1. The following argument applies only to cases where the elements are conceived as making up a *complete set*. It does not apply when the elements have been selected (possibly as a sample) from a larger set. Where the elements are selected on a priority basis, as being the "most important", the argument only applies when this may be interpreted as implying most "fundamental" or "basic"[1]. Ideally the argument should also apply to any numbered list of points in an argument. But, since numbers are usually allocated for convenience to provide a simple structure to a sequence of paragraphs (and only indirectly related to the concepts developed), this is seldom the case. It should however apply wherever the author(s) declare that: "The following points apply", provided "*including* the following points" is not used or implied. The list of points should therefore have been elaborated through a "struggle" to get the best "fit" – a struggle which may have required much more than superficial reflection over a short period of time[2].

2. The sets under consideration contain elements which are essential to the ordering of an equilibrium state or an evolving process (expecially in the psycho-social domain). As such each element is different and has a special part to play. Each complements the others and all are conceived as essential (e.g. in the case of human values or needs). There is a desire that such sets should be well-formed or well-ordered, even if some degree of "fuzziness" must be tolerated as the content is clarified through research and debate.

3. The elements in such sets should be equally distinct from one another or else the question arises whether two or more similar elements should not be redefined as one. This said, however, two cases must be distinguished:
– the set itself may well be made up of sub-sets whose elements have characteristics in common
– some elements may be more directly related to others whilst still being distinct from them.
Any ambiguity implied here should be resolved by the form in which the set is represented (see below).

## 3. Constraints on number of elements in a set

1. There is an implicit assumption that authors are free to include as many elements in a set (of the above kind) as they wish. In fact, 1-element and 2-element sets are seldom of interest to scholars, although there is a tendency reinforced by public policy considerations to identify 1-element sets (e.g. *the* fundamental value, need, problem, principle, etc.). At the other extreme, 1000-element sets are considered unacceptable, as are 100-element, or even 20-element, sets. The implication here would be that the authors have not made an adequate attempt to regroup the elements in the light of common characteristics. An apparent exception is the matrix, but even here the number of columns or rows becomes unacceptable (for other than special cases) in excess of 20, for example. In fact, the probability of encountering a set with a given number of elements seems to decrease rapidly when the number exceeds about 10. It would be interesting to see whether a survey[3] would show any relation to the isotope abundance curve (see Fig. 1) in which the peaks are approximately congruent with the atoms of highest structural stability[4].

2. Authors are therefore constrained, irrespective of the nature of the set, to reduce the number of elements to something in the region of 10. Each such element, however, may in turn be considered as a (sub)set within which a similar number of elements is admissable. In this way, any number of elements can ultimately be incorporated. This coding procedure is considered legitimate because it facilitates comprehension. The consequences of such a procedure have not been examined – and yet

*it is this very procedure which produces the sets of values, principles, problems, needs, concepts, policy elements, etc. in terms of which attempts are made to order social processes and resolve their problems.*

3. The objectivity by which elements are selected on the basis of scientific criteria for inclusion in a set is therefore strongly affected by constraints on the ability of the author/observer to comprehend the set as a whole and to render it comprehensible to others. As Christopher Alexander notes (ref.(2), p.5) it has been shown that there are bounds to man's cognitive and creative capacity. There are limits to the difficulty of a laboratory problem which he can solve (3); to the number of issues he can consider simultaneously (4) (5)[5]; to the complexity of a decision he can consider wisely[6]. In commenting on relevance judgements in priority determination, a Unesco document notes "The number of positions on the scale (of relevance) can be at most 6 or 7, the maximum number of different positions among which the human mind can meaningfully discriminate". (6)

4. This constraint is also reflected in the "embodiment" of such sets in social organization, namely in the limits on the size of an effective committee, on the one hand, or on any small encounter/therapy group, on the other (7). The limit to the number of subordinate bodies which a body can effectively control is of the same kind, particularly as evidenced by the number of divisions reporting to a coordinating or presidential office. Antony Jay has explored many organizational examples of such limits[7]. Note that such organizational sub-division is carried out and limited irrespective of the complexity or diversity of the operations or problems with which the body as a whole has to deal.

5. The constraint is also "embodied" in the category sub-division of the thesauri which govern the manner by which information is obtained from libraries and information systems. Note again that this is so irrespective of the complexity or diversity of the subjects recorded in such systems.

6. The constraint may also be noted in the sets of "key" or "fundamental" problems, values, needs, etc. which are identified as the basis for action programmes. Such a breakdown lends itself readily to institutional embodiment or reinforces institutional structures which already reflect (and are therefore unthreatened) by this structuring. The predilection for sets of 10 key problems is noted by the editors of the Yearbook of World Problems and Human Potential (ref. (19), see especially Appendix 3). An excellent example is Unesco's own exercise to identify the major world problems with which it is concerned. It found 12 and condensed them under 10 objectives in its Medium-Term Plan 1977—1982 (Paris, Unesco, 1977, 19 C/4). Another excellent example is the Assessment of Future National and International Problem Areas (Washington, National Science Foundation, 1977, NSF/STP76-02573). This carries an illustration, reproduced here as Fig. 2, which shows admirably the nature of the process. The document con-

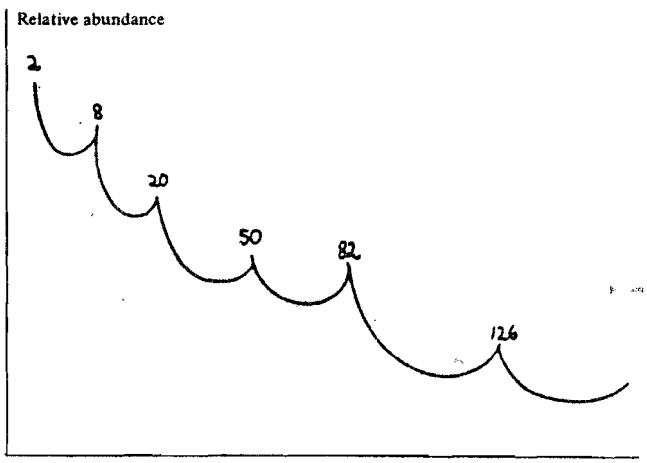



*Fig. 1: Indication of progressive decrease in relative abundance of isotopes of increasing atomic number*

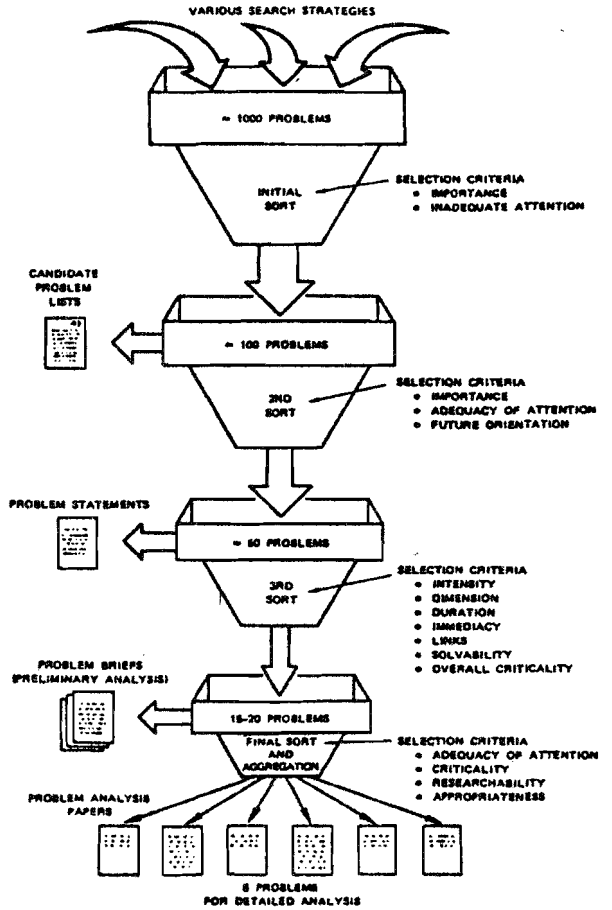



*Fig. 2: Illustration of the project approach of the "Assessment of Future National and International Problems".*
*(Reproduced from a document of that title, published by the National Science Foundation, Washington DC, 1977.)*

centrates on the 6 problems which emerge from this filtering procedure. (It is perhaps naive to ask what attention will be given to the 994 problems excluded by this procedure.)[8]

7. Such is the prevalence of this constraint that it is of interest to identify the conditions under which it is exceeded and the consequences of doing so for the communicability and viability of the set[9].

8. Another aspect of the constraint on the number of elements in a set emerges from recent explorations into the psychophysical significance of number as the common ordering factor of psyche and matter (9). Since this raises the question of the nature of the observer's relation to the observed, this is discussed separately below.

## 4. Representation of sets: Introductory comment

Herbert Simon notes: "An early step toward understanding any set of phenomena is to learn what kinds of things there are in the set – to develop a taxonomy. The step has not yet been taken with respect to representations. We have only a sketchy and incomplete knowledge of the different ways in which problems can be represented and much less knowledge of the significance of their differences." ((5) p. 78)

The problem of representation is generally considered to be of little interest compared with the subject matter of the representation and is seldom a matter of scholarly concern[10]. One reason derives from the prevalence of evidence that the physical and social environment is hierarchically ordered (10)[11]. Now hierarchical structures are those in which the interactions amongst the subsets are weak in comparison with interactions between the elements within the set. They are therefore referred to as "nearly decomposable" and as such the high-frequency dynamics within subsets are distinguished from the low-frequency dynamics between subsets. Herbert Simon relates this property to the comprehensibility of such systems: "The fact, then, that many complex systems have a nearly decomposable, hierarchic structure is a major facilitating factor enabling us to understand, to describe, and even to "see" such systems and their parts" ((5), p. 108). And clearly once it is assumed that the subsets can be represented individually, or separately in relation to the set and to each other, representation is merely a question of a hierarchy of "maps". Each can be made as detailed as necessary and can be comprehended separately.

It may be argued, however, despite the apparent ease of this approach, that widespread understanding of the many systems within which man functions (or with which he interacts) remains elusive. Indeed complaints about "increasing complexity" are now common. And studies of psycho-social systems have not produced insights to make them more manageable, in fact such systems appear to have become less manageable whilst such studies are produced.

There are three weaknesses in the conventional stress on the prevalence of hierarchical ordering. Herbert Simon follows the previously cited remark with: "Or perhaps the proposition should be put the other way round. If there are important systems in the world that are complex without being hierarchic, they may to a considerable extent escape our observation and our understanding." ((5), p. 108). Such systems, possibly exerting a "field effect" or based on non-hierarchically ordered networks may indeed be at the root of our difficulties. It is interesting that the 1970s has witnessed a rapidly burgeoning interest in networks of all kinds and a suspicion of hierarchically coordinated social structures (13). The relationship between sub-sets of different hierarchies is recognized as being increasingly critical (e.g. in environmental systems). The problem of representing such complex patterns of relationship to facilitate comprehension has not been resolved[12].

A second weakness derives from lack of clarity on the nature of the set of which the hierarchical set under consideration is a sub-set – namely the super-ordinate set. Each discipline is responsible for its own hierarchical sets, none is responsible for the super-ordinate set (and the interactions between its sub-sets). This relates back to the first weakness. There is little understanding of what happens at the "top" of hierarchies and especially "above" them[13].

A third weakness derives from lack of clarity on the relation of the person creating or observing the set – to that set. Some aspects of this question are discussed separately below. It is particularly important where one or more such sets are expected to order the comprehension of the individual who therefore has the problem of "juggling" them into a suitable configuration in relation to his own psychic ordering[14]. This raises the question of the iconicity of any representation which is discussed below.

In discussing the description of complexity, Herbert Simon makes a basic distinction between state descriptions and process descriptions[15]. "These two modes of apprehending structures are the warp and weft of our experience. Pictures, blueprints, most diagrams and chemical structural formulas are state descriptions. Recipes, differential equations, and equations for chemical reactions are process descriptions. The former characterize the world as sensed; they provide the criteria for identifying objects, often by modeling the objects themselves. The latter characterize the world as acted upon; they provide the means for producing or generating objects having the desired characteristics. ... Given a desired state of affairs and an existing state of affairs, the task of an adaptive organism is to find the difference between these two states and then to find the correlating process that will erase the difference. Thus, problem solving requires continual translation between the state and process descriptions of the same complex reality." ((5), p. 111–112).

Some of the ways of representing sets are discussed below.

## 5. Representation of sets: review of types

1. *Lists:* As implied above, the most favoured way of presenting a set is in the form of a list of items or points. Such lists may be unstructured or else items may be grouped into subsets. No other aid is provided for the comprehension of the set. It is assumed that any normal mind will be able to grasp the content in a satisfactory manner. Such lists do not identify the nature of the relations between the elements of the set (other than by what is implied by grouping into subsets).

2. *Thesauri:* As mentioned above, when there are many elements these are classified, with the aid of thesauri, into subsets at various depths within a thesaurus structure. Again little is provided to aid comprehension, the assumption being that a person knows which element is required and that the structure of the whole is of little importance. (There are a number of competing thesauri prepared by institutions – themselves competing for resources.)

3. *Tables/Matrices:* The degree of order of a set becomes clearer when it is presented in the form of a table, of which there are various kinds (e.g. the periodic table of chemical elements). These blur into matrices as a more general form of tabular presentation, which may be multi-dimensional. But here again the mind has difficulty in comprehending the whole, although it may distinguish the parts. There is a limit to the tolerance for complex tables or matrices in policy-making circles, for example, and they are seldom suitable for media-oriented presentations.

4. *Diagrams:* As the variety of relationships between the elements of a set is recognized to be of importance a diagrammatic form of presentation may be used – even if it means sacrificing the precision of a matrix presentation. There are many kinds of diagrams (14), from the simplistic to the full detail of a system flow chart. But again the simplistic can only serve momentarily to introduce the set, they cannot carry the detail which a highly ordered set demands; whilst the overall significance of the detailed charts eludes the grasp of most minds[16]. It is also interesting to note that there are constraints on the representation of such diagrams on paper due to the limited acceptability of lines crossing each other, multiple line coding, or the use of many colours.

5. *Yantras/Mandalas:* One form of diagram of special interest, because of its deliberate orientation ofward the observer, is the "yantra" (or "mandala", in its circular form). These have been used extensively in Eastern cultures to integrate many hierarchic levels of information detail concerning the universe in a form designed to be both comprehensible and to have a profound impact on the attentive observer. Indeed special practices have been developed for their preparation and use[17]. Significant in the light of the weaknesses connected with hierarchical representations noted above, is the fact that here hierarchies are bound together within a common framework with detailed elements on the outer edge of the diagram and the super-ordinate sets linking into a common centre – the focal point for the observer[18] through whose awareness (once refined) the disparate sets of experience are integrated. The challenge to the observer is to penetrate into and structure his awareness through the diagram. It is especially noteworthy that diagrams of this type contain a high degree of symmetry, as well as colour coding and symbols of various kinds. (These are in part designed to "trigger" the conditions required of the senses and awareness in order for the "programme" to work.) The symmetry features are of course constrained by the planar representation.

6. *Other techniques:* The paragraphs above would seem to mark out the current ability to represent sets, given the number of elements, the degree of their ordering, and the erosion of comprehensibility as the combination number/degree of order increases in complexity.

There are a number of other techniques of communicating the content of a set. Some are discussed in (16), but they tend to suffer from the defect of being unable to represent the set in a form which can be easily reproduced and which lends itself to detailed examination and review. It is also appropriate to note here that many authors do not summarize their insights as a set of points or insights and may well consider such a representation as damaging to the nature of the insights they seek to communicate. Indeed the pre-logical biases, identified by W. T. Jones (17)[19] against such a representation may in certain cases constitute an ultimate constraint on clearly distinguishing the elements in a set.

7. *Three-dimensional constructs*

7.1 As noted above, diagrams in 2-dimensions are extensively used to represent sets. It is however very rare to see 3-dimensional representations of sets, partly for the obvious reason that it is difficult to see the internal structure of such representations. And, despite the considerably increased facility it offers, 3-dimensional representation creates a barrier to the linear verbal description so essential to the verbal and textual expression on which much research and decision-making is based[20]. However there are techniques for handling the representation of sets in 3-dimensions, of which the most sophisticated are the graphic terminals used in computer-aided design ((19) Appendix 6). But it is interesting that, despite much attention to hierarchical ordering in organic and inorganic systems composed of 3-dimensional entities, it is in terms of a 2-dimensional representation that such hierarchies are studied[21].

This is so even though the champion of the hierarchical perspective, Lancelot L. Whyte, specifically notes that "the real need is for a systematic and exhaustive survey of the types of three-dimensional spatial ordering which characterize the more important levels in both realms" (ref. (10), p. 13). He also remarks that "Where a system is 'sufficiently ordered' and 'sufficiently nearly stationary' (terms to be clarified) three-dimensional *geometrical* relations (i.e. lengths or angles) may play a fundamental role... It is conceivable, in principle, that under certain conditions everything is derivable from angles. It seems that theory may sometimes pass rather

easily from central geometrical hierarchical models to the heterogeneous properties of static, stationary, or near-equilibrium systems, thus opening the way towards a physics of hierarchy" (ref. (10), p. 11). The equivalence in properties between physical and social systems has been repeatedly noted (20).

7.2 A further justification for moving to 3-dimensions is that it increases the *iconicity* of the representation, namely the degree of isomorphism between the structure of the reality represented and the structure of the representation. Where this is high, comprehension is considerably facilitated – which is why architects communicate new concepts to clients via models and not plans.

7.3 The question now arises as to what relation the cognitive elements of the set bear to their representation. This argument is based on the assumption that in the case of the fundamental elements under consideration, there is a strong configurational component to their comprehension as nested concepts. Many of the arguments in support of (and against) this assumption have been developed by Rudolf Arnheim (21), who states, moreover: "The aesthetic element is present in all visual accounts attempted by human beings. In scientific diagrams it makes for such necessary qualities as order, clarity, correspondence of meaning and form, dynamic expression of forces, etc. The value of visual representation is no longer contested by anybody. What we need to acknowledge is that *perceptual and pictorial shapes are not only translations of thought products but the very flesh and blood of thinking itself*. . ." ((21), p. 134). And also: "In the perception of shape lie the beginnings of concept formation." (21,p. 27). He defines "shape" to include 3-dimensional forms, though most of his examples are based on 2-dimensional shapes, especially sketches and diagrams. He does, however, imply that a third dimension (depth) enters into perception, when appropriate (as with pictures). It may therefore be concluded that under certain conditions man thinks in terms of 3-dimensional constructs, whether or not he also thinks in terms of words or 2-dimensional shapes.

7.4 In moving to 3-dimensions a highly significant constraint emerges. In 2-dimensions there is, conventionally[22], a certain freedom in that the planar surface may be extended and divided at will (within the limits of line and colour coding noted above). Whereas, in 3-dimensions, what are known as packing constraints become much more significant (23). The ways in which subsets can be nested within sets may then be severely limited.

The question is then whether such geometric constraints on representation bear any relationship to constraints on the interrelationship between subsets or their elements as concepts in the human mind. On a hypothetical 2-dimensional system flow chart, one can well imagine over 50 input/output lines drawn to a particular process box. There appears to be no restriction (although there must be electro-mechanical and computing limits to their control). But at the conceptual level, the number would be unacceptable (in terms of the constraints noted earlier) and the process box would have to be divided into smaller units. A process box with 50 input/output lines would not be a useful guide to thinking about the system. It is as though each such unit could only have one of a small range of "valencies", to borrow a chemical term (24).

Now in 3-dimensional representations the permissable valencies emerge from the manner in which the sub-components can be packed in contact together (e.g. packing small spheres into a larger one). In fact this is also true in 2-dimensions (e.g. packing small circles into a larger one), but at this level the number of relationships (i.e. points of contact) is more limited than with 3-dimensions. It can of course be argued that in many cases such a representation is adequate to the complexity represented. The search for improved tools is however stimulated by the failure of the existing ones to improve collective, operational understanding of the social condition; the assumption of adequacy may not in fact correspond to the complexity of the environment.

The 2-dimensional model is not rich enough to reflect a 3-dimensional reality adequately (or with the compact elegance and symmetry that one may suspect comprehension of complexity demands). But it may also be argued that a 3-dimensional model is equally inadequate at reflecting higher dimensional realities. However there is little to suggest that man tends to think *in* 4 or more dimensions, even if some can think *about* them and represent their results in mathematical terms[23]. To be comprehensible and widely so (in order to be of relevence to social change), "it seems safe to say that only what is accessible to the perceptual imagination at least in principle, can be expected to be open to human understanding" ((21), p. 293). Hence the value of exploring the conceptual significance of 3-dimensional representation as opposed to other forms.

7.5 The point by Whyte cited above "that under certain conditions everything is derivable from angles" has recently been explored independently in a book by Arthur M. Young. He argues "a whole object or situation is divided into aspects (or, to use Aristotle's word, causes) and that these aspects have an angular relationship to one another" ((25), p. XV). He asks: "Is my opening statement, 'All meaning is an angle', too abstract? Not if one accepts my allegation that meaning is in general a kind of relationship" ((25), p. XV). Despite his unique understanding of 3-dimensions (as the inventor of the Bell helicopter), he only applies his approach to 2-dimensional cases. In a second book (26), published simultaneously, he explores related matters basing them on a 3-dimensional concept – but he does not link this explicitly to the angular concept of meaning.

7.6 For an extensive exploration of the meaning associated with the geometry of 3 dimensions, it is necessary to turn to R. Buckminster Fuller (see note 4). His preoccupation, despite the subtitle of his book, is however with the architectural and concrete material implications of his work (of which one application is the geodesic dome which he invented). Nevertheless, in his work especially, and in that of others, stimulated by it[24] lie the basis for many generalizations in support of the

argument here. In particular, as with Whyte and Young, he is also sensitive to the general significance of angle[25]. This is essential to his basic argument that the focal points for energy events in any system are linked into a closed pattern of relationships which can be effectively represented by an appropriate polyhedron ((1), p.95 and 655). "All the interrelationships of system foci are conceptually represented by vectors. A system is a closed configuration of vectors. It is a pattern of forces constituting a geometrical integrity that returns upon itself in a plurality of directions." ((1), p. 97). No reason is given why this should not apply to a system of conceptual elements constituting the kind of ordered set of interest here.

An attempt by a biologist has in fact been made to use the geometry of the 3-dimensional biological cell structure as a cubic framework in terms of which concepts may be ordered and interrelated (29). This has been extensively developed (using large-scale 3-dimensional models) as an experiential learning tool. Another very interesting approach (30), again using a cubic framework, has been considerably developed – from a model originating in the data-processing industry (31) – in order to provide a way of structuring and representing ideas. Many points relevant to the argument here are discussed, as well as the transition from 2 to 3-dimensions. Whilst interesting and valuable as exercises, these raise further points discussed below.

8. *Mathematical notations and N-dimensional representations:* Much that is of interest with regard to sets and their elements is expressed and represented in mathematical notation which is meaningful to very few (including this writer!). This is the case with the highly relevant argument of Spencer Brown (18). It is also true of the very relevant insights of René Thom who leaves most social scientists, and policy makers behind at his point of departure: "We therefore endeavor in the program outlined here to free our intuition from three-dimensional experience and to use much more general, richer, dynamical concepts, which will in fact be independent of the configuration spaces. In particular, the dimension of the space and the number of degrees of freedom of the local system are quite arbitrary – in fact the universal model of the process is embedded in an infinite-dimensional space." ((32), p. 6). He does however support the geometric representation argued above: "I should like to have convinced my readers that geometrical models are of some value in almost every domain of human thought. Mathematicians will deplore abandoning familiar precise quantitative models in favor of the necessarily more vague qualitative models of functional topology; but they should be reassured that quantitative models still have a good future, even though they are satisfactory only for systems depending on a few parameters." ((32), p. 324). However rich the resultant insights, it is their significance and representation in 3 dimensions which is fundamental to their value for the comprehension and ordering of social processes.

## 6. Involvement of the observer/creator of the set

1. Whenever it is convenient, there is a widespread tendency to avoid consideration of the impact of those involved on research or on the policy-making process in which they participate. Researchers correct for bias in experiments and aim for reproducible results. Efforts are made to balance the interests represented at policy meetings. Consequently, when sets of basic values, problems, concepts, or principles are generated by either, they are conceived to be objective. The relationship between any such objectively determined category sets and the thinking processes of those involved (or on whom those categories are subsequently "inflicted") is not open to rational discussion in the same arenas and may well be perceived both as impolite and threatening. And yet it is recognized that:

> "The categories in terms of which we group the events of the world around us are constructions or inventions. The class of prime numbers, animal species, the huge range of colours dumped into the category "blue", squares and circles: all of these are inventions and not "discoveries". They do not "exist" in the environment. The objects of the environment provide the cues or features on which our groupings may be based, but they provide cues that could serve for many groupings other than the ones we make. We select and utilize certain cues rather than others." (Jerome S. Bruner et al., (33), p. 232.)

And again:

> "Nowadays we concede that the purpose of science is to invent workable descriptions of the universe. Workable by whom? By us. We invent logical systems such as logic and mathematics whose terms are used to denote discriminable aspects of nature and with these systems we formulate descriptions of the world as we see it and according to our convenience. We work in this fashion because there is no other way for us to work." (S S Stevens, (34), p. 93.)

In justifying their own work, Bruner et al. argue:

> "Two consequences immediately become apparent... The characteristic forms of coding, if you will, now become a dependent variable worthy of study in their own right. It now becomes a matter of interest to inquire what affects the formation of equivalent classes or systems of equivalence coding. The second consequence is that one is now more tempted to ask about systematic individual and cultural difference in categorizing behavior." ((33), p. 8).

This point was however made in 1956. Both in the research on which they report and in subsequent research, it would appear that the focus has been on categorization in the case of "laboratory problem" sets which are essentially trivial in comparison with the sets of *fundamental* concepts which are elaborated *consciously* in the course of research (or policy-formulation). The former are laboratory exercises requiring minutes or hours, the latter involve much reflection and a protracted "struggle" for the best "fit", possibly over a period of many months or years. In particular, to give the kind of "uncomfortable" example that is required, the research has *not* been applied to the sets and categories selected by those undertaking research in this very area, as an aid to explaining the differences of opinion which give rise to non-rational behavioural dynamics between the various schools of thought affected. Only "pointed", self-reflexive research of this kind, on the formulators of sets which are fundamental to social policy, can help to

clarify the basis for the opposition between policies which tends to fragment society into hostile camps.

6.2 *Laws of form*

It is not sufficient simply to complain about the widespread tendency to avoid consideration of the impact of those involved in set formation on the sets which they formulate. The reason for such avoidance merits continuing study[26].

Part of the problem seems to lie in a missing link in the relation of mathematics to logic which has been provided, with the encouragement of Bertrand Russell, by G. Spencer Brown (18). Much of science (and that includes classification) makes explicit or implicit use of set theory based on Boolean algebra which was designed to fit logic – but in doing so detaches the observer from any involvement in the logical processes[27]. Spencer Brown argues that: "nobody hitherto appears to have made any sustained attempt to elucidate and to study the primary, non-numerical arithmetic of the algebra in everyday use which now bears Boole's name" (18), p.xi). And again: "That mathematics, in common with other art forms, can lead us beyond ordinary existence, and can show us something of the structure in which all creation hangs together, is no new idea. But mathematical texts generally begin the story somewhere in the middle, leaving the reader to pick up the thread as best he can. Here the story is traced from the beginning." ((18), p.v) And, according to Francisco Varela: "By succeeding in going deeper than truth, to indication and the laws of its form, he has provided an account of the common ground in which both logic and the structure of any universe are cradled . . ." ((42), p. 6).

The result of Spencer Brown's formal exercise to separate what are known as algebras of logic from the subject of logic, and to re-align them with mathematics, is the explicit, and extremely elegant logical re-integration of the observer. His final chapter, entitled "re-entry into the form" commences with: "The conception of the form lies in the desire to distinguish. Granted this desire, we cannot escape the form, although we can see it any way we please" (p. 69). It ends with: "An observer, since he distinguishes the space he occupies, is also a mark . . . In this conception a distinction drawn in any space is a mark distinguishing the space. Equally and conversely, any mark in a space draws a distinction. We see now that the first distinction, the mark, and the observer are not only interchangeable, but, in the form, identical." (p. 76)

Spencer Brown shares the concern of Buckminster Fuller and Keith Critchlow (22), (36) with the initial conceptualization of a whole and its subsequent subdivision. He explores this using a powerful logical notation (18), whereas Fuller and Critchlow explore the structural implications in 3-dimensions. The latter would appear to be fundamental to representation and hence to comprehension. Jay Kelley, in considering the connection between man and his knowledge and the requirements for an adequate information system, arrives at similar conclusions[28].

6.3 *Logical "curvature"*

Spencer Brown may have effectively established a means of encompassing the "curvature" of the logical universe of our science-dominated culture. In Part 1 it was noted that our culture was weak in its ability to handle anything "above" the top of the hierarchies of categories we care to distinguish. His work seems to offer a remedy. For it would appear that there is a "curvature" in the more fundamental hierarchies back to the (otherwise detached) person's involvement: (a) as an *observer* in the elaboration and subdivision of such ordered sets (whether conscious or tacit), and (b) as a *participant* in the reality which such sets encode. It is the observer/participant who links, through his own person, the top and the bottom of a hierarchy. Equally it is the observer/participant who links distinct hierarchies and is therefore challenged or fragmented by any conflict between competing coding systems to which his perception is subject.

Spencer Brown makes the point that "we cannot escape the form, although we can see it in any way we please" (p. 69). However all forms are not equally probable, as was argued above in the discussion of the numerical constraints on the subdivision of sets. His own work[29] explored the ordered emergence of certain forms. René Thom's (32) widely-acclaimed study is concerned with the stability of certain forms (in every domain of knowledge), of which the "islands of stability" encountered in the pattern of isotopes are a well-known example. His analysis extends to forms encountered in social systems and human thought[30,31].

He argues that: "It may seem difficult to accept the idea that a sequence of stable transformations of our space-time could be directed or programmed by an organizing center consisting of an algebraic structure outside space-time itself. The important point here, as always, is to regard it as a language designed to aid the intuition of the global coordination of all the partial systems controlling these transformations." ((32), p. 119) This "algebraic structure" (which he expresses in geometric terms) would seem to play a role in the human psyche which is functionally equivalent to the Jungian "archetype"[32]. Although, even if this possible equivalence is invalid, this does not affect the argument below concerning such archetypes.

6.4 *Self-reference and time*

It is Francisco Varela (42) who has further developed the calculus of indications provided by Spencer Brown in order to deal with the many self-referential situations characteristic of our society. "Stubbornly, these occurrences appear as outstanding in our experience. Particularly obvious is the case of living systems, where the self-producing nature of their entire dynamic is easy to observe, and it is this very fact that can be taken as the characterization for the organization of living systems. Similarly the physiological and cognitive organization of a self-conscious system may be understood as arising from a circular and recursive neuronal network, containing its own description as a source of further descrip-

tions" (p. 5). In citing papers which address themselves directly to the self-referential nature of such systems, he notes that the topic is "normally avoided as undesirable difficulty (or *circulus vitiosus*)," and that such difficulties are rooted in language.

Consistent with the remakrs of René Thom(above), and the preoccupations of von Franz (below), Varela argues that the duality of the producer and the produced (which embodies the producer, as in any category encompassing its user) "can be pictured only when we represent for ourselves a sequence of processes of a circular nature in time. Apparently our cognition cannot hold both ends of a closing circle simultaneously; it must travel through the circle ceaselessly. Therefore we find a peculiar equivalence of self-reference and time, insofar as self-reference cannot be conceived outside time, and time comes in whenever self-reference is allowed." ((42), p. 20) In his own extended calculus based on a 3-valued system, "self-reference, time, and re-entry (into form) are seen as aspects of the same third value arising autonomously in the form of distinction" ((42), p. 21). Use of a third value enables the system to explore self-referential situations which are the basis for the limitations examined by Gödel (43). In his conclusion Varela describes his achievement as follows:

"The starting point of this calculus, following the key line of the calculus of indications, is the act of indication. In this primordial act we separate forms which appear to us as the world itself. From this starting point, we thus assert the primacy of the role of the observer who draws distinctions wherever he pleases. Thus the distinctions made which engender our world reveal precisely that: the distinctions we make – and these distinctions pertain more to a revelation of where the observer stands than to an intrinsic constitution of the world which appears, by this very mechanism of separation between observer and observed, always elusive. In finding the world as we do, we forget all we did to find it as such, and when we are reminded of it in retracing our steps back to indication, we find little more than a mirror-to-mirror image of ourselves and the world. In contrast with what is commonly assumed, a description, when carefully inspected, reveals the properties of the observer. We, observers, distinguish ourselves precisely by distinguishing what we apparently are not, the world.

We then see that we stand in relation to the world by mutual negation, and that the union of us two has therefore an autonomous structure whereby the negation engenders a distinction which leads to its own negation in a ceaseless circular process which is, in fact, the symbol which tradition has chosen to represent the creation of everything since time immemorial.

Autonomy is seen in this light to engender the two stages of the form when this ceaseless process is broken into its constituents. By the introduction of a third autonomous state in the form, we do nothing but restore to our field of view that which was there at the beginning, and which we can only see now reflected as segments of the world or in language itself. Conversely, by taking self-reference and time as our filum ariadnis through a succession of levels, we dwell upon the re-union of the constituents of these levels up to our own union with the world, and thus we find a way to retrieve the unity originally lost." ((42), p. 22–3)

### 6.5 *Number and time*

Marie-Louise von Franz (of the C J Jung Institute, Zurich) has conducted an extensively documented study into the significance of number for mathematicians, in philosophy, and as symbols of psychological significance, in a deliberate effort to bridge the gap between psychology and physics. As she puts it, her remarks "balance to some extent on the razor's edge between philosophical-mathematical and numerical-symbolical statements" (ref. (9), p. 33–34). She deliberately bridges the gap between Western and other concepts of number, which is an aspect of a current debate into the wider interpretations of the concepts of science, space, and time, which have hitherto been supposed to conform conveniently to the Western versions (40)[33].

She notes that Niels Bohr has stressed that an important step had been taken toward realizing the ideal "of tracing the description of natural phenomena back to combinations of pure numbers, which far transcends the boldest dreams of the Pythagoreans" ((9), p. 16). She argues that if we accept Wolfgang Pauli's contention that "certain mathematical structures rest on an archetypal basis, then their isomorphism with certain outer-world phenomena is not so surprising" ((9), p. 19).

She sums up her argument as follows:

"To sum up: numbers appear to represent both an attribute of matter and the unconscious foundation of our mental processes. For this reason, number forms, according to Jung, that particular element that unites the realm of matter and psyche. It is "real" in a double sense, as an archetypal image and as a qualitative manifestation in the realm of outer-world experience. Number thereby throws a bridge across the gap between the physically knowable and the imaginary. In this manner it operates as a still largely unexplored mid-point between myth (the psychic) and reality (the physical), at the same time both quantitative and qualitative, representational and irrepresentational.

Consequently, it is not only the parallelism of concepts (to which Bohr and Pauli have both drawn attention) which nowadays draws physics and psychology together, but more significantly the psychic dynamics of the concept of number as an archetypal actuality appearing in its "transgressive" aspect in the realm of matter. It preconsciously orders both psychic thought processes and the manifestations of material reality. As the active ordering factor, it represents the essence of what we generally term 'mind'." ((9), p. 52–53)

She concludes that: "Most probably the archetypes of natural integers form the simplest structural patterns in . . . (the common unknown confronting both physicist and psychologist) . . . that manifest themselves on the threshold of perception." ((9), p. 56) In order to explore further, it is therefore necessary to return "to the individual numbers themselves, and gather together the sum total of thought, both technical and mythological assertions, which they have called forth from humanity. Numbers, furthermore as archetypal structural constants of the collective unconscious, possess a dynamic, active aspect which is especially important to keep in mind. It is not what we can *do* with numbers but what *they* do to our consciousness that is essential." ((9), p. 33)

Von Franz outlines the recommended programme as follows:

"When we take into account the individual characteristics of natural numbers, we can actually demonstrate that they produce the same ordering effects in the physical and psychic realms; they therefore appear to constitute the most basic constants of nature expressing unitary psycho-physical reality. Because of this I would conjecture that the task of future mathematicians will be to collect their characteristics and analyze, when possible, every number in its logical relationship to all others. This research should be undertaken in collaboration with physicists, musicians, and psychologists who are conversant with the empirical facts about the structural characteristics of numbers in different mediums." ((9), p. 303)

## 7. Number and logic

7.1 *Beyond 2-term logic: multi-term systems*

In the above argument the terms "set" and "system" have been used interchangeably since one of the characteristics of the sets of elements under consideration was identified as the complementarity of their elements. In discussing multi-term systems, a mathematician and director of industrial research J. G. Bennett clarifies further the kinds of sets to which these arguments apply ((45), vol. 2, pp. 3–10). A set of elements taken without reference to any internal connections is called a class. A system is to be distinguished from a class, and he suggests rules for doing so[34,35]. His summary of the characteristics of systems clarifies the definition of the sets considered here (see Annex 1) .

Bennett notes that: "The properties of systems are usually studied in terms of their inner connectedness, but *there is no general doctrine of systems based upon the properties that are associated with the number of terms by which they are constituted.* This is strange, for philosophers have always been deeply concerned with the question whether or not there is a fundamental number system in the basic structure of reality." ((45), vol. 2, p. 4) Such systems have however always had to be studied by using the conventional two-term logic. "Our usual language, though full of inconsistencies and ambiguities, can be adapted to the description of two-term systems. When the meanings of words and sentences are defined with special care, a logic is constructed that turns out to be the law of two-term systems... The ambiguities and inconsistencies of our ordinary speech are not a defect, and recognition of them is a reminder that experience has more dimensions than logic. Analytical and sceptical philosophers have, during a hundred generations, exposed the barrenness of two-term thinking, and it becomes necessary to examine the possibilities latent in higher modes of thought." ((45), vol. 1, p. 25)

In support of this investigation Bennett quotes Bertrand Russell on classical two-term logic: "The extension of the subject-predicate logic is right as far as it goes, but obviously a further extension can be proved necessary by exactly similar arguments. How far it is necessary to go up the series of three-term, four-term, five-term relations, I do not know. But it is certainly necessary to go beyond two-term relations." (46) Bennett draws attention to the widespread dualism in thought, feelings and instinctive reaction[36] and comments, as an example, on the difficulty of triadic (three-term) thinking. "Contemplation of the triad is not merely recognizing a third idea as the reconciliation of two contradictories, but rather seeing in the union of the three an exemplification of the fundamental relationship by which all experience is governed. So long as nothing more is at work than the primitive associating mechanism, to speak of the 'unity of the triad' conveys little meaning. In order to perceive this unity directly, a power of attention is required that comes only with a change of consciousness." ((45), vol. 1, p. 26) He notes Russell's view that it appears to be beyond the ordinary power of man (47), although clearly both Bennett, and others believe that there is a way around the limitation (42), (48), (49).

As an indication of the route to be followed, Bennett remarks that: "The doctrine of logical types indicates that some words do not refer to terms but to systems. For example, a single term may have qualities, but it cannot have relationships. Relationship is the property of a system, and at first sight it might seem that any multi-term system can exemplify relationships. It can readily be seen that a dyad – that is a two-term system – cannot carry a relationship . . . If relatedness is a property or quality that belongs to three-term systems, *the question arises whether there are other properties that characterize systems with different numbers of terms* ((45), vol. 2, p. 5–6).

The point to be emphasized as a result of the above argument is that *the sets fundamental to the social sciences and policy-formulation constitute systems whose characteristics merit investigation irrespective of the nature of the terms in any particular case.* Namely a 5-term set of values (concepts, principles, problems, etc.) has characteristics distinct from a 7-term set, irrespective of the values selected in either case. And, furthermore, *such characteristics are solely dependent upon the total number of terms in the set.*

7.2 *Logic of inter-paradigmatic dialogue*

In proposing a deliberately non-western complement to the Aristotelian logic of western science, Kinhide Mushakoji (49) introduces a *third* pole in the dialogical process to destabilize the intellectual equilibrium which exists between two paradigms dividing a given intellectual community into two opposing poles. He then argues, in the light of complementarity in physics, that:

> "inter-paradigmatic dialogues – not only in natural sciences but also in social sciences – should be concerned not with the determination of who is right or wrong in defining a concept one way or another. It should rather concern itself with the question of what parts of the natural or social realities are best approached by one or the other position. Two formally contradictory definitions of the realities may be both relevant and complementary in shedding light on different aspects of the same social realities. This is why the logic of inter-paradigmatic dialogue cannot be bound by the laws of Aristotelian formal logic: identity, contradiction and excluded middle . . .
>
> A group theoretical treatment of concepts used by a given paradigm is insufficient because it deals only with the structure of the *signifiant* system (the logical level) without touching how the *signifié* realities (the reality level) are decomposed when one relies on a given paradigm (50). This "logico-real" problem of the relationship between the logical and the reality level calls for a study of the morphogenesis of paradigms. Catastrophe theory helps us here since it sheds light on the different logical positions in the morphogenetical space. A major difference between the two levels of *"signifiant"* and *"signifié"* lies in the fact that the former is composed by discrete concepts while the latter is a continuous space. Therefore, it becomes necessary to apply a catastrophe theoretical model relating the continuous reality (i.e. the *"signifié"*) with the discrete set of concepts (i.e. the *"signifiant"*)."

His reference to catastrophe theory, formulated by René Thom (32), relates this argument to that on logical "curvature" (above). Mushakoji then argues for a non-formal logical model developed in oriental logic on the basis of four lemmas (affirmation; negation; non-affirma-

tion and non-negation; affirmation and negation). Such lemmas are concerned with the modalities according to which the human mind grasps reality rather than how human intellect reasons about it (51). He considers the lemmic approach to be a breakthrough in view of the possibilities it provides for overcoming the static ontology of the West inherited from Parmenides and highlighting the limitation of means-end rationality.

Mushakoji's concerns are shared in part by Sallantin (48) and Varela (see above), although they both elaborate on 3-term systems in much greater detail. The relationships between these three is elusive and a broader framework (such as Bennett's) raises questions: (a) of the possibility of 4, 5 or higher term systems, (b) of why the three authors are seemingly insensitive to the qualitative attributes of systems higher in the series and (c) of the implication for classification.

### 7.3 *Number and N-term systems*

In order to make further use of the programmes that Bennett and von Franz respectively set themselves, it is necessary to link the concept of N-term system (Bennett) and that of number as studied by von Franz. What these and other authors have each attempted, in one way or another, is to identify the *qualitative* characteristics to be associated with each term in the series:

one-ness, two-ness, three-ness, etc.
or unity, duality, triplicity, etc.
or one-term, two-term, three-term, etc.
or monad, dyad, triad, etc.
or unitary, binary, ternary, etc.

Bennett argues the case as follows: "Even when enfranchised from the limitations of logic, thought does not reach beyond the triad; yet we cannot doubt that four-term, five-term, and even higher systems must be significant ... Multi-term systems oblige us, therefore, to take account of the significance of *number* as a factor in all experience; and for this we must seek a fuller apprehension than is given by logic. The logical interpretation of number derives from the formation of classes, and is essentially polar or dualistic; that is, it consists in the assignment of an object to a given class in terms of the simple distinction of 'yes, it is a member' or 'no, it is not'. This procedure leads to a view of number according to which there is nothing to be known except the laws of arithmetic. These laws belong, however, merely to a primitive form of logical thought." ((45), vol. 1, p. 26–7). Sallantin also addressed this point (48).

Bennett argues that there are several other ways in which we can think about number, such as lead to cardinal or ordinal numbers. In addition the 'arithmetic quality', based on the inner relationships of a group, may be used to distinguish prime and composite numbers, for example. But even so "the full significance of number is far from being exhausted. Numbers have meaning in their own right. The number two is not merely the symbol of duality; 'twoness' depends upon and defines the separation of opposites. The number three is indissolubly connected with the very idea of relatedness. Three as a class concept is an abstraction from experience – three as a relationship is an integral part of experience itself. This leads us to seek for a property which can be called the concrete significance of number." ((45), vol. 1, p. 28)

Bennett joins von Franz in recognizing that: "The search for the concrete significance of number is very ancient... At some unknown period ... man had already become convinced of this concrete significance, and must, therefore, have seen how a number can enter directly into events as experienced by himself." (p. 28) And: "If we are ever to free ourselves from the limitations of logical thinking, we shall have to discover a new significance in number; for number and logic, as we know them today, are inseparable." (p. 28)

## 8. Comprehension and number

### 8.1 *Problems of comprehension*

It is appropriate to note that work in the well-defined field of "multi-valued logic" does not seem to have had any impact on these concerns[37]. Nor does that on the "theory of numbers"[38]. It is only more recently in studies which face up to *non-quantitative* considerations with propositions for 3 or 4-valued logics that the nature of the link begins to emerge (48), (49). The reason for the lack of progress would appear to be that in both fields named above the problems of comprehension, and the *status of the observer*, are ignored despite the early efforts of Korzybski on general semantics (54). It is here that the questions of self-reference (see above) and the wider implications of complementarity are now significant in legitimating further investigation[39] (55).

Comprehension may be considered purely as a problem of "pattern recognition" in non-verbal data. This is now receiving considerable attention in some branches of information science concerned with the man-machine interface. It is a quantitative response to complexity and is of limited relevance here (although the illusion that the conventional quantitative mode is neutral and "value-free" is now being widely attacked (56,(57)). A much more subtle problem is associated with the comprehension of qualities, and as such necessarily involves the observer actively to a greater or lesser degree.

The question is how qualitative distinctions can be comprehended and communicated. Clearly the problem does not even become apparent until differences in interpretation create difficulties. Even then it may be disguised by explaining differences as characteristic of different schools of thought, social backgrounds, educational levels, or cultures. The effect of such perceived differences *on the medium* used to portray the quality in question may even be the focus of appreciation, where the preoccupation is primarily aesthetic (painting, music, poetry, etc.), thus again disguising the problem. Where deliberate efforts are made to use words to define the meanings to be conveyed by other words, obvious discrepancies can be resolved whilst subtler ones remain. The systematic approaches currently explored by COCTA and INTERCONCEPT (58) may further reduce the problem. Nevertheless, even when the ideal has been achieved of an agreed definition for a qualitative attri-

bute (available in a universally understood language), a core problem still remains. The word-ensemble constituting the definition will be comprehended in different ways according to the capacity and inclination of the reader – even if the definition triggers a *single* gestalt perception of the quality rather than a serial perception of its *multiple* facets. It would be naive to expect that the ultimate definition of "beauty", "justice" or any other quality could be formulated in 1979 – thus depriving the future of any possibility of comprehending, describing or expressing them more appropriately than is now possible. Similarly both a child and an adult may share a verbal definition of "peace" – but their comprehension of it is likely to differ, expecially if one has experienced the realities of war. Further elaboration of a verbal definition does not eliminate the difficulty and is quickly counterproductive.

The nature of the challenge to comprehension can be illustrated by the simple sequence of numbers: 1, 2, 3, 4, etc.:

(a) where 1 denotes any single entity *isolated* from its context, no demand on comprehension is made; higher numbers merely provide an arithmetic total. The totality is never more than an aggregate and the relationships between the elements are irrelevant.

(b) where 1 is used to denote *a totality within which no element has been isolated*, then use of higher numbers indicates successive degrees of subdivision of the original totality[40]. With each higher number the total pattern becomes increasingly difficult to comprehend.

(c) where 1 is used to denote *the totality encompassing the universe as experienced*, then the comprehension demanded is associated in traditional cultures with a supreme being. Higher numbers then reflect hierarchies of such gods, each governing qualities of an appropriately lower level of abstraction[41].

(d) where 1 is used to denote *the totality* encompassing the universe as experienced and *including the experiencer*, the state or level of consciousness of the observer is necessarily affected. Higher numbers then denote successively more multi-faceted levels of consciousness at increasingly lower levels of abstraction.

The focus of this paper is on *complete sets* which in some way aim to encompass a structured totality. These may raise problems of comprehension of type (b), (c) or (d) depending on the level of abstraction of the set elements and the degree of "insulation" of the observer.

There is currently great faith that when verbal descriptors are used as identifiers for such set elements, they will carry some universally understood meaning (e.g. peace, justice, human rights, development, democracy, etc.). As argued above, and as ongoing investigations are demonstrating (59), (60), this is far from the case. Such fundamental characteristics elude complete or even adequate definition by any particular set of words[42]. Clearly the definition or label merely points towards a comprehensible experience. It is not the comprehension of that experience. ("The map is not the territory.")

8.2 *Comprehension, remembering and mnemonic aids*

The special problem in comprehending complete sets lies in the relationships between the interdependent elements. These are seldom explored in any verbal definition, thus detracting from its adequacy. But even if the member elements can be comprehended singly or in groups in serial fashion, remembering them is increasingly difficult and their relationships are lost as is any grasp of the totality.

It is at this point that various mnemonic aids are used in describing such sets in order to provide some reinforcement to memory. The crudest and most prevalent is a simple numbering of elements. At the other extreme are sophisticated diagrams showing their relationships. Attention has even been drawn to the advantages of interactive computer graphics as an aid to maintaining creative "thinking momentum" and obtaining a grasp of a total pattern[43]. But as noted in Part I (Section 5), it is the mandala-type representations which constantly stress this mnemonic function. It is the manner in which they are designed to be used which recognizes the challenge to comprehension and causes attention to be focused (as with an optical lens) through the member elements disposed in an appropriate configuration.

Significantly it is from the continent of the mandala that have come papers on the relevance of mnemonics to classification (62), (63)[44].

> "The basic idea underlying seminal mnemonics is that concepts of objects or phenomena which are apparently unrelated at the phenomenal level, may be seen to be related to each other at a deeper level of perception. Seminal mnemonics consists of assigning the same notational digits to such "seminally equivalent" concepts, regardless of their verbal denotation or class context. But the perception of seminal equivalence of concepts is a difficult process, and demands a high degree of intuitive ability in the classificationist." ((62), p. 16)

The last sentence and other comments in the paper link back to the discussion of the previous section. But the technique is said to have been used only "intuitively and almost unconsciously" by S. R. Ranganathan in developing the Colon Classification.

8.3 *The quagmire of number symbolism: the past*

As a contrast, those preoccupied with number symbolism over the centuries have been quite deliberate in their attempts at seeking to associate numbers and concepts[45]. Such investigations have from time to time been very fashionable, whether from Pythagoras onwards in the West[46], or in the East, as Ranganathan writes in his *Prolegomena:*

> "In the mystic tradition of Chaldea and India, many such equivalences are believed to have been recognized. It gives seminal mnemonic significance to letters as well as numerals. A correct knowledge of it will make the use of digits conform with seminal mnemonics. The forgotten tradition needs to be recaptured. As the deep region of seminal equivalences transcends expression in words alone, communication through the written or printed word is difficult. Seminal equivalences are ineffable, but they get permeated by personal association and communication in a school." (67)

But although there have been many investigations and the literature is vast, the result is a veritable quagmire into which many have ventured and from which few have returned unsullied. This is not to deny that many

of the eminent intellects who have been attracted to this question have not come up with valuable insights, but rather that it is now difficult to filter the significant insights from a rich smorgasbrød of culture-bound speculations and outright nonsense which have accumulated over many centuries.

The investigations of von Franz and Bennett of the qualitative attributes of the simple systems are therefore to be welcomed because they successfully disassociate themselves from number mysticism in its more unfortunate traditional forms. Indeed, working independently, they provide the necessary complementary perspectives of psychologist and physicist which is von Franz's objective (see above). She herself explores material concerning the first four integers. Bennett ambitiously explores up to 12-term systems[47] – thus inviting misunderstanding, however due to the ever present problems of comprehension to which he himself draws attention.

The nature of the danger is illustrated by his results which are summarized in Annex 2 . Although it is the most systematic and disciplined attempt (in the West, at least), its main weakness lies in the verbal descriptors. These are really only useful as tentative signposts lacking any indication as to how the referent is to be experienced. The problem, as with all number symbolism in the past, is that it is only too easy for a reader to assume that his own comprehension of the descriptor as defined is as complete as that which is intended (leaving aside questions of Bennett's own limitations and difficulties of comprehension). Although more limited in scope, the same reservations must apply to the verbal descriptors of seminal mnemonics[48].

This is the basic difficulty with verbal descriptors and their definitions however much effort (*à la* Académie Française) is made to govern their usage and significance. It is worth considering the possibility that those such as Pythagoras were aware of this problem, as well as of the others indicated above: level of comprehension, superseding dualistic logic, the need for mnemonics, and self-reference (including the distinguisher's relationship to the denotative mark). What better way could they use to embody the subtlety of their insights than through numbers and their interplay, specially since the above questions are all number-related? The use of numbers does at least continually confront each user with his responsibility for any verbal descriptors he chooses to associate (temporarily and according to circumstance) with the concept. More important, it continually challenges him to greater levels of understanding of that concept and the manner in which it relates to others. It also leaves the future free to reinterpret the concepts in other ways or to greater depths – a process which is full of pitfalls and discontinuities when culture-bound verbal descriptors are used as at present.

However numbers are sufficient only to the very few. They do not provide a concrete image for those incapable of sustained thought at that level of abstraction. They can however be readily associated with archetypal figures (divinities, etc.) which each constitute a highly complex composite of qualities comprehensible as a whole at many levels of understanding and according to ability (9). Such figures are characteristic of many cultures for which their nature is powerfully clarified by many vivid tales and myths concerning their *relationships*. Their value lies in their ability to clarify or intensify ideas or emotions through appeal to sense experience. By such symbols, "the abstract may be brought into the realm of the concrete, where it is immediately recognizable and meaningful" ((66), p. vii). And indeed studies of the symbolic nature of medieval thought and expression "reveal in the medieval mind a weblike structure of abstract ideas and concrete realities so closely interwoven and interdependent that no serious gap was felt to exist between them." ((66), p. vii)

### 8.4 *The quagmire of number symbolism: the past resurgent*

This is of course essentially a sympathetic assessment and it cannot be denied that much that was produced within this context, if not most, now appears at best as fascinating nonsense – but this is a predictable consequence of using the culture-bound verbal descriptors of interpreters feeding endlessly on one another's products. But it would be a great mistake to believe that modern society has completely resolved the issues to which number symbolism responded.

On the one hand very many scholarly or administrative papers now enumerate lists of fundamental issues, principles, values, problems, etc. (to be compared with the medieval predilection for N virtues, sins, principles, etc.) as discussed. It is then the task of the classifier to prescribe some meaningful order. But for various reasons, society now faces a crisis of meaning which the plethora of studies is instrumental in aggravating rather than alleviating. And, despite the efforts of classifiers (who themselves have various preferences for number-governed ordering systems), such studies tend to achieve quicker oblivion than their medieval counterparts, and are just as meaningless to the uninitiated. Meaningful synthesis is rare and of limited relevance to societal problems. Comprehension and integration tend to result inspite of current enumerations and classifications and not because of them.

On the other hand, in an effort to render meaningful the nature of the complex issues which face society and the importance of the values by which changes should be guided, government agencies, social-change movements and educational authorities are now obliged to resort to symbols which can be satisfactorily communicated through the available media[49].

This requires that such symbols be easily comprehensible, coherent, and that they bear only a simple meaning (irrespective of the complexity of the issue). Because of the low status of "public relations", such symbols are only indirectly linked to the weakly ordered substantive items enumerated in agency programmes or in the scholarly studies on which they may be based. Where complex abstract notions must be communicated (e.g. concerning ecological systems), cartoon "personalities" are often used, appropriately adapted to each cul-

ture. Were it necessary to embody the characteristics of justice, beauty, love, etc. which were a preoccupation of the past, it is not unlikely that public relations would need to resort to symbols of updated versions of the superhuman beings on which that period so successfully projected its beliefs. (As it is, cartoon characters and film stars define the limits of our subtlety.) Were it considered necessary to show the relationships between the concerns of the different United Nations agencies (e.g. education, justice, agriculture, health, etc.), it is not unlikely that public relations would have to resort to an interplay of such personalized symbols in a modern series of "myths". The quagmire of the past has not been avoided. It is in process of being re-evoked, because the problems from which it arises have not been recognized.

8.5 *Encyclopaedic memory systems*

Strangely enough it is only through the recent remarkably insightful work of historian Frances Yates (68) that the role of memory in relation to knowledge classification has been subject to a preliminary investigation. She demonstrates that: "The history of the organization of memory touches at vital points on the history of religion and ethics, of philosophy and psychology, of art and literature, of scientific method" (p. 374). She makes it clear that not only did the Greeks (and possibly the Egyptians) possess an art of memory, but that this art was widely practised, extensively developed up to the Renaissance, and instrumental in the growth of the scientific method. Briefly, the art involved the memorization of an ordered set of "places" (*loci* or *topoi*[50]) which effectively constituted a permanent system of filing locations – whether based on a building, a town, or a set of divinities[51]. Onto these the user "impressed" images (*imagines agentes*, "corporeal similitudes") which would trigger access to the things or ideas to be remembered – a technique reminiscent of that described by memory and calculating prodigies in recent years. It is somewhat disconcerting that this lost art permitted its exponents to use over 100,000 filing locations (p. 120) and that these could be explored in any sequence. The importance of the art for orators, scholars, administrators, merchants, etc. is clear at a time when text reproduction was difficult and paper expensive.

What is even more disturbing is that with the Ramist educational reform in the 16th century, and increasing reliance on the printed medium, it is clear how the seeds were sown for the problems and dichotomy identified in the previous section. This reform explicitly rejected (for religious reasons) the use of memory triggering images, which seem to have been essential to the art, in favour of the present approach and its associated forms of classification. Memory is now considered as a "mechanical" facility only to be tested during examinations and otherwise to be enhanced by data banks processing information in serial order.

Whilst it would be foolish to deny the need for the reform, it seems clear that this cut off some lines of investigation which could have proved fruitful (irrespective of the "nonsense" from which it is difficult to disentangle them). Despite the subsequent interest in memory of Bacon, Descartes and Leibniz, it does appear that insights were lost (or driven underground) with the rejection of the highly complex memory systems developed by Raymond Lull, Giordano Bruno and Robert Fludd. Although Yates acknowledges that, with the available information, their full scope eludes her, she makes it clear that they at least had more or less explicit concerns for:

- maintaining the individual's *encyclopaedic relationship* to the separate categories of knowledge
- discovering better ways of fitting or *packing* such knowledge into memory, of reflecting the world in memory
- improving sensitivity to the possible systems or *patterns of relationships* between such categories of knowledge.
- developing appreciation of the *qualitative aspects* of knowledge
- discovering methods for the organic *unification of knowledge* in memory and understanding
- intensifying and focusing the individual's exposure to knowledge as a means of provoking a beneficial *change in level of awareness.*

All of these attach much greater importance to the status of the "observer" than did the subsequent development of the "objective" scientific method. It is only in recent years that the consequence of negelcting such concerns has become apparent in society's inability to comprehend and manage change, in the light of its elusive values and problems (cf. Renaissance "virtues" and "vices"), in order to facilitate meaningful human development (19).

8.6 *Augmented comprehension*

Having assembled much evidence, Yates bequeathes to others the problem of whether the Renaissance did in fact possess a secret memory technique for stimulating the human psyche to a wider range of creative achievement than ever before (p. 354). That the described techniques claim to provoke memory to retain the interrelationship between many elements in a whole pattern is clear. That this involved a preoccupation with *complete and ordered sets* in also clear, as is their relationship to *number* (in the light of von Franz's review of the same authors). Proportion, harmony and connexion in the *representation* of such sets are considered vital to success in empowering this new *comprehensive grasp* which is consequently intimately related to artistic expression: poetry, painting, music, architecture, theatre[52]. Current research on computer augmentation of intellect lacks this artistic dimension although it facilitates manipulation of categories (69).

The concern with personally meaningful vivid imagery is echoed in recent studies of symbols as signs charged with meaning (70), (71), (72). To exert their psychodynamic organizing effect such symbols presuppose homogeneity of *signifiant* and *signifié* ((70), p. 20–21). Whether and how, such "charging" can be accomplished is presumably the key to the question. Mircea Eliade has studied a primitive approach to this (73). Contemporary

interest is reflected in research on altered states of consciousness (74), and psychotronic research (75) although the process by which advertising and propaganda impart significance to isolated commercial or political signs is also of great importance. Other factors, such as iconicity, merit attention[53]. But for the special significance of the *configuration approach* characteristic of the Renaissance representation of sets of categories-cum-symbols, insight can perhaps best be gained from contemporary use of the mandala as mentioned above (38). This preoccupation has been absent from western thinking until the work of Jung – it was with the *rotae* of Lull, Bruno and company that development ceased. The technique must therefore have aimed to dissolve the dichotomy identified in the previous section, and to move beyond the neatly disciplined relationships of the concept triangle to a condition in which the "uninsulated" observer was impelled to move, change or create by exposure to symbols. That information no longer moves people to act, is a major preoccupation of those attempting to mobilize resources against world problems (76). The perspectives that Yates opens up suggests that the "quagmire" discussed above may conceal some valuable insights.

### 8.7 *Convergence of concept triangle elements: a limiting condition*

Given the preceding remarks, the question is whether anything useful can be obtained from the vast amount of material available on number symbolism in its different forms (see (9)). The answer would seem to be positive in the light of von Franz's approach. But there is an immediate problem of how to handle the subtle differences between authors and the way attribute sets are shuffled into new configurations. As noted by Varela above, the distinctions selected are as much a description of the author as of the subject matter. This question has been studied by W. T. Jones (17). It would seem that authors can get caught in a subtle trap which does not deny the significance of their insights but rather limits the sectors of society (or period) within which their interpretations can be fully communicated and for which they will be valid and socially significant.

Aside from refining methods for sifting and testing complete sets, their relationship to one another must be clarified. This may be alluded to in terms of their relationship to sets of progressively greater "generality", here-and-now "concreteness" or "inclusiveness"[54,55]. There is a qualitative convergence but its nature necessarily escapes verbal delimitation[56]. It is a challenge to the comprehension of the observer and ultimately to the knower-known dichotomy. And, furthermore, whenever "fundamental" sets must be produced, they can only constitute aspects of a more fundamentally integrated understanding which must necessarily emerge progressively if future society is not to be deprived of all possibility of creative insight in this domain – to say nothing of any more mature insights on the part of the author. Clearly only the future can progressively identify and give content to more fundamental sets. Closure cannot be assumed – or, if comprehended, then not communicated. Closure in this domain cannot even be premature; it is impossible with maturing individuals in an evolving society (except for strictly limited purposes which carry the seeds of their own mortality). Any attempt at closure therefore merely sets the stage for production of "improved" versions, with all the resultant non-rational dynamics between the advocates of each and the hiatus as one version replaces another.

It is the argument of this paper that in such complete sets of a given number of elements, the latter are characterized by a qualitative pattern independent of the nature of the set elements. And as the set becomes more fundamental this qualitative characteristic predominates. For, as such sets become more fundamental or general, the *characteristics of the elements* (labelled by words) are increasingly affected (in their significance to the observer) by the semantic field associated with the *numbers* (whether explicit or implicit) characteristic of the representation of a given set. The label *words* therefore introduce increasing confusion, since the degree of precision they are expected to carry is severely eroded in comprehension by a cloud of polysemantic associations that are progressively more irrelevant to the elements distinguished. And, the more fundamental the set, the more probable it is that the numbers characteristic of the representation would more effectively label the set elements which in any case increasingly approximate the semantic fields of the numbers embodied in the representation[57]. There is in fact a convergence or melding of the elements of the "concept triangle" (word, meaning, referent) with the observer, who is necessarily incorporated into the referent by the set, if it is fundamental. The 4 terms form a "concept tetrahedron"[58]. Recent work has shown the link between the status of the observer and information systems viewed in the light of relativity theory (81).

This recalls Spencer Brown's point cited above that "We see now that the first distinction, the mark, and the observer are not only interchangeable, but, in the form, identical." ((18), p. 76) This is clearly however a limiting condition and for less fundamental sets identity is necessarily not assumed, particularly since it is not experienced[59].

Even though the limiting condition may be ignored, the fundamental sets of interest here are sufficiently close to it, that any use of words must be viewed with great caution[60]. What the words attempt to label is better coded by numbers with their qualitative associations. Differences in formulation of fundamental sets arise because each assumes it is containing fundamental elements but is effectively only containing those evident from an aspect of an even more fundamental domain (e.g. associated with a particular numeric quality). And each tends to focus on different aspects without being able to incorporate others even if the formulator is aware of them. At this level, however, there is a high degree of isomorphism between the numeric qualities characteristic of the sets centred on different aspects. This may be used to clarify the content of sets, and to

identify more fundamental sets, without relying too heavily on the words used to carry meaning in any particular case.

## 9. Qualitative characteristics of sets

### *9.1. Characteristics of multi-term systems*

The remarks of the previous section provide a context within which efforts at establishing the characteristics of multi-term systems can be considered as defined in Annex 1. This question cannot be explored here. It serves as an indication only therefore, that the results of J. G. Bennett's exercise are summarized in Annex 2. This suffers from the disadvantage of not establishing explicit links to the rich variety of cultural and mathematical material reviewed by von Franz in her study of the first four integers. Such material should be used to interpret and broaden the meanings, otherwise Bennett's (or any other) particular orientation is too easily assumed to exhaust the meaning associated with each system – thus subjecting the approach to the difficulties raised in the previous sections.

Bennett points out that "no one system taken alone can exemplify the organized complexity of real structures. We usually need to take more than one system into account in order to gain the insights needed for understanding any existing structure that we find. According to the aspect of structure that happens to be relevant to a given purpose, a system of one order may be more useful than another." ((45), vol. 3, p. 11–12).

Also (bearing in mind the limited value of label words for the system attributes identified in Annex 1): "The series of multi-term systems is a progression such that each system implies all the earlier ones and requires those that follow. We cannot understand the triad unless we already group the notions of universality and complementarity, and the dynamism of the triad is not realized without the activity of the tetrad. The later systems are not only more complex and more highly organized than the earlier ones; they embody an understanding of reality that is more comprehensive and practical. The progression is from abstractness to concreteness." ((45), vol. 3, p. 12).

But: "Not all structures exemplify all stages of the progression to the same degree. A given structure may exemplify one attribute strongly and others weakly.... One other general property of systems remains to be considered. This we shall refer to as term-adequacy. If the terms of a system cannot be clearly discerned in a given structure, the required characters will be lacking and the system in question is then inadequately represented." ((45), vol. 3, p. 13). Namely the set is weak in that attribute.

In the light of this argument, attempts should be made to explore a 3-term set re-interpreted as a 4-term system or more, particularly in the case of fundamental sets. In Bennett's study of systematics[61], he finds that: "for purposes of practical utility, the systems fall naturally in groups of four. The first four from the monad (1-term) to the tetrad (4-term) help us to see *how* structures work. The systems from pentad (5-term) to octad (8-term) show *why* they work and how they enter into the pattern of reality. The third group from the ennead (9-term) to the dodecad (12-term) is mainly concerned with the *harmony* of structures: that is, the conditions that enable them to fulfill thier destined purpose." ((45), vol. 3, p. 12)

### *9.2. Clarification of specific sets*

Two procedures are outlined (in Annex 3) for the clarification of material on complete sets. Both procedures ensure that any given set is embedded in a context. In the first case, this is in relation to alternative (or more superficial) possibilities. In the second, it is in relation to more fundamental possibilities.

By such procedures the set is being tested and refined in a manner which should establish the constraints on its meaningfulness and communicability to those who – in contrast to its vigorous advocates – may be sensitive to other aspects of the context in which it is embedded[62]. The procedures necessarily highlight the extremely limited value of dependence on the univocal, unambiguous meaning of any words (in definitions) used to label such sets or their elements.

It should be stressed that, in contrast to the usual competitive preoccupation, the concern is not with establishing any particular set as the most valid. Rather it is to give some understanding of the probability that any such set will be advocated, perceived as valid, or widely comprehended and communicated. At the same time it supplies a context for elucidating the meaning underlying whatever marks (words, numbers, codes, etc.) are used to identify a set and its elements.

## 10. Representation of multi-term sets

10.1 The above sections have identified: the constraints on set formulation imposed by number: the importance for comprehensibility of representation in 3 dimensions; the impact of particular number choices on the consciousness of those exposed to such sets; the problems of comprehension and the role of memory; and the properties exemplied by sets of a given number of terms. These are brought into focus by the problems of representing and comprehending multi-term sets. The problems have been strongly emphasized. Even a brief perusal of Annex 2 makes it clear that a verbal explanation in linear text form dos not come near capturing the gestalt quality of most of the systems identified. Just as when the elements of a set are listed, the *sequential* presentation introduces the time dimension to an extent determined by the number of terms. Von Franz notes: "Detailed investigation revealed, however, that number, understood as a psycho-physical motion-pattern, is intimately connected with the problem of time" ((9), p. 235). The linear scanning required is not consistent with holistic

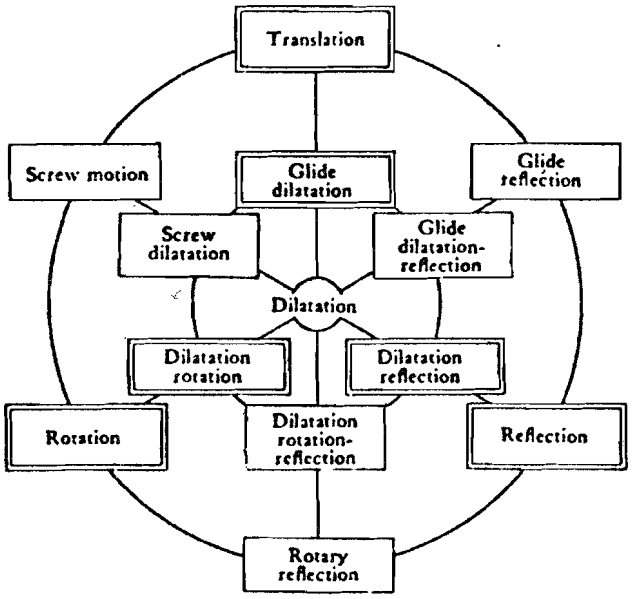



Fig. 1: Schema of positional space relations. The relations of the outer ring contain identity as their inner relation. The relations of the inner ring contain delatation. Dilatation itself is a purely inner relation. (Source: W. von Engelhardt, "Sinn und Begriff der Symmetrie". In: Studium Generale 6 (1953) No. 9, S. 524 reproduced by permission of Springer Verlag and W. von Engelhardt)

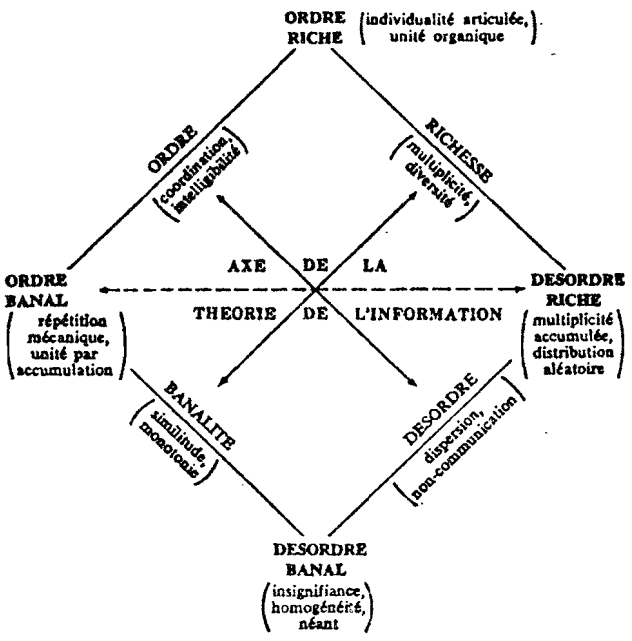



Fig. 2: Schema used to interrelate information of different qualities of order in experimental music. (Reproduced with permission from: Henri Pousseur: Fragments theoretiques I sur la musique experimentale. Bruxelles: Ed. de l'Institut de Sociologie de l'Universite Libre de Bruxelles.

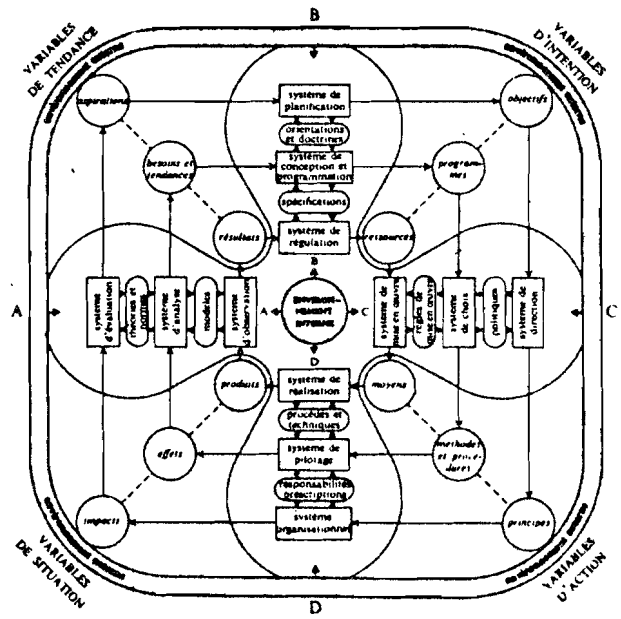



Fig. 3: Interrelationship of 4 activities at 3 levels in an organization resulting in 12 sub-systems linked by imput-out variables and by control variables. (Reproduced with permission from: Bernard Walliser: Systèmes et Modèles; introduction critique à l'analyse des systèmes, Paris: Editions du Seuil 1977, p. 115.

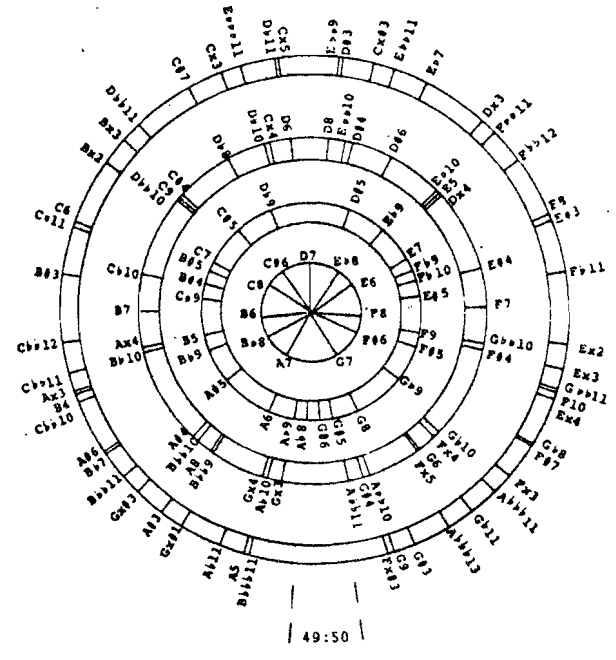
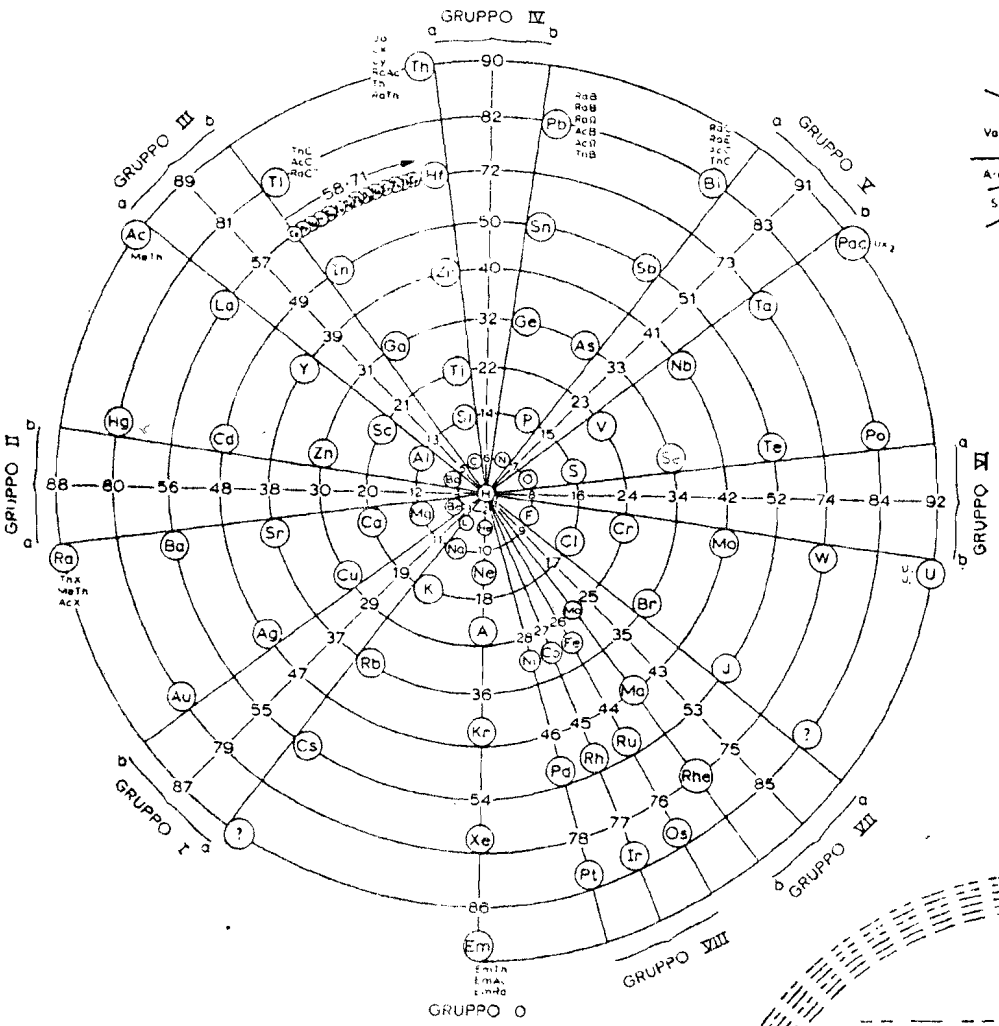
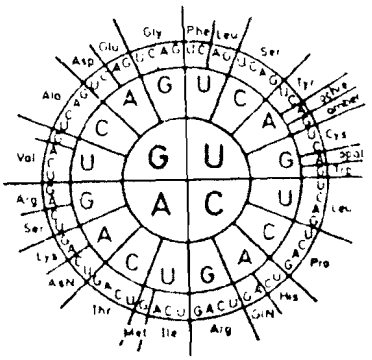



Fig. 4: Emergence and classification of tones governed by some integer ratios (deduced from Platonic texts) and their representation in concentric mandala form. (Reprinted by special arrangement with Shambhala Publications, Inc., 1123 Spruce Street, Boulder, Colorado 80302 U S A, from: "Myth of Invariance" by E. G. McClain, p. 168. Copyright 1978 by Shambhala Publications, Inc.)

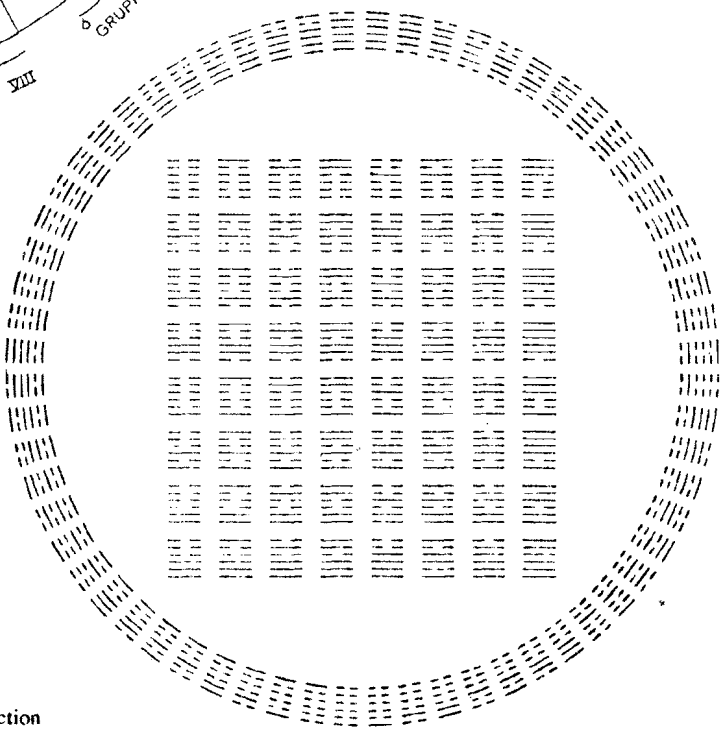
Further examples



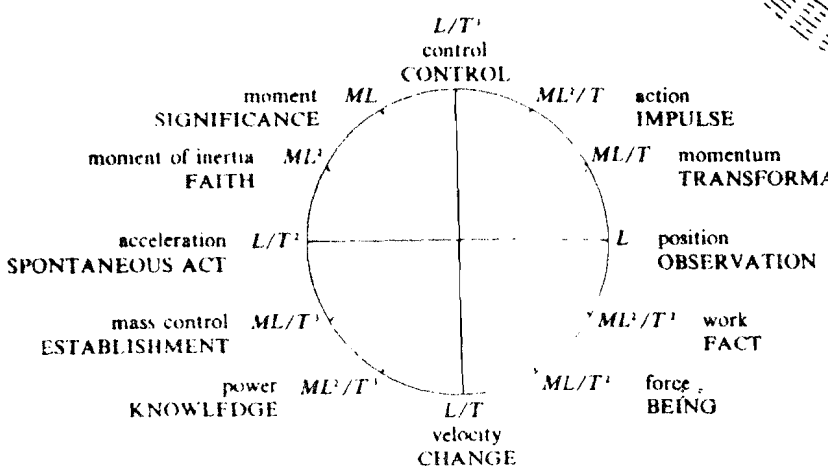

Periodic classification of the chemical elements (Piutti, 1925) (see also ref. 116)



"Sun-code" of the genetic codones (Bresch and Hausmann) (see also ref. 123)



Hexagrams of the Chinese I Ching system (see also ref. 123)




Physical quantities correlated with English language meanings (Young, ref. 26)

comprehension of the single underlying concept. The manner in which the elements stand as "un-time-bound" aspects to the set as a whole is lost[63].

10.2 It is for such reasons that Bennett, in his presentation of the systems in Annex 2, relies heavily on 2-dimensional diagrams with a high degree of symmetry. And indeed many complex structures are open to comprehension if they have a high degree of symmetry (82). The emergence of symmetry in science is also frequently considered an indicator of the adequacy of a description. As Rudolf Arnheim notes: "In a broader sense, symmetry is but a special case of fittingness, the mutual completion obtained by the matching of things that add up to a well-organized whole" ((21), p. 64–65). Symmetry has the special merit of enabling the mind to regenerate constantly those aspects of a pattern which fade from comprehension when they are not the focus of attention[43]. It is in part for these reasons that asymmetric diagrams are seldom used for these purposes. Lack of symmetry limits the comprehensibility of conventional concept maps (83). Figs. 1–3 are thus interesting examples of "representational classification".

10.3 Given that symmetry is richer in 3 dimensions and that representation is then naturally more compact, the basic question still remains whether such packing of 3-dimensional structures should bear any isomorphic relationship to the manner in which concepts are "packed" in comprehension. Is it irrelevant that the geometry of such packing is fundamental to so many natural structures in the environment and to the design of artefacts? The argument may be made that concepts require an N-dimensional space as Rene Thom would seem to imply (see above). And yet he himself recognizes isomorphism between natural and social systems[30]. And it is those very same natural systems requiring an "infinite-dimensional space" which are so elegantly and symmetrically ordered (to one perception) in relatively simple *3-dimensional* arrays (84) (131). Agreed, the N-dimensional space is required to order *transformations* and conflicts between such structures. But it would seem to be highly probable (particularly in the light of the ordering role of number) that there be a certain degree of isomorphism with "concept packing", at least in 3 dimensions and if only with regard to the iconicity of representations[31,64] (The very interesting question, of whether Thom's N-dimensional space can reflect the transformations and conflicts between such structures, namely the social dynamics of ideas and the organizations based upon them, is not an immediate concern here.)

10.4 Bennett, in presenting his schema (see Annex 2), makes use of several different 2 and 3-dimensional diagrams to symbolize a system of a given number of terms. He does this to bring out different qualitative aspects of the system in question. This suggests a much more general approach to the problem of representation using work in graph theory (see Annex 4)

10.5 Although the graph theory convention of points and lines may only be meaningfully representative to a segment of the population in western cultures[65], it is possible that symmetric patterns and solids are much more widely acceptable. Whatever the case, such structures may be used to order or classify the elements of a meaningful representation which could (and does traditionally) employ other forms and media, e.g. animation[66], dance[67], drama, ritual, music ((90)–(95)). Part of the general inability to perceive such underlying structures lies in the widespread "visual illiteracy" discussed by Arnheim ((21), p. 294–315) – although "structural illiteracy" draws attention to an even more neglected aspects of it. (It is likely that there is a whole series of unrecognized configurative classificatory "handicaps" equivalent to such "hidden" disabilities as dyslexia, discalcula, arhythmy, etc).

10.6 There is also good ground for arguing with Fuller (1) that ideal forms such as polyhedra conceal a basic design problem which must be solved to obtain a more complete representation in concrete reality. He does this by generating dynamically stable "tensegrity structures" each based directly on a given polyhedral form[68] (96). In this design problem and its solution may well lie the clue to the limited utility of ideal forms for representation, comprehension and (above all) effective implementation. For this reason, the author has explored the possibility of using tensegrity structures as a basis for new approaches to the representation of concept and problem complexes, and the creation of new kinds of organization (97, 102). Clearly this is relevant to the representation of the sets of interest here (98, 99, 101).

10.7 The above procedures result in the generation of a multitude of symbols which may be enrichened in various ways (e.g. colour coding, etc). The question arises as to whether this multiplicity is not undermining the original objective of representing and communicating the governing central concepts – particularly since it is what already characterizes the representations of sets of various kinds. However, in remarking on the apparent divergencies of representation between traditional cultures, von Franz states: "In this field, too, knowledge of the part played by the psychic unconscious in the formation of the concept is lacking. Of the archetypes of the collective unconscious we know that they – like a crystal lattice in the mother liquid – form structural dispositions in the unconscious, invariable in themselves, although their pictorial and representational appearance in human consciousness exhibits variations. Behind these variations lies a basic archetypal pattern that can be descriptively reconstructed." ((9), p. 31–2). Tucci effectively makes the same point in explaining how different users benefit from different kinds of mandalas to arrive at the same understanding: "Hence, as the ritual is adapted to individual adepts, the mandalas are very great in number. In some of the Tantras of the Yogatantra class they can be counted in hundreds" ((38), p. 80). There is a great deal to be said for adapting symbols and representations to the user, rather than imposing a "standard" pattern.

But the point is that these divergent forms, and those arising from the procedures above, are *generated by rules governed by numbers.* The variations emerge from a general pattern or number field which we are slowly coming to understand (e.g. von Franz has a chapter on

"Archetypes and numbers as 'fields' of unfolding rhythmical sequences" in which she grapples with the question).

## 11. Implications

11.1 The purpose of this paper is to demonstrate the importance of number in the complete sets fundamental to social science and policy formulation. It is fairly obvious that formulation of a 2-term set of concepts (values, problems, etc.) establishes a dynamic for the advocates of each term, or those involved in any institutionalization of the dyad – namely a dynamic having any or all such aspects as: active/passive, right/wrong, we/they, dominant/subordinate, conflict, complementarity. For example:

> "By the very nature of scientific logic which is binary, intellectuals tend to form bi-polar structures with two opposed camps rallied under two paradigmatic banners. The polarization often takes place even within each of the two poles which then divide themselves into subpoles, and so on, and so forth" (49).

It is equally, obvious that promulgation of a 1-term set (e.g. *the* problem, *the* value, *the* method, etc) gives rise to another kind of dynamic. It is however less obvious what kinds of dynamics tend to arise from sets with a larger number of terms. Yet sets with larger numbers are frequently produced and usually it is considered convenient to ignore how the elements of the set interact at the conceptual level or through organizations (departments, programmes, laws, information systems, etc) on implementation. This paper implies that, like it or not, *certain interaction qualities are built in by the choice of the number of set elements.* If ignored, they will erode or completely undermine the effectiveness of any action based upon them. They define the problem to which the initiative is vulnerable and by which it will be counteracted, or nullified.

11.2 Implicit sets of a given number of terms usually engender particular *styles of debate.* For example : 1-term, promulgation and propaganda; 2-term, pro and con argument as in some legal, parliamentary and scholarly arenas; 3-term, mediatory and reconciliatory debate. Given that issues currently exceed the capabilities of such forms of debate or are exacerbated by them, other higher-term forms may be envisaged to contain and facilitate the interactions between a greater number of distinct viewpoints. This would also be relevant to the interactions within *interdisciplinary teams* and the design of the classification systems which serve them (98). A sense of *issue configuration* would stabilize understanding of the complete sets of "logically incompatible" problems which such teams are increasingly obliged to confront. This could lead to the emergence of methods based, on a *non-dualistic complementarity.* A need for an improved approach is becoming evident (132), even in unexpected places: "The mosaic theory of intelligence has focused attention on collection, the gathering together of as many pieces as possible for the analyst to work with. A more psychologically oriented view would direct our concern to problems of analysis, and especially to the importance of mental models that determine what we collect and how we perceive and interpret the collected data. . . there are important implications for the management of intelligence resources" (133).

11.3 *Research on complete sets* is required to clarify their nature and variety. Complementary approaches include: research on number, as advocated by von Franz; research on symbols in traditional cultures, of any well-ordered sets and their elements; and research on modern sets elaborated in scholarly and action-oriented texts. This should lead to better understanding of: (a) how sets can be formed and their elements classified, (b) how the relationships between their elements can be rendered comprehensible, (c) how the nature and value of higher term sets can be demonstrated, and (d) the nature of the totality, they are intended to encompass. It is in the East that qualities and attributes have been so carefully distinguished and ordered, whereas sophisticated number-based frameworks have been elaborated in the West. This research should bring out the points of contact. An excellent point of departure would be the problems of "classifying" tones in music as explored in two complementary studies by a philosopher (134) and a musicologist (135) faced with the challenge of the alternative patterning possibilities within the Rg Veda: "Rg Vedic man, like his Greek counterparts, knew *himself* to be the organizer of the (musical) scale, and he cherished the multitude of possibilities open to him too much to freeze himself into one dogmatic posture. His language keeps alive that "openness" to alternatives, yet it avoids entrapment in anarchy." (134, p. 31) This appears to amount to a degree of order beyond that attained in classification today; the flexibility and the challenge to musical creativity are illustrated by Fig. 4. It is perhaps no accident that P A Heelan's work on the logic of changing classificatory frameworks (139) cites the Rg Vedic example and is considered of fundamental importance by these two authors[84].

11.4 It is not recognized, when advocating or imposing the use of particular sets (e.g. of values, needs, etc), that these effectively compete as *functional substitutes in traditional societies* for other sets of qualities represented by hierarchies of gods or spiritual beings governing those qualities (or some of them). The fundamental sets society now attempts to generate are indeed designed to perform many of the regulatory functions previously ascribed to supernatural beings or potencies. Given the relative rapidity with which such sets are now formulated – compared to the long cultural refinement of a pantheon – it is not surprising if they are viewed as superficial, "bloodless" and unrelated to the cultural refinement of the traditional sets. These are so meaningfully represented (with nested levels of interpretation) through richly decorated beings and memorable tales exemplifying their relationships – to the point that the quality and its representation are difficult to distinguish in a particular culture. The lack of success of public information programmes of national and international agencies, in substituting modern intellectualized versions (of somewhat ersatz quality) using product marketing techniques, is understandable. The new versions lack

credibility and durability even if the traditional versions are destroyed by the process[69,70].

11.5 Comprehension of the qualitative characteristics encompassed by higher-term sets has been shown to be no easy matter despite their vital importance for a more adequate grasp of our current social crisis[71]. Problems of *classification, comprehension, memory aids and representation need to be considered together.* There is every indication that conventional methods do not have an adequate degree of complexity to embody, and reflect for comprehension, the complexity of multi-term systems[72,73]. Research is required: (a) on the generation of *iconic symbol sets of high mnemonic value,* (b) on the consequence of disposing them in configurations so that the pattern of relationships may be comprehended as a whole, and (c) on any paradigm shift or change of awareness which this may facilitate. There is no reason why this should not include an investigation of the traditional memory technique and its intimate relationship to classification systems[74]. To what extent were traditional symbol ssystems, or associated numbers, successfully used for their powerful mnemonic value?

11.6 Intriguing lines of investigation emerge from recognition of the intimate relationship between brain operation and classification. Varela notes: "the contents of our reality are truly a reflection of the recursive biological and cognitive computations, in contradistinction to the more commonsense view that our knowledge is a map of the out-there. From this point of view, there is more a construction than a map. These are tantalizing possibilities for a cross-connection between epistemology and science, for the design of knowledge representation systems, and for management and societal problems." (106)[75]. This is related to current investigations of the transformation of the categories of conscious experience associated with shifts in characteristic EEG frequencies. For example, it is suggested that: "the felt shift and the reorganization of conscious experience is a multi-level phenomenon, involving a reorganization of concepts, a choice of principles consistent with these concepts. . . as well as the appropriate reorganization of all lower levels of the hierarchy consistent with these changes . . . . The transformation, then, is not merely a reorganization, but at a deeper level is a re-creation" (107)[76]. EEG data may even provide a link between characteristic frequencies (1 – 3, 4 – 7, 8 –12 Hz), the preferences mentioned in Part 1 for sets of a given number of elements, the ability to comprehend them, as well as the quality of that comprehension. A better understanding of the conventional separation of subject and object can be obtained by exploring, as does R. Fischer, ecstatic and meditative states in which "the separateness of object and subject gradually disappears and their interaction becomes the principal content of the experience. . . meaning is "meaningful" only at that level of arousal at which it is experienced, and every experience has its state-bound meaning" (136). Relevant to the "concept triangle" question (see Part II and Fig. 5), Fischer in a section on "sign-symbol-meaning transformations", discusses evidence of the transformation of sign to symbol in the visual realm "where the constancies of space and time are replaced by geometric-ornamental-rhythmic structures", namely hallucinatory form constants. These are visible metaphors, otherwise uncommunicable, within a structure of symbolic logic and language whose non-visual equivalents also govern the order of poetic and musical rhythm in such experiences. Once again the importance of number becomes apparent. This question is set in a wider framework in studies initiated by Erich Jantsch (137, 138), to which the argument of this paper links at points too numerous to mention here.

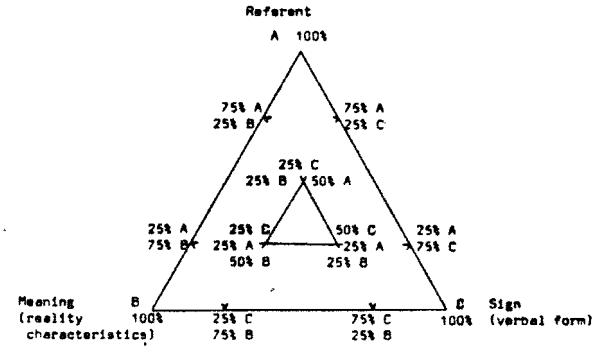



Fig. 5: Convergence of concept triangle elements: Diagram indicating a particular condition of "adulteration" of the absolute distinction between the pure elements of the concept triangle. (Such diagrams are used to indicate the variety of equilibrium conditions of 3 elements in physical chemistry.) Such a presentation may be used to clarify the nature of other kinds of "blurring" of distinctions between the three elements.

11.7 This paper attempts to show the basic role of number and configuration in overcoming limitations to man's ability to perceive (and denote through classification schemes) the patterns which affect him and in which he is embedded. Biologist Gregory Bateson's central thesis is: "The pattern which connects is a metapattern. It is a pattern of patterns. It is that metapattern which defines the vast generalization that indeed it is patterns which connect" (112, p. 11). He asks: "How is logic, the classical procedure for making chains of ideas, related to an outside world of things an creatures, parts and wholes? Do ideas really occur in chains, or is this lineal structure imposed upon them by scholars and philosophers? How is this world of logic, which eschews "circular argument", related to a world in which circular trains of causation are the rule rather than the exception? . . . we shall see as every schoolboy ought to know that logic is precisely unable to deal with recursive circuits without generating paradox, and that quantities are precisely not the stuff of complex communicating systems" (p. 20). And: "as of 1979, there is no conventional way of explaining or even describing

the phenomena of biological organization and human interaction". It is through study of *number-governed qualitative configurations* that responses to Kelley's related questions should be sought: "And the ultimate question is, *what nature of pattern or system of patterns will enable the human mind to retain familiarity with the maximum number of patterns*? And what is the maximum number of patterns the human mind can hold if the patterns are of this type? What other attributes of patterns are conducive to greater retention by the mind?" (41). Finally, how is this related to the level of awareness or maturity of the observer? (107)[77,78]

11.8 The ability of the mind to retain elements of information long enough for it to form memorable patterns with other elements (e.g. of the set) can be enhanced by the use of *mnemonic aids*. Whilst these may be viewed with disdain by those familiar with the subject matter, it must be recognized that classification schemes are not memorable to the uninitiated (e.g. the public, its representatives and those from other disciplines) who ultimately determine through the democratic process whehter resources will be allocated to the maters ordered by such schemes. The same applies with regard to any argument presented in a linear sequence in an article or book. There is a strong case for interrelating the points made in a *non-linear* presentation. This goes beyond the seminal mnemonic serial structure described by Neelemeghan (63)[79]. Furthermore, in view of the increasing resistance to written arguments of any length there is a case for investigating the possibility for their partial replacement by mnemonically structured diagrams which may provide the detailed pattern for dramatized portrayal necessary for communication to a wider audience. Three dimensional centred mnemonic structures may offer possibilities for memory reinforcement and comprehension beyond those of the two dimensional variety.

11.9 In considering contemporary efforts in the West to allocate qualities and attributes to multi-term systems[80] one is particularly struck by the "bloodless" nature of the resulting categories (however innovative the exercise, such as in the case of Bennett). Such frameworks are generally conceived as mutually exclusive, the advocates of each ignoring the others in favour of their own particular slant on reality. There is much misplaced confidence in the ability of words to label qualitative concepts without ambiguity[81]. It is not recognized and that, as such, each constitutes a representational aspect of a more subtle and more comprehensive framework (cf. Rene Thom's approach). In fact, however apparently distorted or inadequate the attempt, its degree of "distortion" identifies the location of its advocates in relation to other perspectives, challenges, and problems of comprehension. Such relationships are governed by numbers indicative of qualitative distinctions.

11.10 It is to be hoped that this paper has demonstrated the importance of a new approach to representation and the possibilities for it. It may indeed be argued that Johan Galtung's emphasis (56) on the need to switch from the conventioinal "facts-theory" to a "facts-theory-value" (i.e. from 2-term to 3-term) approach, should be extented to "facts-theory-value-*representation*" (4-term), or beyond[82]. The dynamics resulting form facts-theory are too well-known, but the difficulties are not eliminated by his 3-term suggestion. Basically, if insights cannot be meaningfully represented, they are incomprehensible and therefore irrelevant to the period in which they are formulated.

11.11 Finally in the words of Bennett: "For a long time, men have looked for ways of getting beyond the dyad: but mankind as a whole remains bound by sentiments of exclusion and contradiction. Meanwhile the progress of science and technology is leading us towards structured notions of greater and greater complexity. The same is true of nearly all branches of life: psychology and sociology, art, history and religion; all are moving away from naive expectations of simple unstructured solutions to human problems and towards the recognition that we and the world in which we live are an organized complexity that can be understood – even to the limited extent that we do understand – only by discerning the structures that bind us together". ((45), vol. 3, p. 74–5). Or in the words of Bateson: "Break the pattern which connects the items of learning and you necessarily destroy all quality". (112, p. 8) Unrelated set elements break patterns.

## Annex 1

### Clarification of terminology

The definitions given below are those of J. G. Bennett ((45), vol. 3, p. 10–11) and are given as a basis for his elaboration of a multi-term sequence in Annex 2. In the main part of this article "set" has been used to signify what Bennett defines as "system", although the two terms have been used interchangeably.

"1. A *system* is a set of independent but mutually relevant *terms*. The relevance of the terms requires them to be *compatible*. No one term of the system can be understood without reference to all the others.
2. The *order* of a system is given by the number of terms. . . .
3. In systems, there are no fixed meanings attributable to the term, which depend upon the structure of the system as a whole, so the various *connectivities* are common to all systems of the same order.
4. Every system exemplifies modes of connectedness that are typical of the number of terms. Thus there are zero connectivities in a monad (one-term), one in a dyad (two-term) . . . If the connectivities are distinguished according to direction, the number is doubled. All the connectivities are significant and must be taken into account if the structure represented by the system is to be understood.
5. Each order of system is associated with a particular mode of experiencing the world, called the *systemic attribute*. . . .
6. The mutual relevance of all the terms of a system requires that they should be of the same logical type and make contributions to the systemic attribute of

one and the same kind. This we shall indicate by a common *designation*. . . .

7. The independence of the terms of a system requires that each should have a distinctive character. An important part of the study of systems consists in identifying the term *characters* of systems of a given order . . . .
8. The mutual relevance of terms of a complex system can be found, to a first approximation, by taking all the terms in pairs. These are called the *first-order connectivities* . . . Connectivities of a higher order can be studied as sub-systems from the tetrad (4-term) onwards . . ."

## Annex 2

### Example of an elaboration of a number-based sequence of systems

The series below was developed by J. G. Bennett (45) to replace the Aristotelian and Kantian categories, with their dualistic characteristic. His definitions of systematic features are given in Annex 1. The characteristics given here summarize the extensive descriptions of Bennett (vol. 1, p. 31–48, vol. 3, p. 14–75).

Comprehension of the systems proceeds in a definite sequence, given their order of emergence into awareness and the minimum number of terms required to exemplify their attributes. Only 12 systems are identified here, although systems of any number of terms may be considered in order to encompass whatever degree of concreteness one is capable of grasping. The limitation is one of understanding.

A particular system never exhausts the possibility of description and comprehension for, whatever number of terms is reached, some degree of abstraction remains and additional terms must be admitted in order to move towards a greater concreteness. Growth in understanding requires recognition of the representational power of successive systems and a deepening appreciation of their significance. *As implied here and as stressed in the main text, Bennett's word labels and comments are only indicative and do not encompass or exhaust the meanings to which they refer.* Their indicative power may be severely eroded by irrelevant polysemantic associations and increasingly so for the 3-term case and above. Conversely the richness of meaning in a given case is indicated by the symbol complexes which cultures produce to exemplify such systems. The symbols may facilitate a better intuitive grasp of each system *as a whole*, in contrast to the fragmented comprehension resulting from the following descriptions presented as linear text.

*1-term representation and comprehension* ("Wholeness")

Systemic attribute: universality. Term designation: totality. Term character: diversity in unity.

Any situation to which we direct our attention is a monad, but some exemplify the systemic attribute of universality more strongly than others. The monadic character of the universe as a totality is present in all its parts. Wholeness is universal and omnipresent but relative; it may be transformed into identity. The combination of confused immediacy and the expectation of finding an organized structure gives the monad a progressive character; it is what it is, but it holds the promise of being more than it appears to be.

Aspects of wholeness: unity, coherence, togetherness, completeness, order, organization.

*2.-term representation and comprehension* ("Polarity")

Systemic attribute: complementarity. Term designation: poles. Term characters: positive, negative. Connectivity: force.

Any pair of terms between which both connection and disjunction are recognized, although few pairs stand in more than weak opposition to one another or with more than insignificant connection. Through polarity, everything is in a state of strain which polarity itself can do nothing to relieve. If gives rise to force which may be transformed into direction. It can neither show how oppositions arise nor how they may be resolved. Its closure is not that of completeness.

Aspects of polarity: active/passive; pleasant/unpleasant, like/dislike, etc.

*3-term representation and comprehension* ("Relatedness")

Systemic attribute: dynamism. Term designation: impulses. Term characters: 1, affirmation; 2, receptivity; 3, reconciliation. Connectivities (1st order): acts (1–2, generation, 2–3, consent; 3–1, decision). Connectivities (2nd order): actions (1–2–3, expansion; 1–3–2, interaction; 3–2–1, freedom; 2–1–3, concentration; 2–3–1, identitiy; 3–1–2, order).

Every dynamic structure has the form of a triad and the three independent impulses found are those to which all relationships are reducible. Such relatedness may be transformed into interaction. The triad shows how acts enter into the structure of the world and resolve contradictions.

*4-term representation and comprehension* ("Subsistence")

Systemic attribute: activity. Term designation: source. Term characters: motivational (1, ground; 2, goal); operational (3, direction; 4, instrument). Connectivities (1st order): interplays

Subsistence is the limitation of existence within a framework and may be transformed into maintenance. The tetrad specifies an event. It is the form of all activities that lead to a change of order and as such is inherently inflexible. Its very nature is to be an activity of transformation. Its lack of central emphasis allows activity to be studied as ordered diversity, but prevents the association of the activity with a particular entity. Indeed it does not allow for the existence of separate entities.

*5-term representation and comprehension* ("Potentiality")
Systemic attribute; significance. Subsidiary attributes: potentiality and meaning. Term designation: limit. Term characters: 1, intrinsic; internal limits (2, lower; 3, upper); external limits (4, upper; 5, lower). Connectivities (1st order); mutualities (10 dyads). Connectivities: (2nd order): 10 triads. Connectivities (3rd order); 5 tetrads.

Meaning and potentiality must be added to activity, if the significance of a structure for itself (and for the totality that contains it) is to be specified. Only then does a structure become a bounded significant entity. Such entities have limits of significant connectedness with the outer world and limits of connectedness with their inner range of meaningful potentialities. Everything that exists has potentialities for actualization that outstrip the relationships that it can sustain within any concrete situation.

*6-term representation and comprehension* ("Repetition")
Systemic attribute: coalescence. Subsidiary attributes: recurrence, progress and self-realization, independence, form of events. Term designation: law (governing the coalescence of events). Term characters: 1, order; 2, expansion; 3, identity; 4, freedom; 5, concentration; 6, interaction. Connectivities (1st order): steps.

Coalescence is understood as the property of structure, whereby significance acquires depth and enrichment and yet retains the unique character associated with a particular event. The hexad, as progressive cyclicity, is the system most appropriate for sutdying structures in a step-by-step process of realizing their significance as events. It expresses the two-fold character of creation and counter-creation and also the movement of the entire process towards a goal. Although potential energy can be stored up indefinitely, it can only renew itself through the repetitive two-fold action of a disturbing and a restoring force. Success in action requires a balance between attention to what actually is and what potentially might be; events continue to transform themselves even when their actualization is completed. However the hexad does tend to emphasize the separateness and isolation of such events from one another.

*7-term representation and comprehension* ("Structure")
Systemic attribute: transformation. Subsidiary attributes: structure, history. Term designation: state. Term characters: 1, initiation; 2, involvement; 3, separation; 4, harmonization; 5, insight; 6, renunciation; 7, completion. Connectivities (1st order): intervals. Connectivities (2nd order): harmonies.

A structure is a self-regulating system capable of relatively independent existence. Such a system is no longer closed and changes in the environment accompany changes in the entity. A transformational superstructure is therefore provided by the heptad to reconcile the self-realization requirement of the well-defined entity (namely the acquisition of new properties that were previously neither potential nor possible) and the dissolution of identity required for integration as a part within a whole. A heptadic system is required whenever there is change involving a real gain or loss in significance. By such transformation, significant events are integrated into the stream of universal history.

*8-term representation and comprehension* ("Individuality")
Systemic attribute: completedness, organized totalities. Term designation: element. Term characters: active (1, summit; 3, atom; 5, base; 7, totality); structural (2, states; 4, functions; 6, necessities; 8, ideals). Connectivities (1st order): components (dyads). Connectivities (2nd order): initiations (triads). Connectivities (3rd order): fields (tetrads). Connectivities (4th order): significant substructures (pentads).

Individuality (whether actualized or potential) is the source of initiative residing in organized structures; it may be transformed into endurance, and is also a unique centre of conscious subjective experience. The octad is able to represent organized structures and historical processes ranging in scale from unity to totality. Its value is classificatory, interpretative, heuristic and predictive. It is however only applicable to structures organized in depth.

*9-term representation and comprehension* ("Pattern")
Systemic attribute: harmonization. Term designation: sources (3), steps (6).

Experience would lose all coherence if there were not always active sources of order residing in the patterns of organized structures. The ideal completion of the octad does not take into account the uncertainty and hazard encountered in actual experience. The ennead permits the representation of everyday working structures (disturbed by unpredictable environmental factors) in which harmony is established and maintained. The harmonization is dynamic and indeterminate.

*10-term representation and comprehension* ("Creativity")
Systemic attribute: integrative complementarity.

In all experience there is evidence of a creative (pattern generating) activity that is not only the source of order but also the vehicle of disorder – a polarity exemplified by the decad. At this level several sets of processes are able to compensate for one another's defects and produce an overall harmony that reacts on, and sustains, the individual structures.

*11-term representation and comprehension* ("Domination")
Systemic attribute: synergism

This is the highest form of relatedness and is the power, subject only to the law of necessity, that reconciles order and disorder through the agency of creativity. It provides the conditions for mutual completion of structures of different kinds.

*12-term representation and comprehension* ("Autocracy")

Systemic attribute: perfection.

The dodecad is significant as a master pattern for understanding all total structures of the universe, because it is the first system in which the main elements of experience can all be represented. It combines dynamism and diversity, or relativity and relatedness. It is the culmination of the transformations whereby the structure of existence is first disordered, then corrected, then redeemed and finally perfected. Autocracy is the primary affirmation by which all possible experience is brought into existence whether as potential pattern or as the actual process of the universe. It is the element that acts without dominating, wills without reacting, and unifies all possibilities.

## Annex 3

### Clarification of specific sets

*1. Eliciting subordinate sets: relating distinctions*

If a set is named (e.g. "development"), the question may be asked in how many ways possible elements may be distinguished by subdividing the set.

*2-level distinction:* The set may, for example, be split into *2 subsets*, but in how many ways may this be done in a particular case? Depending on the level at which the distinction is made, there may be 1, 2, 3, 4, or N recognized 2-level distinctions; namely the most fundamental, and successively less fundamental levels of distinction. Clearly these are not unrelated, since the less fundamental distinctions are regrouped in distinctions at more fundamental levels. For example, at the level at which only 4 distinctions can be recognized, the regrouping would tend to bear a relationship to the level at which only 8 distinctions are made (by regrouping pairs of distinctions). On initial examination of all such 2-level distinctions, there would tend to be some confusion as to the level to which they should be allocated in order that the most fundamental should not be embedded in a set of less fundamental distinctions. The probability of any particular 2-level distinction being advocated as most fundamental is likely to be higher, the greater the number of possible distinctions at that level. (Namely it is less likely that the more fundamental 2-level distinctions would be recognized.)

On the other hand this tendency is counter-balanced by the lower stability, viability and acceptability of the less fundamental distinctions. Over longer periods of time they are meaningful to fewer and are of less value to the ordering of perceptions, however vigorously the use of any particular one may be advocated.

In sorting out to which level each 2-level distinction belongs, reference may be made to the pattern of relations between the various distinctions at that level in the light of the underlying qualitative characteristics of the number associated with that level (see Annex 2, for example).

*3-level distinction:* The set may however be split into 3 subsets. As before, it is a question of the number of ways in which this may be done in a particular case. The argument above applies again.

*N-level distinction:* Clearly the argument may be generalized for N-level distinctions although, in the light of earlier arguments, N is unlikely to exceed about 10.

Now the procedure adopted to clarify the ordering at any particular N-level, effectively clarifies the nature of the most fundamental distinction for N = 2, 3, 4 . . . N. This in turn provides an ordered configuration of aspects which exemplify the nature of the original totality (i.e. N = 1) which was explored by subdivision.

*2. Eliciting superordinate sets*

In addition to proceeding by subdivision, clarification concerning a named set (e.g. development) may be sought by determining of what sets it may be considered to be a part. Note that many of the existent fundamental sets are identified or named by enumerating their elements. The name of the set, if any, derives from them in their *plurality* and not from any concept of the *singular* totality they constitute as a set (e.g. human values, human rights, etc.)

*2-level combination:* The set may, for example, be paired with one other set to form a 2-element set. But in how many ways may this be done in a particular case, given that the pairing cannot be arbitrary but must be based on some aspect of the quality associated with the number 2 (see Annex 2, for example). Such combinations could be ordered and clarified as suggested by the previous section.

*3-level combination:* The set could be grouped with 2 other sets to form a 3-element set. As before it is a question of ordering the ways in which this may be done to clarify the many possible aspects of the superordinate set.

*N-level combination:* Again the argument may be generalized, although it is unlikely, as before, that the total in the resulting set would exceed about 10. In this procedure it may well be that particular combinations are not meaningful or useful. Clearly it becomes increasingly difficult, as N increases, to integrate the original set into a combination. But at any stage, a further procedure may be adopted to identify, for an N-level combination, what, successively, the elements of an N–1, N–2. . . .N–M combination are. This clarifies the aspects of the nature of the more fundamental superordinate sets (where N–M = 1) which may underly any given set. Again the qualitative characteristics of number (Annex 2) may be used as a guide.

## Annex 4

### Symbol generation

1. In a system with P terms, it should be possible to identify by analysis (with computer assistance and graphic output) configurations of the P terms (linked by Q relationships), *selected in order of their degree of sym-*

*metry* for a given value of P. Constraints on the maximum and minimum value of Q in each case could also be partially determined in terms of symmetry requirements. Tables of such configurations, *without* considering symmetry, have been produced by Frank Harary (124). The less symmetrical structures, for a given P value, should then prove to be those of less probable value in the representation of the central concept – although possibly of more value in representing an aspect of it. And indeed the "traditional" diagrams are those which are likely to be prominent in the results – although valuable new ones may well be discovered by this procedure.

2. The same procedure may now be applied for the representation of P-term systems in 3 dimensions. Here the symmetry constraints are more severe. This procedure should preferentially select the regular and semi-regular polyhedra (when P is even) or less well-known structures (when P is odd) (22), (23), (125).

3. The procedure may be made more powerful if, for a given P-term system the structure selected is based upon P equal to:

either – number of edges of the structure
or – number of sides of the structure
or – number of vertexes of the structure (as above)
or – number of axes of symmetry.

For, in terms of representation, it may be as meaningful to associate an aspect of the P-term system with any such numerable features. The emphasis is on ordering structures in terms of probable iconicity, with the expectation that families of decreasing iconicity will be distinguished by the procedure for a given value of P. Such families may be more valuable for representing aspects of the central concept, although the highly asymmetric structures in any family probably reflect the various forms of *pre*-comprehension, *mis*-comprehension or *non*-comprehension of the concept. Information gaps of this kind in education have been modelled in graph theory terms (126 -129).[83]

4. A variation on the procedure in 2 dimensions is to allow each term to be represented:

– by, the *same* simple shape (circle, square, etc) and to select symmetric configurations in which the relationships are represented either by the points of contact between shapes or from implicit symmetry features (see (22), (30), and (36) on net diagrams for example).

or by *different* simple shapes, each characterizing a different aspect.

This procedure should select out many well-known symbols (130).

5. Again this variation may be applied in 3 dimensions using simple solids instead of flat shapes. As mentioned earlier the possible configurations are then governed by well-known packing constraints (22), (23).

## Notes:

1 Further attention should be given to 0-element sets and their significance.

2 Obtaining a "good fit" is essentially a problem of design and indeed in his influential book on the subject, Christopher Alexander (ref. 2) devotes several chapters to the question. *Deciding on the boundaries of a set and distinguishing its elements is a problem of design* as Alexander would see it (as is the problem of elaborating a suitable representation, particularly when the relationships between the elements are taken into account). He notes:
"The ultimate object of design is form . . . every design problem begins with an effort to achieve fitness between two entities: the form in question and its context. The form is the solution to the problem; the context defines the problem. In other words, when we speak of design, the real object of discussion is not the form alone, but the ensemble comprising the form and its context. Good fit is a desired property of this ensemble which relates to some particular division of the ensemble into form and context." (p. 15–16)
And also:
"What does make design a problem in real world cases is that we are trying to make a diagram of forces whose field we do not understand. Understanding the field of the context and inventing a form to fit it are really two aspects of the same process. It is because the context is obscure that we cannot give a direct, fully coherent criterion for the fit we are trying to achieve; and it is also its obscurity which makes the task of shaping a well-fitting form at all problematic. . . I should like to recommend that we always expect to see the process of achieving good fit between two entities as a negative process of neutralizing the incongruities, or irritants, or forces, which cause misfit." (p. 21–24)

3 It would be a simple matter to select, from papers of a wide range of disciplines or administrative activities, lists of "basic points" made (possibly with sub-point coding if any). Irrespective of content, the number of points should follow a pattern which could suggest interesting lines for future research. A rich source of popular material is *The Book of Lists*, edited by David Wallachinsky, et al. (New York, William Morrow, 1977) from information supplied for *The People's Almanac*. It contains 377 lists on all topics. Even if biased toward a particular format (of the Almanac) or to conform with the style of earlier lists, the results are still indicative. (*1–10* items per list, 54.6%; *11–20*, 35.0%; *21–30*, 7.2%; *31–40*, 1.3%; *41–50*, 0.5%; *51–60*, 0.5%; *61–70*, 0.3%; *71–80*, 0%; *81–90*, 0%; *91–100*, 0.3%; *100+*, 0.5%. With *10* items, 39.3%; *15*, 8.0%; *20*, 6.4%). A new edition is in production.

4 For a comment on the *general structural* significance of the peaks in the curve, see ref (1), p. 604–607.

5 Herbert Simon (ref. (5), p. 39–40) notes that such constraints can now be less plausibly explained by a single parameter and that under certain circumstances the value falls from 7 to 2 (on which point see the peaks in the curve of Fig. 1). It appears that it is *short-term* memory which can only handle information by chunks of 7. This constraint does not apply to long-term memory. However this does not change the fact that the sets under discussion usually contain about 7 chunks or less – possibly because access to such sets and their representations is necessarily via short-term memory.

6 Alex Bavelas and Howard Permutter, classified work done at the Center for International Studies, MIT, quoted in "The relation of knowledge to action", by Max Millikan (see (40) p. 164).

7 Antony Jay, in (8), identifies size limitations for organizations: "ten group" of 3–12 (work group, project group, task force); "camp" of 20–60 (work group plus those dependent upon their activity or servicing their requirements); "tribe" of 300–1000 (identity group, mutual recognition); "kingdom" of 5,000–60,000 (administrative, social, cultural or military coherence); "empire" of 100,000+. It would be interesting to explore the change in the nature of government once the number of ministries and cabinet ministers exceeds the critical number for small groups (see (7)) and the usual constraints on span of control.

8 In the light of the NSF exercise, it will be interesting to note the organization of the results of the exercise launched in 1978 by the US Office of Technology Assessment "on the identification of major long-range problems and opportunities facing American society".

9 An intergovernmental meeting may give rise to a many-pointed declaration as the basis for a programme of action. This is then progressively condensed into a programme grouped under a number of headings within the number constraint noted. (Consider the evolution of the UN Environment Programme from 1972, for example.) Where an action programme does not emerge, the number of points remains unconstrained by the limit, particularly in legalistic declarations of principles such as the Universal Declaration of Human Rights (31 articles). But even here, such a declaration would be unacceptable if it had 131 articles, so a new constraint may be in operation.

10 From which arises the whole problem of communication with the non-scholar and between scholars of different disciplines.

11 Magoroh Maruyama has consistently argued that the hierarchical orientation is only one of four culturally determined epistemological standpoints and is characteristic of the following cultures: European (and American), Islamic, Hindu, Japanese, Yamato, Kwekiutl, for example (see (11) and (12)).

12 "It appears that the attention paid hitherto in exact science to increasing precision of analysis into smaller and smaller parts needs now to be supplemented by a method capable of representing the processes of complex systems composed of many parts. But there is no sign as yet of a simple comprehensive method of describing the changing form or structure of a complex of relationships." (ref. (37), p237)

13 This point is discussed in further detail in a later section.

14 Problems also arise when creation of the set is expected to improve the status and prestige of the producer at the expense of others – who may have produced their own or may thereby be challenged into doing so. Such dynamics cannot be discussed rationally in the same arena as for the content.

15 Note that this "basic distinction" constitutes a 2-element set which is subject to many of the points made in this paper.

16 An interesting example is the single sheet chart of the biochemical metabolic pathways in living systems: see (15).

17 "The neophyte can ... grasp this unstable universe of powers which are both within and without. For him the symbol is like a magical and irresistible admission into this formless and tumultuous tangle of forces. With the symbol he grasps, dominates and dissolves it. Through the symbol he gives form to the infinite possibilities lying in the depths of his subconscious, to inexpressed fears, to primordial impulses, to age-old passions." (See (38), p. 22.)

18 Although it is very seldom done, any conventional hierarchical structure (e.g. an organization chart of a corporation) can be curved into a circle with the superordinate element at the centre.

19 Jones discusses seven pre-logical axes of bias and their application to scholarly debates in the arts and in the sciences. (17)

20 "The main difficulty in translating from the written to the verbal form comes from the fact that in mathematical writing we are free to mark the two dimensions of the plane, whereas in speech we can mark only the one dimension of time" (ref. (21), p. 92). And in conventional text, where subscripts and superscripts are not permitted, writing becomes as restricted as speech.

21 "Any aggregate that is neither completely ordered nor completely disordered must have hierarchical aspects, but the

perception of the levels of the hierarchy requires the recognition of a two-dimensional surface to define each three-dimensional unit in accordance with Euler's Law" (ref. (10), p. 81).

22 Of special interest in the 2-dimensional case, is the situation when line coding is not permitted and ways have to be found to fit shapes together. The book by Critchlow (22) explores the variety of regular patterns which result. These patterns can be important when any attempt is made to represent sets and their subsets by nested areas.

23 "If a fourth spatial dimension cannot be visualized, it is probably because geometry is concerned with relations that can use perceptual and physical space as a convenient image up to the third dimension, but no further. Beyond that limit, geometrical calculations – just as any other multi-dimensional calculations, such as factor analysis in psychology – must be content with fragmentary visualization, if any. This also means probably putting up with pieces of understanding rather than obtaining a true grasp of the whole." (ref. (21), p. 292.) Note that in ref. (39) it is argued that higher dimensions can be suitably visualized.

24 See ref. (22) and (23).

25 "When man employs nature's basic designing tools, he needs only generalized angles and special-case frequencies to describe any and all omnidirectional patterning experience subjectively conceived or objectively realized. For how many cycles of relative-experience timing shall we go in each angular direction before we change the angle of direction of any unique system-describing operation?" ((1), p. 248–9).

26 It seems to be time to recognize the extraordinary resistance of each social science profession to the application of the insights of its own discipline to itself as a social group, and to integrate this into the research process. There is a real blindspot, as has been noted with respect to one discipline at least (but not necessarily by many of its practitioners): "But sociologists have been reluctant to test empirically the relevance of many hypotheses... for the development of knowledge in sociology. Studies on the impact of the social organization of the discipline, the prevailing climate of opinion, and the social background and personal values of researchers have been out of fashion..." (p. 45) and "sociologists are notorious for studying everything except their own discipline and its institutional patterns" (p. 55) from the introduction to *The Sociology of Knowledge*, edited by J. E. Curtis and J. W. Petras. London, Duckworth, 1970.

27 Jay Kelley remarks on an associated phenomenon: "When an investigator acquires data and facts, he is improving order within his own sphere. The entropy of the experimenter and his data pad and records is improving, but the moment the observer separates *himself* from his data, he no longer can claim the full possession of value of the information; the information is continually devalued as the observer accumulates other knowledge and as time passes. These observations lead to deeper questions of the nature of order and its human implications ((41), p. 179). For him: "Value implies accessibility to information, which reflects how it is ordered or its entropy."

28 "It seems to be quite evident that oneness stands out as the origin of the structure from whence feasible patterns can emerge as rigidly hierarchical, associative, or sequential. Of these the hierarchical patterns appear to have lasting qualities while associative and sequential features may confer richness and flexibility... Thus, whether negotiating a computer or a sociological system the human conceives patterns from his singular frame of reference and must see and interpret the learned pattern from this state of oneness. Language and other standard ordered patterns tend somewhat to alleviate the plausible dilemma of a human having to interpret for himself from oneness to many independent patterns." ((41), p. 195)

29 "The theme of this book is that a universe comes into being when a space is severed or taken apart. The skin of a living organism cuts off an outside from an inside. So does the circumference of a circle in a plane. By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical, and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance. The act is itself already remembered, even if unconsciously, as our first attempt to distinguish different things in a world where, in the first place, the boundaries can be drawn anywhere we please. At this stage the universe cannot be distinguished from how we act upon it, and the world may seem like shifting sand beneath our feet." ((18), p. v)

30 He argues in favour of the fundamental validity of the ancient philosophical intuition that the "dynamical situations governing the evolution of natural phenomena are basically the same as those governing the evolution of man and societies, profoundly justifying the use of anthropomorphic words in physics. Inasmuch as we use the word "conflict" to express a well-defined geometrical situation in a dynamical system, there is no objection to using the word to describe quickly and qualitatively a given dynamical situation. When we geometrize also the words "information", "message", and "plan", as our models are trying to do, any objection to the use of these terms is removed." ((32), p. 323)

31 In the light of the theme of this paper, it is curious to note that Thom's catastrophe theory identifies only 7 distinct forms of catastrophic discontinuity.

32 René Thom himself develops a set of "archetypal morphologies" ((32), p. 307)

33 Note Marcel Granet's extensive study ((44), p. 127–248) of the use and significance of number in Chinese thought as a means of classifying and expressing qualitative distinctions.

34 The rules given ((45), vol. 2, p. 7) are in effect incorporated into the system definition given in Annex 1. He also makes the following points. A class is an externally determined set of members and a system is an internally connected set of terms. When the internal connections are disregarded, the set degenerates from being a system to being a class. No actual class is wholly free from inner connections so that classes are abstractions whereas systems are concrete (although to different degrees).

35 "it is also possible to have an 'ordered' class or series, such as the first ten numbers. This is not a true system, for it does not take any account of the mutual relevance of the terms except their order. Nevertheless, since the ordinal numbers are in certain respects intermediate between classes and systems, we cannot regard the distinction between class and system as wholly free from ambiguity." ((45), vol. 2, p. 4)

36 "In the realm of ideas, man can count up to two and sometimes, in specially favourable circumstances, as far as three. He has no notion at all of what would be required for entertaining richer combinations. This limitation applies not only to man's thought but also to his feelings and to his instinctive processes. His judgements of feeling reduce always to the choice between like and dislike, attraction and repulsion, interest and boredom. His instinctive reactions have the same dualism of pleasure and pain, of activity and repose, of stimulus and inhibition." ((45), vol. 1, p. 21)

37 Varela ((42), p. 21) notes that to introduce more than two values in a calculus or a logical system has been a current field of investigation since Lukasiewicz (52). Such additional values are usually interpreted in terms of probability or necessity. Günther (53) has been alone in pointing out another possible interpretation of many-valued logics, namely as a basis for cybernetic ontology, that is for systems capable of self-reference.

38 Matila C. Ghyka however draws attention (64) to the Hamiltonian Principle of Least Action as fundamental to further reflection on these matters. He and Bennett ((45), vol. 1) also refer to the implications of transfinite numbers in which the whole can be seen as reflected within the part.

39 "Far from restricting our efforts to put questions to nature in the form of experiments, the notion of *complementarity* simply characterizes the answers we can receive by such inquiry, whenever the interaction between the measuring instrument and the objects forms an integral part of the phenomena." (Niels Bohr, in Essays 1958–1962; on atomic physics and human knowledge. New York, Wiley, 1963).

40 *Subidvision* of a set (by the act of distinguishing elements) has been used rather than *articulation*, although the latter is preferable. It implies a respect for the functional relationships between the system elements (and an expression of them), whereas the former is solely concerned with their classical *logical* relationships.

41 See (9), p. 144. Jungian psychology regards such gods as archetypal figures representing energies locked within the individual human psyche.

42 There is a difference between archetype and archetypal image. The latter is always variable, but behind these variants stands a constant, nonperceptual pattern. According to Jung a conscious and invariable definition of its meaning is not possible.

43 "Unfortunately, my abstract model tends to fade out when I get a circuit that is a little bit too complex. I can't remember what is happening in one place long enough to see what is going to happen somewhere else. My model evaporates... In all fields there are such abstractions. We haven't yet made any use of the computer's ability to 'firm up' these abstractions. I think that really big gains in substantive scientific areas are going to come when somebody invents new abstractions which can only be represented in computer graphical form." (61)

44 I am indebted to Ingetraut Dahlberg for drawing my attention to refs. (62) and (63) and the question of seminal mnemonics in general.

45 For a historical review and bibliography, other than that of von Franz (9), see Ghyka (64), Butler (65), and Hopper (66).

46 It is Ghyka (64) who has traced the pythagorean developments, recording the modern mathematician's tendency to dissociate himself from that perspective. However Sallantin notes: "D'ailleurs est pythagoricien quiconque percoit un lien naturel entre le nombre Un et l'idée d'Unité, entre le nombre Deux et l'idée de Dualité" (48). He demonstrates that conventional arithmetic is in effect one of four types of arithmetic; the others have increasing degrees of indeterminacy and are more suited to handling problems in biology and physics. He proposes that one of them should be used as the basis for trialectic logic.

47 Although Bennett's analysis is used by him as a basis for a much wider investigation which is not a matter of concern here.

48 It is interesting to compare Bennett's exercise (in Annex 2) with Neelameghan's (63) application of seminal mnemonics as a pattern for systems analysis, which makes an attempt to associate ideas with the numbers 1 to 7. Although quite independent, there would appear to be some similarity between them.

49 "Our community life is perhaps so structured that the very moment we seek to grasp reality in all its concreteness we run after simulacra. The present set of texts takes as its hypothesis that illusion and simulation have assumed in the Twentieth Century a power hitherto without parallel. We have entered, perhaps, the age of the simulacrum." Special issue summary of *Traverses* (Paris, Centre national d'art et de culture), 10, février 1978.

50 "Topics are the 'things' or subject matter of dialectic which came to be known as *topoi* through the places in which they were stored" ((68), p. 46)

51 Yates quotes a pre-Socratic text on memory, dated about 400 BC: "For things [do] thus: for courage [place it] on Mars and Achilles; for metal-working, on Vulcan; for cowardice, on Epeus" ((68), p. 44)

52 Amongst others, Yates quotes Marsilio Ficino: "Aristotle and Simonides [the inventor of the memory technique] think it useful to observe a certain order in memorizing. And indeed an order contains proportion, harmony and connexion." ((68), p. 163)

53 The implications of the "imprinting" process of learning should also be considered as well as the role of portraits in political, religious and cultural personality cults.

54 There is of course a paradox associated with any such ultimate set. The act of distinguishing it necessarily establishes at least two subsets, for it necessarily incorporates the distinguisher as Spencer Brown demonstrates (18).

55 This relates to Jung's concept of "unus mundus" as an expression of the unity of existence founded: "on the assumption that the multiplicity of the empirical world rests on an underlying unity, and that not two or more fundamentally different worlds exist side by side or are mingled with one another. Rather, everything divided and different belongs to one and the same world, which is not the world of sense but a postulate whose probability is vouched for by the fact that until now no one has been able to discover a world in which the known laws of nature are invalid" (77).

56 One is reminded of the possibility of a *qualitative* analogue to the "big bang" cosmological theory which postulates the universe as having been elaborated from a single homogeneous ball of proto-matter. That the analogue might operate on standing wave principles, also merits reflection (note (78)).

57 Von Franz ((9), p. 77) notes the Chinese use of numbers as qualitative fields whose internal numerical structures "represent time phases of the fields dynamic internal structure." She quotes: "The ontological and logical ordering (of numbers) is translated into rhythmical and geometrical images. On account of their descriptive power, as exponents of concrete analysis, numbers are classificatory, and for that reason used to identify concrete sets. They can serve as rubrics, for they indicate the various types of organization which are imposed on things when they are manifest in their proper order in the cosmos." ((44), p. 123)

58 In the light of the scheme presented in Annex 2, the 3-term "concept triangle" (see (59)) is preceded in the series by the traditional 2-term "knower-known". It may be followed by the 4-term "word-meaning-referent-observer" (and it is this which blurs into a single set at the limit condition). This series bears an interesting relationship to that derived from Galtung's "theory-fact-value" triangle as discussed in the conclusion. Note the terms change significance with addition of a term (see note 73). Zeman (80) specifically proposes a "gnoseological triangle": objective reality, the observing subject (i.e. conscious man), and expression. This combined with the concept triangle, constitutes a tetrahedron (4-term).

59 Except possibly through peak experiences (see (79)). Von Franz stresses Jung's view "that there is little or no hope of illuminating this undivided existence except through antinomies. But we do know for certain that the empirical world of appearances is in some way based on a transcendental background." ((9), p. 9). Historically this has been represented by symbols (p. 303).

60 It is rather as though different witnesses to a crime were to attempt separately to describe the criminal by establishing an Identikit portrait (a definition) using the kit components (words). Not only do the portraits differ from one another, but possession of a portrait however good does not magically result in the capture of the person identified.

61 *Systematics* 1963–1970 (Institute for Comparative Study of History, Philosophy and the Sciences, UK)

62 Only by viewing an N-term set as an N–1 term and an N+1 term system can its significance be established.

63 On this point, the relationship of *time* to the variety of *standing wave* configurations of sand particles vibrated on thin plates of metal merits attention (see ref. (78)).

64 René Thom, on the first page of his study, makes the point that: "recognition of the same object in the infinite multiplicity of its manifestations is, in itself, a problem (the classical philosophical problem of concept) which, it seems to me, the Gestalt psychologists alone have posed in a geometric framework accessible to scientific investigation" ((32), p. 1).
Rudolf Arnheim in discussing the same question, notes that Gestalt psychologists recognize a tendency to "good form" or "well organized structure" (88). L. L. Whyte sees all mental processes such as memory, classification, choice, and will as "displaying a movement toward greater three-dimensional spatial order, symmetry, or form". And such morphic processes "are directly responsible both for the existence of forms, and of brain-minds themselves generating forms and being responsive to forms." ((85), p. xvi)
Jean Piaget also makes points which could be interpreted to be in support of this position: "As a result, spatial structures, from the biological point of view, bridge the gap between logico-mathematical structures, the nature of which is still unknown, and those structures which are either hereditary or, as is sometimes the case, acquired by learning" ((86), p. 309). Also: " . . . cognitive functions are an extension of organic regulations and constitute a differentiated organ for regulating exchanges with the external world. The organ in question is only partially differentiated at the level of innate knowledge, but it becomes increasingly differentiated with logico-mathematical structures and social exchanges or exchanges inherent in any kind of experiment." ((86), p. 369).

65 I am indebeted to Colin Cherry (*On Human Communication*, 1968) for this insight (87).

66 It could be interesting to explore the possibilities of portraying each term in a multi-term system by a human or animal figure and animating their interaction on graphics devices to produce a cartoon effect, using a computer programme governed by the original structure. (Supposedly many folk tales are based on such structures)

67 Rudolf Arnheim notes ((88), p. 207–8) that: " . . . one must assume that structural characteristics of visual form are spontaneously related to similar characteristics in human behaviour. We have called this type of symbolism 'isomorphic' because this is the term used by gestalt psychologists to describe identity of structure in different media. . . . The gesture of a dancer. . . contain(s) structural features whose kinship with similarly structured mental features is immediately felt." Ritual dances are based on this insight and even have their modern advocates: Steiner's eurythmy. Gurdjieff's movements, Ichazo's Arica movements, and the like. The aim being to penetrate and express the more fundamental forms and to use them as a means of classifying experiences within a functional whole. It is no accident that Keith Critchlow in a book on design (22) incorporates Laban's use of the icosahedron for dance notation (89).

68 It is interesting that in order to solve the problem Fuller has effectively had to confront the constraints of the basic duality with which our culture is faced as it is reflected in material forms. The "primitive" structuring effects of the duality have to be bypassed within a larger whole which depends on them for its integrity. This requires many more elements than the ideal forms, thus conforming to Bennett's insight that a higher number of terms is required to provide a better approximation to reality. (Although the higher number is effectively reduced by the encoding properties of the underlying polyhedron in each case).

69 In terms of the status in society of fundamental sets, there would seem to be an amusing parallel between the role of temples to different deities in the Roman Empire and that of international agencies with respect to global society. Both the temples and the agencies each base their actions on well-defined sets of qualities.

70 Possibly only by anthropomorphizing the representation of "world problems" which society faces will their nature and interplay be communicable to an adequate degree – particularly in terms of how they are ordered or governed.

71 Interpreting Bennett's scheme (Annex 2), It can be very tentatively suggested that sets of the following numbers of terms are required to encounter these current issues: mediation, relationships (3-term); retraining, resource *renewal* (6-term); organizational systems (7-term); worker *individuality* and human development (8-term); environmental *processes* (9-term); social innovation and *creativity* (10-term). Each stage requires more subtle skills in organization and governance in order to tolerate the additional freedom (i.e. reduction in imposed order) it implies and demands; in fact the challenge to policy at this time seems to lie with the 11-term approach of balancing order and disorder, rather than attempting to eliminate the latter (100). But understanding, if there is any, in terms of such multi-term sets seems to be only instinctive or intuitive, aided by frantic "rational" (2-term) attempts to order the component elements in isolation from each other, and a "fire-fighting" response to problems arising from their interactions – when they can no longer be ignored.

72 Chinese philosophy, as exemplified by Lao Tzu and Chuang Tzu, is full of references to the attitude implied by the 12-term approach. This is also evident in the attitude advocated in Eastern martial arts, see Herrigel (103). It would be interesting to examine the Study of S. Boorman in this light (104). Clearly a strategy based on thinking in N+1 terms is bound to out-manoeuver one based on only N terms, as well as appearing unpredictable and disorderly to the latter.

73 Clearly Ashby's Law (105) concerning the necessary complexity for a *control* system also applies with regard to the complexity of a *representational* device. However there is the paradox that representations which are as complex as that which they represent are of questionable value.

74 Yates presentation (68) concerning *rotae* suggests the possibility of an approach intermediate between conventionally static classification schemes and computer-based mathematical models (e.g. of social systems), namely a memorable pattern of classification possiblities implying the complete range of relationships between a set of categories.

75 I am considerably indebted to Ira Einhorn for drawing my attention to references: (42), (106–107), (112).

76 Don (107) discusses a model of the brain put forward by Powers (108) and based on ten hierarchical levels of control: musculoskeletal intensity, sensation, configuration, transitions, sequence, relationships, control of patterned logical processes, principle, concepts. Again this bears comparison with a scheme such as Bennett's (Annex 2).

77 Recent work needs to be related to that of Zipf (109), used by Kelley (41), for despite revision by Mandelbrot (110), it is strongly critized by Rapoport (111). There may be a link in this context between Zipf's Principle of Least Effort and the Hamiltonian Least Action Principle (see note (38)).

78 Margalef (113) suggests that it is possible to measure the "maturity" of an eco-system as closely related in one respect to its diversity or complexity, and in another to the amount of information that can be maintained with a definite spending of potential energy. This is a question of patterning. A highly diversified community has the capacity for carrying a high amount of organization and information, and requires relatively little energy to maintain it. Conversely, the lower the maturity of the system, the less the energy required to disrupt it. Anything that keeps an eco-system oscillating (or "spastic") retains it in a state of low maturity. (Hence the danger of simplistic reorganization of organizational, conceptual or value systems.) A mature ecosystem has a maximum number of trophic levels of which, curiously in the light of this paper, the number rarely exceeds 7.

79 From Yates presentation (68), one may suspect that Giordano Bruno's "seals" served this purpose in relation to his own texts. A similar role may be ascribed to the lapidiary seals collected by Rziha (114) as reported by Ghyka (64)

80 Interesting examples, which have never been cross-linked, include Abellio (115), Buckminster Fuller (1), Haskell (116), Dodd (117), Lock Land (118), Langham (29), Young (25) and (26), Bennett (45). The Eastern equivalent which has attracted the most attention is the I Ching: see Needham (119), Blij (120), Gardner (121), Sung (122). The recently remarked link between the I Ching code and the genetic code raises many questions, see Schönberger in (123).

81 Bennett notes ((45), vol. 3, p. 25) that: "Many of the difficulties in the interpretation of natural phenomena arise from treating qualities as if they remain the same in passing from one system to another." (e.g. from a 2-term system to a 3-term system, the added third term modifies the qualities originally expressed by the other two terms)

82 Addition of "representation" as a *fourth* element is almost certainly insufficient simply as a passive pattern, at the best inviting to the attention. As with language in the West, it may simply classify experience without opening the observer to the action it suggests. Here lies a danger. Already with crude representations users of the flood of text information are overloaded to the point of blockage or effectively insulated from experience by suitable explanation and depiction. Some more iconic sophisticated representation may only reinforce the user's passivity, whereas appropriate representation may offer the user the visual configuration through which to *act* participatively and experientially (cf. the contrast between McLuhan's "hot" and "cool" media). "Activating potential" would thus seem to be a *fifth* element in the series and an appropriate constraint on representation. (I am indebted to Anthony G. E. Blake, for provoking these insights.).

83 See (128) "Both geometry and topology deal with the notion of space, but geometry's preoccupation with shapes and measure is replaced in topology by more abstract, less restrictive ideas of the qualities of things. . . (giving). . . a richer formalism to adapt as a tool for the contemplation of ideas. . ."

84 The fruitful area identified is the use of a non-Boolean (non-distributive) lattice structure of complementary or dialectically developing languages (perspectives, categories) which reflects the logic of quantum mechanics (140, 141). A developmental sequence may emerge either as the result of research *or* of comprehension (cf. programmed learning pathways) through stages which appear mutually incompatible for some period. From the diagrams used by Heelan and de Nicolas, both sequence and complementarity can simultaneously be represented by developmental pathways of polyhedral form which, in their examples, privilege a single vertex (e.g. in a cubic structure) as the "least upper bound element". Richer possibilities, corresponding to non-dualistic complementarity of multi-term sets, could well become comprehensible in the light of the full range of polyhedral structures – nesting polyhedral pathways to distinguish levels of co-existing incompatible perspectives (possibly linked by experiential or non-cumulative learning pathways, as might be represented by a circular chain of overlapping Venn circles) from levels at which complementarity is evident. Such polyhedral encirclement, of an unknown to be defined progressively without closure, could facilitate the relationships between viewpoints as discussed elsewhere (142).

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